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"CRITICAL ANALYSIS OF TEMPERATURE AND THERMAL STRESSES VARIATION ON FGM CERAMIC BEAM"

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ABSTRACT:

Functionally graded materials (FGM) are most commonly used for barrier coating against large thermal gradient. Now a day's FG materials are replacing the composite materials because in high temperature environment various discontinuities like cracks, unbending, delaminating etc. are accounted in composite material. In FGM, variation of material properties are continues across the thickness and this material property variation is obtained by three function laws that is exponential law, sigmoid law and power law and then comparing it. In this work explores the effects of heat conduction variation in the thickness direction of the proposed 3-layer FGM composite. Then micromechanical modeling of functionally graded thermal barrier coating is considered to predict stresses under thermal and mechanical loading. Residual stress is calculated for assumed FGM beam model by using mathematical tool MATLAB and than comparing the results with the previous work and in last section see the nature of neutral shift and deflection in a assumed FGM beam model.

Keywords:

FGM, Residual thermal stresses, function laws, heat conduction

1.INTRODUCTION:

Functionally graded materials (FGMs), Which are advanced multiphase composites and have a smooth spatial variation in material. Functionally graded materials (FGMs) are made from a chemical-alloy mixture of metals and ceramics. FGMs are useful for many engineering sectors such as the aerospace, aircraft, automobile, and defense industries, spring and most recently the electronics and biomedical sectors.

A functionally graded material (FGM) is made from metal & ceramic. Ceramic have mechanically brittle and good high-temperature behavior. Another may be a metal which is exhibits better mechanical and heat-transfer properties but cannot withstand to high temperatures. But ceramic is the hotter region and metal is the cooler region. In the high-temperature condition, the strength of the metal is reduced.

The ceramic materials have better characteristic in heat resistance property and ceramic applications have a low toughness. FGMs are stable in ultrahigh temperature variations or large thermal gradients & thermo-mechanical loading. Typical examples include towers, movable arms, and antennae that can be reduced to elastic beams with variable cross-sections, and beams used in high-performance surface and air vehicles. e.g., FGM sensors and

Actuators, FGM metal/ceramic armor, FGM photo-detectors and FGM dental implant In "Functionally Graded Materials" (FGMs) the material properties are varied in a particular way.

Thus, Functionally Graded Materials (FGMs) are

locally varied so that a certain variation of the local material properties is achieved. FGM is also defined as the volume fraction of twoor more materials which are achieved continuously as a function of positional on g certain directions of the structure.

E.g. mixture of ceramic and metal by grading the material properties in a particular manner and the effect of interlaminate stresses developed at the interfaces of the laminated composite beam.

1.1 DrawbacksofLaminatedComposites:

The laminated composite materials provide the design flexibility, stiffness and strength. The anisotropic constitution of laminated composite structures often result in stress concentrations near material and geometric discontinuities that can damage in the form of matrix cracking and adhesive bond separation. FGMs alleviate these problems because of a continuous variation of material properties from one surface to other.

2. FGM Material Structure Composition:

2.1 Effective Properties of FGM:

Effective properties of FGM are obtained by basic three laws i.e. Power Law (P-FGM), Exponential Law (E-FGM) and Sigmoid Law (S-FGM).

Material property	**Property related formula**
Thermal conductivity (k)	$k(z) = k_t\left(1+\dfrac{3(k_b-k_t)V_m(z)}{3k_tV_m(z)+(k_b+2k_t)V_c(z)}\right)$
Modulus of elasticity(E)	$E(z) = E_t\left(\dfrac{E_t+(E_b-E_t)\big(V_c(z)\big)^{2/3}}{E_t+(E_b-E_t)[\big(V_c(z)\big)^{2/3}-V_c(z)]}\right)$
Poission's ratio (v)	$v(z) = (v_t-v_b)V_c(z)+v_b$

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Coefficient of thermal expansion (α)	$\alpha(z) = (\alpha_t - \alpha_b)V_c(z) + \alpha_b + \left(\frac{V_m(z)V_c(z)(\alpha_t - \alpha_b)(k_b - k_t)}{(k_b - k_t)V_c(z) + k_b + (3k_b k_t/4G_m)}\right)$
Density (ρ)	$\rho(z) = (\rho_t - \rho_b)V_c(z) + \rho_b$
Yield strength (σ_y)	$\sigma_y(z)=(\sigma_{yt} - \sigma_{yb})V_c(z) + \sigma_{yb}$

In Table, *K* and *G* are the bulks modulus and modulus of rigidity, respectively. Also, the undefined parameters are given by

$K_t = \frac{Et}{3(1-2Vt\ )}$; $G_t = \frac{Et}{2(1+Vt\ )}$

$G_b = \frac{E_b}{2(1+V_b\ )}$; $K_b = \frac{E_b}{3(1-2V_b\ )}$

The subscripts *t* and *b* stand for the material property at the top and bottom, respectively for the corresponding property. *t* corresponds to the material property of the pure ceramic, and *b* corresponds to the material property of the pure metal.

