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Study of Total Hip Arthoplasty Using Mathematical Model: A Review

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ABSTRACT

The Complications in the hip replacement are often caused the failure of implants. Several experiments have been performed and investigated with different combination of load, cements, and implants to find the cause of failures. Performing the experiment several times is quite expensive and time consuming whereas mathematical molding is cheap and essential tool. The following study consist the Mathematical model for heat transfer, fluid flow, and stress distribution using sets of governing differential equations and weak form to analyze the hip arthoplasty.

Keywords:Mathematical molding, FEM, Total Hip Replacement, Artificial implants

Total hip replacement is the replacing hip joint with a Artificial implant called prosthesis. This surgery usually done in case of hip diseases like pain, trauma, arthritis, necrosis, deformity, fracture and tumors.

The two mostly used techniques in such surgery are Cemented arthroplasty and uncemented arthroplasty[3].Cemented arthroplasty is mostly used technique due to its fast recovery. Even, the cement mantle stresses are reduces by 35-60% in intact cement-implant[1]. Even after the high success rate, still there are the cases of prosthesis failure or prosthesis loosening [2]. The average life span of prosthesis is around 15 years. This is why it leads the problems for younger patients

To get rid of such problem several studies of acrylic bone cement were done using various combination of implants cements with different bone stiffness, patient weight by Yettram and Markolf (1976, [18]), and Urist (1975, [8,10]).A 2D finite element methods used to calculate the normal and shear stress distributions on the implants-cement and cement-bone interfaces. The result shows that stresses increased with the decreased stiffness of the stem [16]. A very less effect found due to Cement stiffness and stem shape. It was also noted that nerve palsy and Musculoskeletal loading are also play important role in failure. [5,6]. Moreover the failure of prosthesis is found to be associated with stress caused by load sharing between bone and prosthesis. Turner (2005, [9,11]) also according to mechanical theory, aseptic loosening is assumed to be result of cyclic stress generated by daily activities on the implants and hip joint assembly.[12,13]

The Complications of the hip replacement are often caused by the distribution of mechanical stresses over the implant-bone. Several mathematical model of bone are investigated using nonlinear differential equations. As the performance and success of cemented total hip arthroplasty is related to the stress fields in the hip prosthesis cement mantle-bone assembly.

Performing the experiment several times is quite expensive and time consuming whereas mathematical molding is cheap and essential tool. In the following study a mathematical model is generated to study the stress distribution in the prosthesis-hip joint assembly.The following sets of equilibrium equations are governed the 2D stress distribution in the artificial hip assembly.

$$\rho \frac{\partial^2}{\partial t} u - \left(\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y}\right) = p_{11} \qquad \text{(i)}$$

$$\rho \frac{\partial^2}{\partial t} v - \left(\frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y}\right) = p_{22} \qquad \text{(ii)}$$

And strain is different direction is given as

$$\varepsilon_{11} = \frac{\partial u}{\partial x} \qquad \text{(iii}$$

$$\varepsilon_{22} = \frac{\partial v}{\partial y} \qquad \text{(iv)}$$

$$2\varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \text{(v)}$$

From the generalized Hook's law for 2D plane stress condition

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} \qquad \text{(vi)}$$

Here σ_{ij} is stress tensor in different direction, ε_{ij} is strain tensor, p_{ij} used for body forces, u and v are the displacement in x and y direction. C_{ij} are stiffness component, ρ is density

Using the equations (iii) to (v), equation (i) and (ii) can be rewritten as

$$-\frac{\partial}{\partial x}\left(C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left[C_{33}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] = p_{11} - \rho\frac{\partial^2}{\partial t^2}u \qquad \text{(vii)}$$

$$-\frac{\partial}{\partial x}\left(C_{33}\frac{\partial u}{\partial y} + C_{33}\frac{\partial v}{\partial x}\right) - \frac{\partial}{\partial y}\left[C_{12}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y}\right] = p_{22} - \rho\frac{\partial^2}{\partial t^2}v \qquad \text{(viii)}$$

Weak form of the governing differential equation using weight function W_1

$$\int W_1\left[-\frac{\partial}{\partial x}\left(C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left\{C_{33}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right\} - p_{11} + \rho\frac{\partial^2}{\partial t^2}u\right]dx\,dy = \oint W_1\begin{bmatrix}\left(C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y}\right)n_x + \\ n_y\left\{C_{33}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right\}\end{bmatrix}ds$$

(ix)

