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GENERALIZED DIJKSTRA'S ALGORITHM FOR SHORTEST PATH PROBLEM IN INTUITIONISTIC FUZZY ENVIRONMENT

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ABSTRACT

In this paper, a generalized Dijkstra's algorithm is proposed to handle the Shortest Path Problem under intuitionistic fuzzy environment. This algorithm gives the shortest path from a single source vertex to every other vertex and the shortest distance estimate values of each vertex from source vertex. The effectiveness of the algorithm is illustrated by means of an example.

Keywords: *Intuitionistic fuzzy number, Intuitionstic fuzzy set, Intuitionisti c fuzzy shortest path , Dijkstra's algorithm.*

1.INTRODUCTION

Segmentation plays an important role in image analysis where you want to divide an image into two or several different regions of interest. For example in a CT image of the abdomen you can segment it into two parts; the liver and the rest of the abdomen, respectively, or you can segment several organs. Segmentations are often represented as a curve, that separates the region of interest from the rest of the image, by assigning each pixel/voxel a label. Images can be represented by graphs and there exist several methods that can solve a segmentation problem using this representation. The shortest path problems arises in many applications, e.g. finding the shortest route from one city to another, but segmentation can also be formulated as a shortest path problem. There are number of algorithms that solves the shortest path problems and one of the most used method to solve this problem is the Dijkstra's algorithm. Here in this paper, an attempt is made to find the shortest path from a source vertex to all the other vertex by Dijkstra's algorithm using trapezoidal intuitionistic fuzzy number. Consider the edge weight of the graph as uncertain, which means that it is either imprecise or unknown. In 1965 Zadeh [8] introduced the concept of fuzzy set theory to meet these problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line. The fuzzy shortest path problem was first analyzed by Dubois and Prade [2] using fuzzy number instead of a real number is assigned to each edges. There are very few works in the literature on finding an intuitionistic fuzzy shortest path in a graph.

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This paper is organized as follows. In section 2, preliminary concepts and definitions are given. The pseudocode for finding the shortest path using trapezoidal intuitionistic fuzzy number and an example is given in section 3. The last section draws some concluding remarks.

II PRELIMINARIES

2.1Intuitionistic Fuzzy Set(IFS)

Let X be an universe of discourse, then an intuitionistic Fuzzy Set A in X is given by $A=\{x, \mu_A(x), \gamma_A(x) / x \in X\}$ where the functions $\mu_A(x): X \to [0,1]$ and $\gamma_A(x): X \to [0,1]$ determine the degree of membership and non membership of the element $x \in X$ respectively for every $x \in X$, $0 \le \mu_A(x) + \gamma_A(x) \le 1.$

2.2 Intuitionistic Fuzzy Number(IFN)

Let $A=\{x, \mu_A(x), \gamma_A(x) / x \in X\}$ be an IFS, then we call the pair $(\mu_A(x), \gamma_A(x))$ an intuitionistic fuzzy number. We denote it by $(\langle a,b,c\rangle, \langle l,m,n\rangle)$, where $\langle a,b,c\rangle \in F(I), \langle l,m,n\rangle \in F(I), I=[0,1], 0 \le c+n \le 1.$

2.3 Trapezoidal Intuitionistic Fuzzy Number

A trapezoidal intuitionistic fuzzy number A in R, written as $(a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ has the membership function

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{b_1-a_1} & if \quad a_1 \le x \le b_1 \\ 1 & if \quad b_1 \le x \le c_1 \\ \dfrac{d_1-x}{d_1-c_1} & if \quad c_1 \le x \le d_1 \\ 0 & otherwise \end{cases}$$

And the non-membership function

$$\gamma_A(x) = \begin{cases} \dfrac{b_1-x}{b_1-a_1'} & if \quad a_1' \le x \le b_1 \\ 1 & if \quad b_1 \le x \le c_1 \\ \dfrac{x-c_1}{d_1'-c_1} & if \quad c_1 \le x \le d' \\ 0 & otherwise \end{cases}$$

2.4 Defuzzification [3]

Accuracy function of a Trapezoidal Intuitionistic fuzzy number $A=(a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ is defined as

$$H(A) = \frac{(a_1 + b_1 + c_1 + d_1) + (a_1' + b_1 + c_1 + d_1')}{8}$$

The image is represented by a directed graph, $G = (V, E)$, where each vertex, $v \in V$, corresponds to a pixel in the image. Neighboring pixels will be adjacent in the graph with an edge, $e \in E$, connecting them. A neighborhood of a pixel can be defined in different ways; e.g. 4-connected and 8-connected in the 2D case (Fig. 1). Each edge has a weight, w, which may be derived from e.g. the intensity difference between the neighboring pixels.