2.2 Volume fraction distribution laws of FGMs:

In Power Law (P-FGM), a model is created that describes the function of composition throughout the material. In Figure 3.3b, the volume fraction V_c, describes the volume of ceramic at any point *z* across, the thickness *h* according to a parameter *n* which controls the shape of the function [44].

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \qquad (3.1)$$

In the law of FGM denoted the volume fraction of metal, V_m (*z*), in the FGM is 1-V_c (*z*). A graphical representation of volume fraction of ceramic for various values of the parameter *n* can be seen in Figure 3.4.

The area to the right of each line represents the amount of metal, and the area to the left represents the amount of ceramic in the material. It should be noted that *n*→0, the material approaches to a homogeneous ceramic, while as *n*→∞, the material becomes entirely metal. For 0 ‹ *n* ‹ ∞, the metal will contain both metal and ceramic. When *n* = 1, the distribution of ceramic and metal is in equal portion. According to Nakamura and Sampath [50], the values of *n* should be taken in the range of **(1/3, 3),** as values outside this range will produce an FGM having too much of one phase.

Table: Effect of Power Law Index (n) on the Volume Fraction

Thickness z(m)	**Power index (n=.4)**	**Power index (n=.8)**	**Power index (n=1)**	**Power index (n=1.8)**	**Power index (n=2.8)**

.015	1	1	1	1	1
.01	1	1	1	1	1
.0075	.9480	.8987	.8750	.7863	.6881
.005	.8913	.7944	.7500	.5958	.4469
.0025	.8286	.6866	.6250	.4291	.2682
0	.7579	.5743	.500	.2892	.1436

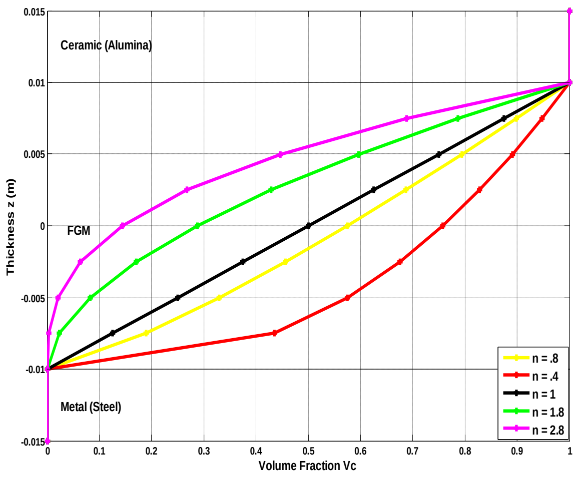

Fig. (a) Effect of power law index (n) on the volume fraction.

Table: Effect of Power Law Index (n) on the Young modulus E

Thickness z(m)	**Power index (n=.4)**	**Power index (n=.8)**	**Power index (n=1)**	**Power index (n=1.8)**	**Power index (n=2.8)**	**Power index (n=3.8)**
.015	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.01	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.0075	3.80×10^{11}	3.71×10^{11}	3.67×10^{11}	3.51×10^{11}	3.338×10^{11}	3.18×10^{11}
.005	3.70×10^{11}	3.53×10^{11}	3.45×10^{11}	3.17×10^{11}	2.904×10^{11}	2.70×10^{11}
.0025	3.59×10^{11}	3.33×10^{11}	3.22×10^{11}	2.87×10^{11}	2.582×10^{11}	2.40×10^{11}
0	3.46×10^{11}	3.13×10^{11}	3.00×10^{11}	2.61×10^{11}	2.358×10^{11}	2.22×10^{11}
-.0025	3.31×10^{11}	2.92×10^{11}	2.77×10^{11}	2.40×10^{11}	2.2155×10^{11}	2.14×10^{11}
-.005	3.13×10^{11}	2.69×10^{11}	2.55×10^{11}	2.24×10^{11}	2.137×10^{11}	2.109×10^{11}
-.0075	2.88×10^{11}	2.44×10^{11}	2.32×10^{11}	2.14×10^{11}	2.105×10^{11}	2.1007×10^{11}
-.01	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}
-.015	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}