Equation (ix) rewritten as

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$$\int \left[-\frac{\partial W_1}{\partial x}\left(C_{11}\frac{\partial u}{\partial x}+C_{12}\frac{\partial v}{\partial y}\right)-\frac{\partial W_1}{\partial y}\left[C_{33}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-W_1 p_{11}+\rho W_1\frac{\partial^2}{\partial t^2}u\right]dx\,dy-\oint\left[W_1\left(C_{11}\frac{\partial u}{\partial x}+C_{12}\frac{\partial v}{\partial y}\right)n_x+\right.$$

$$nyW1C33\partial u\partial x{+}\partial v\partial yds{=}0 \qquad \text{(x)}$$

And using weight function W_2

$$\int W_2\left[-\frac{\partial}{\partial x}\left(C_{33}\frac{\partial u}{\partial y}+C_{33}\frac{\partial v}{\partial x}\right)-\frac{\partial}{\partial y}\left\{C_{12}\frac{\partial u}{\partial x}+C_{22}\frac{\partial v}{\partial y}\right\}-p_{22}\,\rho\frac{\partial^2}{\partial t^2}v\right]dx\,dy=\oint W_2\begin{bmatrix}\left(C_{33}\frac{\partial u}{\partial y}+C_{33}\frac{\partial v}{\partial x}\right)n_x+\\ n_y\left\{C_{12}\frac{\partial u}{\partial x}+C_{22}\frac{\partial v}{\partial y}\right\}\end{bmatrix}ds$$

(xi)

Equation (xi) rewritten as

$$\int\left[-\frac{\partial W_2}{\partial x}\left(C_{33}\frac{\partial u}{\partial y}+C_{33}\frac{\partial v}{\partial x}\right)-\frac{\partial W_2}{\partial y}\left\{C_{12}\frac{\partial u}{\partial x}+C_{22}\frac{\partial v}{\partial y}\right\}-W_2 p_{22}+\right.$$

$$W2\,\rho\partial 2\partial t2vdxdy{-}W2C33\partial u\partial y{+}C33\partial v\partial xnx{+}nyC12\partial u\partial x{+}C22\partial v\partial yds{=}0 \qquad \text{(xii)}$$

Displacement functions u and v in term of lagrangian functions given as

$u(x,y,t)=u_1(t)N_1(x,y)+ u_2(t)N_2(x,y)+ u_3(t)N_3(x,y)\ldots\ldots u_n(t)N_n(x,y)$(xiii)

$v(x,y,t)=v_1(t)N_1(x,y)+ v_2(t)N_2(x,y)+ v_3(t)N_3(x,y)\ldots\ldots v_n(t)N_n(x,y)$ (xiv)

$W(x,y,t)=W_1(t)N_1(x,y)+ W_2(t)N_2(x,y)+ W_3(t)N_3(x,y)\ldots\ldots W_n(t)N_n(x,y)$ (xv)

Above equation rewritten in form of generalized equations

$u(x,y,t)=\sum_{i=1}^{n}N_i(x,y,t)\,u_i(t)$ (xvi)

$v(x,y,t)=\sum_{i=1}^{n}N_i(x,y,t)\,v_i(t)$ (xvii)

$Wi(x,y,t)=\sum_{i=1}^{n}N_i(x,y,t)\,W_i(t)$ (xviii)

Put the equations (xvi) - (xvii) in equations (x) and (xii) and convert the resultant equations into matrix form, we obtain set of ordinary nonlinear differential equation

$M\ddot{u}+A(u)u=P$ (xix)

Equation (xix) can be solved by any suitable solving method like MOL, GDM, DEM, QNM.

To analyise the effect of heat transfer the following unsteady heat equation can be used

$\rho H\frac{\partial\theta}{\partial t}+\nabla.(-\alpha\nabla\theta)=0$ (xx)

Here , θ used for temperature, H is heat capacity, α is thermal conductivity, t is for time.

Naviour-stoke and Brinkman eqquations use for the cemented flow in the hip replacement and is governed by the following equation

$\nabla . u = 0$ (xxi)

$\rho \frac{\partial u}{\partial t} - \nabla . \tau_m + \rho(u. \nabla)u + \nabla p_m = p$ (xxii)

Where u is velocity of fluid, p_m is the pressure, p represents force, τ_m shear tensor, D is deformation tensor

Also

$$\tau_m = 2\eta \Upsilon D$$

$$D = \frac{1}{2}(\nabla u + (\nabla u)^T)$$

To model flow in porous media Brinkman equation used

$\nabla . v = 0$ (xxiii)

$\rho \frac{\partial v}{\partial t} - \nabla . \tau_c + \frac{\eta}{k} v + \nabla p_c = p$ (xxiii)

The viscosity (η) and shear rate (Υ) is given as

$$\eta = m\Upsilon^{n-1}$$

$$\Upsilon = \sqrt{2tr(D^2)}$$

II.CONCLUSIONS

A set of some governing differential equations with proper boundary conditions can give a good mathematical model. The mathematical model, presented in this study can be solved any above mention numerical techniques and stress distribution and other field variables can be obtain. Although some factors are still not consider in the development of model.

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