

Overview of PAPR Reduction Coding Techniques for OFDM

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ABSTRACT

OFDM is a transmission scheme that offers diversity in frequency selective fading environment but suffers from high PAPR. Many researchers have studied the appropriate coding scheme to reduce PAPR and offer good error control properties as well. This article reviews the major results obtained up to date.

Keywords: Peak to Average Power Ratio; OFDM, Reed-Muller Codes; Peak power (PEP); Golay codes.

I. INTRODUCTION

OFDM is considered a good candidate for wireless systems because it offers diversity gain in frequency selective channels [1]. It uses the FFT to multicarrier modulate a signal and thus can take advantage of advances in DSP and digital circuitry. As in other multicarrier schemes, however, OFDM suffers from high PAPR. This is a major drawback of the scheme and ways of minimizing the PAPR have been researched through many years. This report covers the major developments. It first reviews the idea behind OFDM and the meaning of PAPR. It then presents some simple block coding schemes and moves on to the more elegant coding schemes using Complementary Sequences and Reed-Muller Codes.

II. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

OFDM is a form of multicarrier transmission that sends information simultaneously over N orthogonal carriers. It introduces frequency diversity by making the bandwidth of each carrier smaller than the coherence bandwidth of the channel. Each carrier may still suffer from flat-fading, however. OFDM is considered a good candidate for high data rate wireless systems and is currently used for the HyperLAN II standard [7]. The transmitted signal over a symbol duration T is [4].

$$s(\bar{c}, t) = \operatorname{Re} \left(\sum_{i=0}^{N-1} c_i \exp(j2\pi(f_0 + if_s)t) \right) \quad 0 \leq t \leq T \quad (1)$$
$$\bar{c} = (c_0 c_1 \dots c_{N-1})$$

The codeword c consists of N symbols chosen from an M-ary modulation method. All of the codewords form the set C. For MPSK.

$$c_i = e^{j\frac{2\pi}{M}(a_i)} \quad a_i \in Z_M \quad (2)$$

The duration of an OFDM symbol T is N times the duration of the symbols c_i plus the duration of the cyclic prefix or guard band. The complex envelope of the transmitted signal, sampled at $1/T$, is:

$$\tilde{s}(\bar{c}, n) = \sum_{i=0}^{N-1} c_i \exp(j2\pi ni / N) \quad (3)$$

This equation can be recognized as the IDFT of the sequence $c_0 \dots c_{N-1}$.

III. PEAK AVERAGE POWER RATIO

An important limitation of OFDM is that it suffers from a high Peak-to-Average Power Ratio (PAPR) resulting from the coherent sum of several carriers. This forces the power amplifier to have a large input backoff and operate inefficiently in its linear region to avoid intermodulation products. High PAPR also affects D/A converters negatively and may lower the range of transmission. PAPR is defined as:

$$PAPR = \frac{\max |s(t)|^2}{E[|s(t)|^2]} \quad (4)$$

Theoretically, the PAPR can be as high as N , but the occurrence of such peaks is rare. The summation of a large number of carriers assumes a Gaussian distribution. The numerator, $\max |s(t)|^2$, is also known as the PEP (Peak Envelope Power). It is also equal to:

$$PEP = \tilde{s}(t) \tilde{s}^*(t) \quad (5)$$

Several methods have been devised throughout the years to limit the PAPR of a multicarrier signal. These methods include clipping, filtering, and coding. Clipping methods are the most widely used but at the cost of degradation of performance [10]. Some filtering and coding methods modify an OFDM symbol to lower its PAPR [3][4]. The more sophisticated methods form error-correcting codes with inherently low PAPR. [10]

IV. PAPR REVISITED

As mentioned previously, the PEP of an OFDM symbol can be found from its complex envelope. It can be shown that the PEP can also be written as (unit amplitude PSK symbols): [8]

$$\tilde{s}(t)\tilde{s}^*(t) = N + 2 \operatorname{Re} \left\{ \sum_{i=1}^{N-1} R_{c_i}(i) e^{j2\pi i t / T} \right\} \quad (6)$$

$$R_c(i) = \sum_{k=0}^{N-1-i} c_i c_{k+i}^* = \sum_{k=0}^{N-1-i} e^{j(\phi_k - \phi_{k+i})} \quad (7)$$

R_c is known as the aperiodic autocorrelation function of the sequence $c_0 \dots c_{N-1}$. Sequences with low sidelobes will give a small PAPR. Golay binary complementary sequences are one of the sequences that give a PAPR of no more than 3 dB [10].

V. REED MULLER CODES AND PAPR

In [10], the authors use a generalized Boolean function, mapping into Z_2^h instead of Z_2 ($h \geq 1$), to describe Complementary Pairs over Z_2^h of length 2^m . The two sequences $a(v_1 \dots v_m)$ and $b(v_1 \dots v_m)$ form a Complementary Pair and are given by (π is a permutation of the symbols $\{1, 2, \dots, m\}$ and c, c' are members of Z_2^h)

$$a(v_1 \dots v_m) = f(v_1 \dots v_m) + c \quad (8)$$

$$b(v_1 \dots v_m) = f(v_1 \dots v_m) + 2^{h-1} v_{\pi(1)} + c' \quad (9)$$

$$f(v_1 \dots v_m) = 2^{h-1} \sum_{k=1}^{m-1} v_{\pi(k)} v_{\pi(k+1)} + \sum_{k=1}^m c_k v_k \quad (10)$$

A new sequence formed from a by any permutation of the symbols $\{1, \dots, m\}$ and adding any c also is a Complementary Sequence. This gives $2^{h(m+1)} m! / 2$ Golay sequences.

The authors then generalize the Reed-Muller codes from binary ($h=1$) to $h \geq 1$ by defining them over a ring instead of a field. $RM_2^h(r, m)$ over Z_2^h is generated by the same way as before. In addition, the r th order linear code $ZRM_2^h(r, m)$ is generated by the monomials in the v_i of degree no greater than $r-1$ along with twice the monomials in the v_i of degree r (monomials of degree -1 and $m+1$ equal to 0). Each of the $m! / 2$ cosets of $RM_2^h(1, m)$ that are in $ZRM_2^h(1, m)$ make up $2^{h(m+1)}$ Complementary Sequences of length 2^m . The cosets have a representation of the form:

$$2^{h-1} \sum_{k=1}^{m-1} v_{\pi(k)} v_{\pi(k+1)} \quad (10)$$

Using these cosets in OFDM transmission limits the PAPR to at most 2. Using more cosets would increase the transmission rate, but would also increase the PAPR. The authors were able to partition the cosets into increasing values of PAPR for small values of h and m . In [11], the authors generalize these results by using q alphabets (an even number) instead of 2^h and consider general second order cosets of $R_q(1, m)$.

The authors in [14] used these codes in an indoor wireless environment. The codes offered protection against amplifier nonlinearities and offered error protection, but fell short of the gain that could be provided by convolutional codes.

VI. CONCLUSIONS

This article has reviewed several coding techniques to reduce the PAPR in OFDM transmission. One approach to reduce the PAPR tries to minimize the PEP of a signal by combining it with a weight vector. This method can lower the PAPR and its complexity depends on the algorithm used. The more sophisticated approach uses sequences that inherently have low PAPR. Golay Complementary sequences are in this class and are related to the well-known Reed Muller codes. Like these, codes, using a higher number of carriers decreases the coding rate, but offers the gain obtained from these codes. There is a fundamental trade-off between coding rate and PAPR.

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