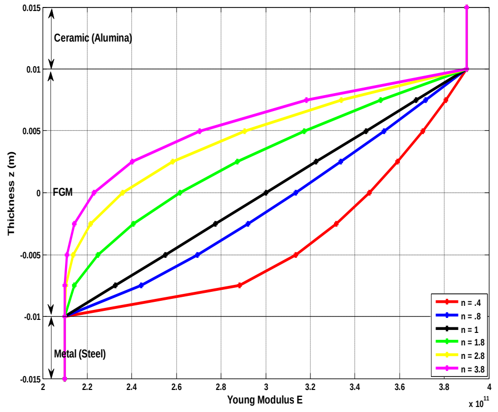

Fig. (b) Effect of power law index (n) on the young modulus E

One of most common methods to determine the effective properties of FGM is the rule of mixtures and is given by

$$P(z) = (P_t - P_b)V_c(z) + P_b \qquad (3.2)$$

$$E(z) = (E_t - E_b)V_c(z) + E_b \qquad (3.3)$$

One of most common methods to determine the effective properties of FGM is the rule of mixtures and is given by

$$P(z) = (P_t - P_b)V_c(z) + P_b \qquad (3.2)$$

$$E(z) = (E_t - E_b)V_c(z) + E_b \qquad (3.3)$$

Where P_t and P_b represent the material property of the top and bottom respectively. P_t corresponds to P_c or the material property of the pure ceramic and P_b corresponds to P_m or the material property of the pure metal. This equation holds true for the modulus of elasticity, density, thermal expansion, thermal conductivity and Poisson's ratio.

These equations become important when determining the material properties for discrete steps in a modeling process as current finite element software cannot handle a true functionally graded structure. The material property for the center of the discrete layer within the beam should be determined using the above approaches and applied to the entire layer of the FGM. Using the power law index (n), total composition of ceramic percentage in a composite material can be determined with the help of Equation 3.4 as seen below. This composition helps to understand the basic characteristics of materials.

$$V_{total,ceramic} = \frac{1}{n+1} \qquad (3.4)$$

Another valuable parameter which can be determined from the value of n is the average material property across the thickness. This is derived from Equation 3.2 and Equation 3.4. This expression can be used for quick calculations treating the material as homogeneous such as solving for the volume fraction of a functionally graded structure. The average material property can be determined as [46].

$$P_{average} = \frac{P_t - P_b}{n+1} + P_b \qquad (3.5)$$

Another quantity that might be interested in for modeling or judging the basic properties of the material is the point within the FGM at which the volume fraction transition from mostly metal to mostly ceramic is given as [47].

$$\frac{z_{transition}}{h} = 0.5^{1/n} - \frac{1}{2} \qquad (3.6)$$

An engineer can use any of these values as a goal and solve for the value of n which satisfies that need in the particular application. The value of n along with the materials used defines the characteristics of the material composition and can be tailored to produce desired result. Structural designers requiring significant thermal protection should consider low values of n which will yield a ceramic rich panel. Conversely the opposite holds true for the designer who wishes to produce a structurally sound material with slightly less thermal protection.

Many researchers used Exponential Law (E-FGM) to describe the material properties of FGM. This function is more convenient than power law because there is no need to take power index and the properties of FGM are totally dependent on ceramic and material properties. It directly generates the young's modulus across the thickness and change according to exponential law as given below. $E(z) = Ae^{\beta\left(z+\frac{h}{2}\right)}$

(3.7)

$A = E_m$ and $\beta = \frac{1}{h}\ln\left(\frac{E_c}{E_m}\right)$

$$E(z) = E_m e^{\frac{1}{h}\ln\left(\frac{E_c}{E_m}\right)\left(z+\frac{h}{2}\right)}$$

Here, E_c and E_m represent the modulus of elasticity of top (ceramic) and bottom (metal).

Chung and Chi (2001) defined the volume fraction using two power-law functions to ensure smooth distribution of stresses among all the interfaces. The two power law functions are defined by: $V_1(z) = 1 - \frac{1}{2}\left(\frac{h/2-z}{h/2}\right)^n$ for $0 \le z \le h/2$ (3.8) $V_2(z) = \frac{1}{2}\left(\frac{h/2+z}{h/2}\right)^n$ for $-h/2 \le z \le 0$ (3.9)

By using rule of mixture, the Young's modulus of the S-FGM can be calculated by: $E(z) = V_1(z)E_c + [1 - V_1(z)]E_m$ for $0 \le z \le h/2$ (3.10) $E(z) = V_2(z)E_c + [1 - V_2(z)]$ for $-h/2 \le z \le 0$ (3.11)

Table: Variation of Young modulus in an S-FGM Beam across the Thickness.