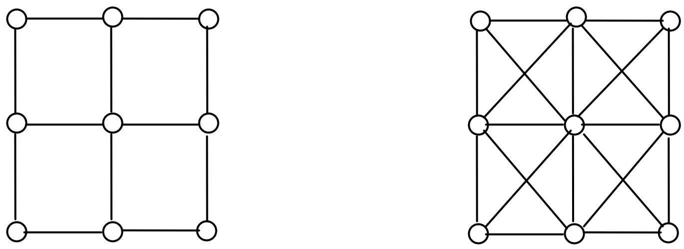

Figure 1: Graph representations of a 3 x 3 matrix with a 4-connected (*left*) and a 8-connected (*right*) neighborhood [5].

The shortest path problems arises in many applications, but segmentation can also be formulated as a shortest path problem. The objective in the shortest path problem can be summarized as

$$\min_p w(p) = \sum_{i=1}^{n} w(v_{i-1} - v_i),$$

for a path $p = (v_0, v_1, ...v_n)$. Many of the algorithms that solve the shortest path problems rely on the property that each shortest path contains several shortest paths within it. This can be stated as in the following lemma.

Lemma.

Given a weighted, directed graph $G = (V, E)$ with weight function $w : E \to R$, let $p = (v_0, v_1, ...v_n)$ be a shortest path from vertex v_0 to vetex v_n and , for and i and j such that $0 \le i \le j \le n,$ let $p_{ij} = (v_i, v_{i+1}, ..., v_j)$ be the subpath of p from vertex v_i to vertex v_j. Then p_{ij} is a shortes path from v_i to v_j.

Proof:

The path $p = (v_0, v_1, ...v_n)$ can be decomposed into the subpaths $p_{0i} = (v_0, v_1, ...v_i)$, $p_{ij} = (v_i, v_{i+1}, ..., v_j)$ and $p_{jn} = (v_j, v_{j+1}, ..., v_n)$ which gives $w(p) = w(p_{0i}) + w(p_{ij}) + w(p_{jn})$

.

Now assume that there is a path p'_{ij} from v_i to v_j with $w(p'_{ij}) < w(p_{ij})$ then the path $p_{0i} \to p'_{ij} \to p_{jn}$ is a path from v_0 to v_n whose total weight , $w(p_{0i}) + w(p'_{ij}) + w(p_{jn})$, is less than $w(p)$, which contradicts that p is the shortest path from v_0 to v_n.

Dijkstra's algorithm [1] can be used on a weighted, directed graph with non-negative weights. It solves the single-source variant of the shortest path problem.

III INTUITIONISTIC FUZZY(IF) DIJKSTRA'S ALGORITHM

In the proposed algorithm, S is the set of all visited vertices and Q is the set of all unvisited vertices that are to be removed. In set Q, u represents the vertex with the smallest distance from the source node. For each vertex v, $v.d$ denotes an upper bound on the weight from the source s to v (a shortest path estimate) and $v.\pi$ which is assigned either NIL (no vertex) or another vertex and it is the predecessor of the current vertex v. The step by step procedure is as follows:

Step 1: Initializing.

Step 2: Check if Q is empty. While Q is not empty, seek u in Q. If $u.d$ is not infinity, then from Q, u is removed.

Step 3: After finding v , each neighbor node of u , if $v.d > u.d + w(u,v)$, then set $v.d = u.d + w(u,v)$ by using defuzzification of a trapezoidal intuitionistic fuzzy number and $v.\pi = u.$

Step 4: Choose the one among each neighbor v with shortest distance $v.d$ and replace it with u and goto step 2.