Thickness z(m)	Power index (n=.4)	Power index (n=.8)	Power index (n=1)	Power index (n=1.8)	Power index (n=2.8)	Power index (n=3.8)
.015	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.01	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.0075	3.38×10^{11}	3.603×10^{11}	3.675×10^{11}	3.825×10^{11}	3.884×10^{11}	3.895×10^{11}
.005	3.21×10^{11}	3.383×10^{11}	3.45×10^{11}	3.641×10^{11}	2.770×10^{11}	2.835×10^{11}
.0025	3.09×10^{11}	3.185×10^{11}	3.225×10^{11}	3.363×10^{11}	2.497×10^{11}	2.598×10^{11}
0	3×10^{11}	3×10^{11}	3×10^{11}	3×10^{11}	3×10^{11}	3×10^{11}
-.0025	2.902×10^{11}	2.815×10^{11}	2.775×10^{11}	2.636×10^{11}	2.502×10^{11}	2.401×10^{11}
-.005	2.782×10^{11}	2.616×10^{11}	2.55×10^{11}	2.358×10^{11}	2.229×10^{11}	2.164×10^{11}
-.0075	2.616×10^{11}	2.396×10^{11}	2.325×10^{11}	2.174×10^{11}	2.118×10^{11}	2.104×10^{11}
-.01	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}
-.015	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}

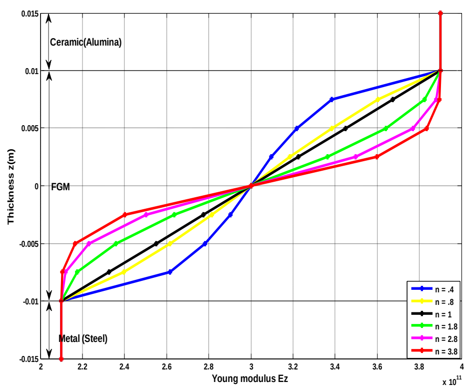

Fig. variation of young modulus in an s-fgm beam across the thickness.

T (°C)	**k (W/m K)**	**E (GPa)**	**$\alpha \times10^6$ (1/K)**	**σ_Y (MPa)**
0	61.8	194	11.4	420
27	60.7	204	11.6	397
100	57.8	195	12.1	381
200	53.5	204	12.7	362
300	49.0	193	13.3	380
400	44.5	188	13.9	359
500	40.2	183	14.4	313
600	35.7	167	14.8	284
700	31.2	141	15.0	167
800	27.3	106	14.8	72
900	26.0	74	12.6	44

2.3 Consideration of Temperature Dependency of Material Properties for FGMs:

FGMs are generally used in applications where high temperature environments fields are involved. In these high temperature environments, some material properties (thermal conductivity (k), coefficient of thermal expansion (α), modulus of elasticity (E), and yield strength ($\square_Y$) are of particular pertinence to this work) become temperature-dependent [58].

Basically two materials will be used, which are steel and alumina (Al_2O_3). The thermal properties for the materials are shown in Table 3.6 and Table 3.7. These material property data were collected from engineering manuals, material handbooks [59], and an online database of material properties [60].

Table: Thermal properties of alumina [60]

T (°C)	**k (W/m K)**	**E (GPa)**	**$\alpha \times 10^6$ (1/K)**	**σ Y (MPa)**
0	50.45	415	4.75	459
27	42.00	408	5.55	455
100	29.5	393	6.86	442
200	21.56	380	7.42	424
300	16.92	373	7.79	407
400	13.54	371	8.15	390
500	10.62	370	8.43	375
600	8.77	368	8.72	363
700	7.80	364	9.02	355
800	7.08	353	9.29	350
900	6.45	336	9.53	349

Fig. temperature dependence of the thermo-elastic properties of steel.

2.3.1 Temperature profile Modeling:

In this part present the mathematical formulation and solution of the heat conduction steady-state problem for composite FGMs beam model under thermal loading.

2.3.1.1 One-dimensional Heat Conduction Steady-State Exact Solution for 3-Layer FGM beam:

This part considers the solution of the conduction steady-state problem in a composite beam consisting of 3 layers, which are assumed to be in prefect thermal contact. This section is a formulation to find the one dimensional temperature distribution for a 3-layer beam with a middle FGM layer.

2.3.1.2 The mathematical formulation of heat conduction problem is given with boundary condition as:

$\frac{d}{dz}\left[\frac{k_1 dT_1(z)}{dz}\right]=0$ $(h_1$+a)< z < -a4.1 $\frac{d}{dz}\left[\frac{k_2 dT_2(z)}{dz}\right]=0,$ a< z < a 4.2

$\frac{d}{dz}\left[\frac{k_3 dT_3(z)}{dz}\right]=0,$ $\quad a< z < (a+h_2) \quad$ 4.3

Subject to boundary and interface condition

$T_1 = T_b \quad$ at $\quad z = -(h_1 + a) \quad$ 4.4

$\left.\begin{aligned} &\frac{k_1 dT_1(z)}{dz} = \frac{k_2 dT_2(z)}{dz} && 4.5 \\ &T_1 = T_2 \quad \text{at} \quad z = -a && 4.6 \end{aligned}\right\}$

$\left.\begin{aligned} &\frac{k_2 dT_2(z)}{dz} = \frac{k_3 dT_3(z)}{dz} && 4.7 \\ &T_2 = T_3 \quad \text{at} \quad z = a && 4.8 \end{aligned}\right\}$

Where k_1, k_2 and k_3 are the thermal conductivity coefficient for metal (steel), graded layer, and ceramic (alumina).the solution to the equation (4.1-4.3) subjected to the boundary and interface condition given by Eqs.(4.4-4.8) can be found the numerically. In Special cases can results in exact solution such as when $k_1 = k_b$ and $k_3 = k_t$ are constant throughout layers 1 and 3, while $k_2(z)$ is assumed to vary only in direction of the beam thickness according to

$$k_2(z) = k_t e^{-.5\ln(\frac{k_t}{k_b})(1-\frac{z}{a})} \qquad 4.9$$

The solution of the ordinary differential equation (4.1-4.3) for each layer is given in form

$T_1(z) = C_1 z + C_2 \qquad$ 4.10

$T_2(z) = C_3 (\frac{k_t}{k_b})^{-.5z/a} + C_4 \qquad$ 4.11

$T_3(z) = C_5 z + C_6 \qquad$ 4.12

2.3.1.3 Beam Theory for Stress Calculations:

In this section, mechanical stress distribution has been determined for a three layered composite beam having a middle layer of functionally graded material (FGM), by analytical methods. Beam is subjected to uniformly distributed transverse loading with continuous and smooth grading of metal and ceramics based on P-FGM Law, E-FGM Law and S-FGM Law are considered for study and Poisson ratio is to be held constant across FGM layer. Analytical solution is based on simple Euler-Bernoulli type beam theory.

The dimension of FGM beam of width unity and thickness h are considered, where the material property varies continuously in thickness direction (z). FGM beams have their volume fraction of ceramics V_c defined according to the power law function, sigmoid law function and exponential law function and the volume fraction of metal V_m is obtained as

$$V_m(z) = 1 - V_c(z) \qquad (4.13)$$

The mathematical modeling for evaluating the properties of functionally graded materials (P(z)) or P_b is the bottom layer property and P_t is the top layer property, which are chosen from any of the three laws expresses as per the Equations 3.2, 3.7, 3.9-3.11.

$$P(z) = P_b + (P_t - P_b)V_c(z) \qquad (4.14)$$

The basic assumptions are derived by that laws which is:

The beam is assumed to be in a state of plane strain, it is normal to the xz plane.

Euler-Bernoulli type beam theory is applied.

There is no variation in thickness along the length of beam.

Poisson's ratio is to be held constant along FG layer.

Material properties are independent of temperature gradient.

For a cantilever beam, the displacement field can be written as:

$$w(x, z) = w(x)$$

$$u(x, z) = u_0(x) - z\frac{dW(x)}{dx}$$

In above equations, u and w are denoted as horizontal and vertical displacement of beam across the thickness. It may be noted that u_0 denotes displacement of points on the middle surface of the beam along the x direction. It is assumed that σ_{zz} is negligible. Then the stress-strain relations take the form:

$$\sigma_x(z) = \breve{E}(z)\varepsilon_x\,,\ \tau_{xz}(z) = \breve{G}(z)\gamma_{xz} \qquad (4.15)$$

Where the plane strain Young modulus is given by:

$$\breve{E} = \frac{E}{1-v^2}$$

The expressions for axial strain and stress can be derived as:

$$\varepsilon_x = \frac{du(x,z)}{dx} = \frac{d}{dx}\left(u_0(x) - z\frac{dW(x)}{dx}\right) = \varepsilon_{x0} + zk_x$$