The pseudocode of the IF Dijkstra's algorithm is shown in Fig. 2.

for each vertex $v \in G.V$ **do** *// Initializations*
$v.d = \infty$ *// Unknown distance function from source to v*
$v.\pi = NIL$ // Previous node is optimal path from source
end for
$s.d = 0$ *// Distance from source to source*
$S = \Phi$
$Q = G.V$ *// All the nodes in the graph are unoptimized*
while $Q \neq \phi$ **do** *// The main loop*

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$u=$ the vertex v in Q with the lowest shortest path estimate

$S=S\cup\{u\}$

for each vertex $v\in G,Adj[u]$ **do** *// where v has not yet been removed from Q*

if $v.d>u.d+w(u,v)$ **then** *// A shorter path to v has been found*

$v.d=u.d+w(u,v)$

// Defuzzification of trapezoidal intuitionistic fuzzy number is used

$v.\pi=u$

end if

end for

$Q=V\setminus S$

end while

Figure.2: Pseudocode of IF Dijsktra's algorithm.

3.1. EXAMPLE

Consider the graph $G=(V,E)$ with 5 vertices and 10 edges (Fig. 3). The edge weights are considered as a trapezoidal intuitionistic fuzzy number, as given below.

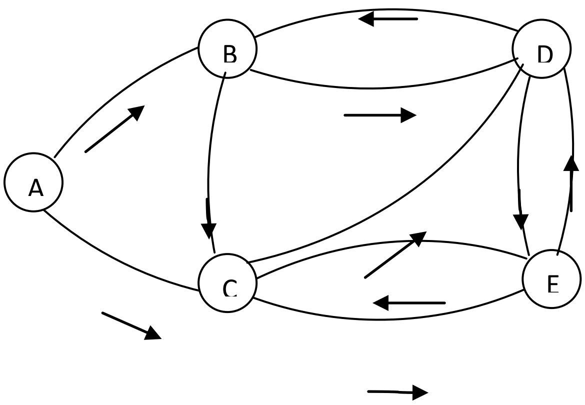

Figure. 3

Edge	Edge weight	Edge	Edge weight
(A, B)	(2, 4, 5, 7; 3,5,7,8)	(C, D)	(3, 4, 6, 7; 5,6,7,10)
(A, C)	(4, 6, 8, 9; 5,7,9,11)	(C, E)	(2, 3, 4, 6, 3,4,6,9))
(B, D)	(2, 3, 5,6; 4,5,7,8)	(E, C)	(1, 3, 5 ,6, 2,5,6,8)
(D, B)	(2, 2, 4, 7; 3,4,5,8)	(D, E)	(0, 1, 3, 5; 2,2,4,7)
(B, C)	(0, 1, 2, 4; 2,3,4,5)	(E, D)	(2, 3, 5, 7; 3,4,7,10)

Execution of the above algorithm for the graph given in Fig.3 [5], The source vertex is the left most vertex, the shortest path from the source vertex to each other vertex is obtained as in Fig. 4. (The shortest path edges are shown in dark lines).

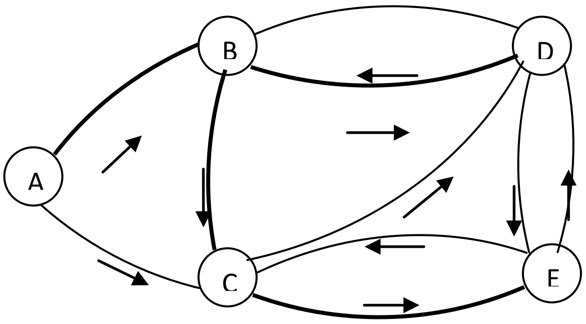

Figure. 4: Dark edges form the shortest path from the source vertex to every other vertex

The shortest path estimate from the source vertex to every other vertex are given below.

The shortest path estimate from A to B : 5.125 (Path : A→B)

The shortest path estimate from A to C: 7.375 (Path : A→B→C)

The shortest path estimate from A to D: 10.125 (Path : A→B→D)

The shortest path estimate from A to E: 12 (Path : A→B→C→E)

IV CONCLUSION

Shortest path problem in image segmentation has many applications in medical image analysis. This paper extended the Dijkstra's algorithm to solve the shortest path problem with intuitionistic fuzzy arc lengths. The proposed method is also computes the shortest path estimate from the source vertex to every other vertex. A numerical example was used to illustrate the efficiency of the proposed method. This algorithm provides the better output for different types of graphs and network and needs light computational load.

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