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$\varepsilon_{x0} = \frac{du_0}{dx}$, $k_x = -\frac{d^2w(x)}{dx^2}$

$$z(z) = \breve{E}(z).\varepsilon_{x0} + z.\breve{E}(z).k_x \quad (4.16)$$

$$\begin{bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{13} & 0 \\ Q_{13} & Q_{33} & 0 \\ 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{bmatrix} \quad (4.17)$$

$$[\bar{Q}_{ij}] = [Q_{ij}]$$

$$Q_{11} = \frac{E}{1-v^2} = Q_{33}, \qquad Q_{13} = \frac{vE}{1-v^2}, \qquad Q_{55} = \frac{E}{2(1+v)}$$

Here,$[\bar{Q}_{ij}]$and $[Q_{ij}]$ both are stiffness matrices and ε_{x0}, k are axial strain in the middle surface and the beam curvature. According to Euler-Bernoulli beam theory, the axial force and bending moment, N and M, are defined

$$(N, M) = \int_{-h_1}^{h_1} [\breve{E}(z).\varepsilon_{x0} + z.\breve{E}(z).k](1, z)dz \quad (4.18)$$

Deformation case considered the axial force resultant is zero for analytical solution so that the expressions for the deformation take the form

$$C_0\varepsilon_{x0} + C_1 k = 0$$

$$C_1\varepsilon_{x0} + C_2 k = M_{maximum}$$

C_0, C_1, and C_2 are the coefficients of mid-plane strain and curvature. Using Equation 4.16, the axial stresses in ceramic, metal and FGM section across the thickness of proposed model are obtained.

2.3.1.4 Temperature Profile modeling for thermal stress formulation:

When proposed FGM beam model is subjected to uniform temperature change (ΔT), the total strain under a small strain assumption, can be taken as made up of elastic and thermal part. For a beam under plane strain condition, the only non-zero stress component is σ_x:

$$\sigma_x = E(z)[\varepsilon_{x0}^T + z.k^T - \alpha(z)\Delta T] \quad (4.19)$$

Where ε_{x0}^T is the strain at the mid-plane (z = 0) of the FGM layer and k^T is the laminate curvature due to temperature gradient. Since only thermal loading is considered here:

$$\sum F_x = 0, \qquad \sum M_x = 0$$

On the other hand

$$(N, M) = \int_{-h_1}^{h_1} \sigma_{xx}(1, z)dz = 0$$

The axial force and bending moment in thermal gradient can be obtained as given below:

$$N^T = (\Delta T)\sum_{k=1}^{m}[\bar{Q}]_k[\alpha]_k(h_k - h_{k-1}) \qquad (4.20)$$

$$M^T = \frac{1}{2}(\Delta T)\sum_{k=1}^{m}[\bar{Q}]_k[\alpha]_k(h_k^2 - h_{k-1}^2) \qquad (4.21)$$

Here, m is the number of lamina and in proposed model three laminas is considered. Further thermal strain, mid-plane strain and curvature, mechanical strain and thermal stresses are calculated by below formulas:

$$\{\varepsilon^T\} = (\Delta T)\{\alpha\}$$

$$\begin{bmatrix}\varepsilon_{x0}^T \\ k^T\end{bmatrix} = \begin{bmatrix}A & B \\ B & D\end{bmatrix}^{-1}\begin{bmatrix}N^T \\ M^T\end{bmatrix}$$

$$\{\varepsilon\} = \{\varepsilon_{x0}^T\} + z\{k^T\}$$

$$\{\varepsilon^M\} = \{\varepsilon\} - \{\varepsilon^T\}$$

$$\{\sigma^T\} = [\bar{Q}]\{\varepsilon^M\} \qquad (4.22)$$

The coefficient of thermal expansion for FGM is obtained by rule of mixture

$$\alpha(z) = (\alpha_c - \alpha_m)(V_c) + \alpha_m$$

3. PERFORMANCE EVALUATION:

The Performance Evaluation of FGM beam under Distribution of temperature in one dimension, stress generation according to depth of mechanical and thermal loading which is performed using a formulation presented in the previous chapter and the mathematical tool MATLAB is used for coding and generation of graph between different properties of FGM. The beam responses are compared with earlier studies. Also a study to determine the influence of neutral surface position on deflection of functionally graded beam under uniformly distributed loading is presented.

3.1 Temperature Distribution:

A three layered composite system of Al_2O_3 – FGM – Steel, Sic – FGM – C and αAl_2O_3 – FGM–(W, Ti) C is considered in which middle layer is FGM and other two are ceramic (Al_2O_3, Sic,αAl_2O_3) and metal (steel, C, (W, Ti) C). Acantilever beam of 1 m length made of the composite system of thickness 0.03 m is considered. The topmost material is ceramic (Al_2O_3, Sic,αAl_2O_3) which has a thickness of 0.005 m and the bottom layer is metal (steel, C, (W,Ti)C). of same thickness as ceramic. In between these two layers there is an FGM layer of 0.02 m and the beam has unit width. According to Nakamura and Sampath, the value of *n* should be taken in the range of [1/3,3], as values outside this range will produce an FGM having too much of one phase. Here *n* is represented the grading parameter which will be taken (n=1.4) for solve the problem and FGM thickness (.02m). First, fixed-free boundary condition is considered and beam is subjected to a uniformly distributed load (q =200 kN/m) and there is no rise in temperature ($\Delta T = 0°C$). Next, free-free boundary condition is considered and the effect of thermal loading is studied were beam is subjected to a temperature gradient ($\Delta T = 200°C$). Effect of temperature rise/fall is considered by augmenting the thermal strain to the mechanical strain. The coefficients of thermal expansion of FGM beam are calculated by rule of mixture from the properties of metal and ceramic are given in Table.

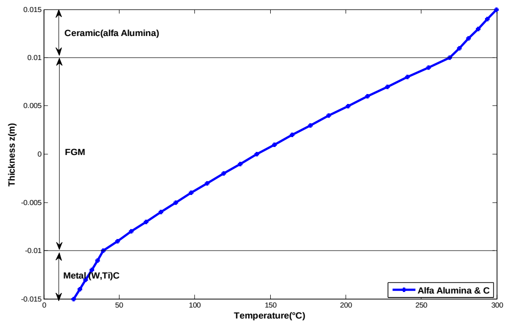

Fig. temperature distribution graph between αal_2o_3& (w, ti) c (present work)

Axial Stress distribution of three layered Composite FGM Beam across the thickness & according to the volume fraction distributions in composite (Metal & Ceramic) and FGM beam table:

Table: Axial Stress of Three Layered Composite FGM Beam across the Thickness

Thickness (z)m	**With FGM**			**Without FGM**		
	Ceramic (σ_x) Pa	**FGM (σ_x) Pa**	**Metal (σ_x) Pa**	**Ceramic (σ_x) Pa**	**FGM (σ_x) Pa**	**Metal (σ_x) Pa**
0.015	7.9909×10^8	----	----	7.9909×10^8	----	----
0.01	4.88×10^8	----	----	4.88×10^8	----	----
0.0075	----	3.24×10^8	----	3.35×10^8	----	----
0.005	----	1.66×10^8	----	1.83×10^8	----	----
0.0025	----	0.255×10^8	----	$.302\times10^8$	----	----
0	----	-0.94×10^8	----	-1.22×10^8	----	-1.22×10^8
-0.0025	----	-1.905×10^8	----	----	----	-1.48×10^8
-0.005	----	-2.676×10^8	----	----	----	-2.301×10^8

-0.0075	----	-3.316×10^{8}	----	----	----	-3.13×10^{8}
-0.01	----	----	-3.946×10^{8}	----	----	-3.946×10^{8}
-0.015	----	----	-5.59×10^{8}	----	----	-5.59×10^{8}

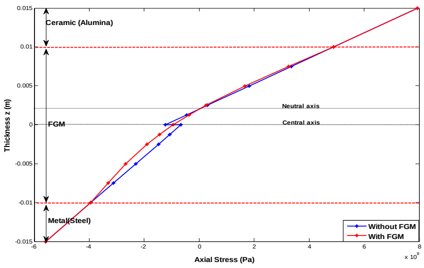

Fig.(a) axial stress with fgm & without fgm (present work)

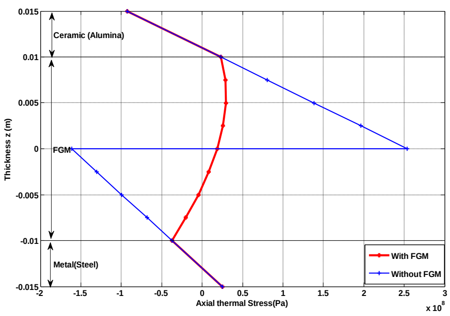

Fig. (a) axial thermalstress with fgm & without fgm (present work)

3.2 Comparisons of Performance for Three Function Laws:

Properties of proposed FGM model are obtained from P, S and E function laws and the variation of young's modulus for three laws is presented in Figure.

TABLE : Comparisons of Axial Mechanical Stresses in Three Layered Composite FGM

Thickness Z (m)	**POWER LAW**			**Sigmoid Law**			**Exponential Law**		
	Ceramic (σ_x) Pa	**FGM (σ_x)Pa**	**Metal (σ_x)Pa**	**Ceramic (σ_x) Pa**	**FGM (σ_x)Pa**	**Metal (σ_x)Pa**	**Ceramic (σ_x) Pa**	**FGM (σ_x)Pa**	**Metal (σ_x)Pa**
.015	8.09×10^{8}	–	–	7.9×10^{8}	–	–	7.9×10^{8}	–	–
.01	5.03×10^{8}	–	–	4.8×10^{8}	–	–	4.9×10^{8}	–	–
.0075	–	3.22×10^{8}	–	–	3.2×10^{8}	–	–	3.1×10^{8}	–
0.005	–	1.66×10^{8}	–	–	1.6×	–	–	1.5×	–

					10^8			10^8	
0	–	$-.78\times10^8$	–	–	$-.94\times 10^8$	–	–	$-.87\times 10^8$	–
-.005	–	-2.5×10^8	–	–	-2.6×10^8	–	–	-2.7×10^8	–
-.0075	–	-3.2×10^8	–	–	-3.3×10^8	–	–	-3.3×10^8	–
.01	–	–	-3.8×10^8	–	–	-3.9×10^8	–	–	-3.9×10^8
-.0015	–	–	-5.5×10^8	–	–	-5.5×10^8	–	–	-5.5×10^8

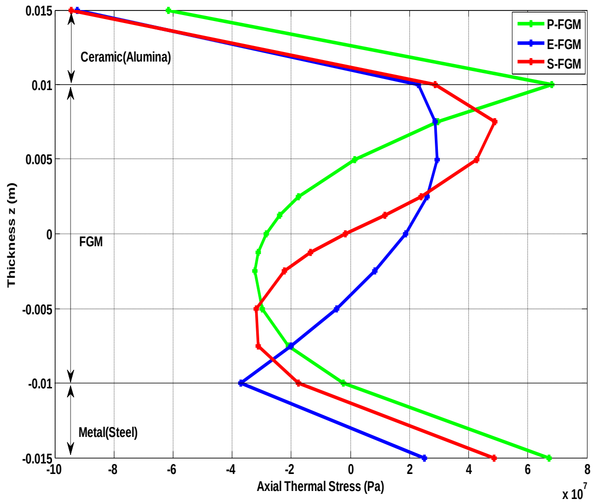

Figure: Comparisons of Axial Thermal Stresses in Three Layered Composite FGM (present Work)

4. CONCLUSIONS:

Functionally graded materials are good replacement of composite materials because they overcome the unbinding type problems. These materials are commonly used in aerospace industries where the harsh temperature is major issue. The basic properties of FGM can be obtained by any of the three function laws, power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM).

In this work comparing all these function law's and fined out that E-FGM law is better than the other two function law. Because in FGM region, property is change continuous, not in a parabolic nature.

After that taking three FG materials and doing comparative study in relation to the heat conduction and concluded that alumina and carbon has good head conduction because the temperature difference in their FGM region is low than the other two FG materials.

Generating residual stress and mechanical stress for three layer composite FGM by using a mathematical tool MATLAB and compare the obtained results with the previous work and predict that FGM is more suitable in thermal loading than mechanical loading.

Nature of neutral surface shift is determined for variation of power law index and also the deflection of FGM beam with changing power law index is examined. When the power law exponent is increased the maximum deflection of the beam is increased.

FUTURE SCOPE OF WORK

- We suggest further investigation of functionally graded beam structures with material properties varying in directions other than through the thickness
- A further investigation regarding the techniques for estimating effective material properties of functionally graded materials is desirable. In the graded layer of real FGMs, ceramic and metal particles of arbitrary shapes are mixed up in arbitrary dispersion structures. Hence, the prediction of the thermo-elastic properties is not a simple problem, but complicated due to the shape and orientation of particles, the dispersion structure, and the volume fraction. This situation implies that the reliability of material-property estimations becomes an important key for designing a FGM that meets the required performance.
- The thickness of the middle FGM can be optimized.

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