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CONNECTIONS BETWEEN THE MATHEMATICAL SCIENCES AND OTHER FIELDS: A REVIEW

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ABSTRACT-

Mathematics is the science of patterns and relationships. As a theoretical discipline, mathematics explores the possible relationships among abstractions without concern for whether those abstractions have counterparts in the real world. Applied mathematics is the application of mathematical methods by different fields such as science, engineering, business, computer science, and industry. Thus, applied mathematics is a combination of mathematical science and specialized knowledge. The term "applied mathematics" also describes the professional specialty in which mathematicians work on practical problems by formulating and studying mathematical models. In the past, practical applications have motivated the development of mathematical theories, which then became the subject of study in pure mathematics where abstract concepts are studied for their own sake.

In research work the modelling is used to predict behaviour and in doing so validate the theory or raise new questions as to the reasonableness of the theory and often suggests the need of sharper experiments and more focused observations. Thus, observation and experiment, theory, and Modelling reinforce each other and together lead to our understanding of scientific phenomena. As with data mining, the other approaches are only successful if there is close collaboration between mathematical scientists and the other disciplinarians.

Keywords: Computer Algebra Systems(CAS), MuPAD, MathCAD, NRC.

I. INTRODUCTION

THE NATURE OF MATHEMATICS:

The mathematical sciences aim to understand the world by performing formal symbolic reasoning and computation on abstract structures. One aspect of the mathematical sciences involves unearthing and understanding deep relationships among these abstract structures. Another aspect involves capturing certain features of the world by abstract structures through the process of modeling, performing formal reasoning on the abstract structures or using them as a framework for computation, and then reconnecting back to make predictions about the world. Often, this is an iterative process. Yet another aspect is to use abstract reasoning and structures to make inferences about the world from data. This is linked to the quest to find ways to turn empirical observations into a means to classify, order, and understand reality the basic promise of science. Through the mathematical sciences, researchers can construct a body of knowledge whose interrelations are understood and where whatever understanding one needs can be found and used. The mathematical sciences also serve as a natural conduct through which concepts, tools, and best practices can migrate from field to field. [1].

II. THE MATHEMATICAL SCIENCES AND THEIR INTERFACES

Over the past decade or more, there has been a rapid increase in the number of ways the mathematical sciences are used and the types of mathematical ideas being applied. Because many of these growth areas are fostered by the explosion in capabilities for simulation, computation, and data analysis (itself driven by orders-of-magnitude increases in data collection), the related research and its practitioners are often assumed to fall within the umbrella of computer science. But in fact people with varied backgrounds contribute to this work. The process of simulation-based science engineering is inherently very mathematical, demanding advances in mathematical structures that enable modeling; in algorithm development; in fundamental questions of computing; and in model validation, uncertainty quantification, analysis, and optimization. Advances in these areas are essential as computational scientists and engineers tackle greater complexity and exploit advanced computing. These mathematical science aspects demand considerable intellectual depth and are inherently interesting for the mathematical sciences. [1].

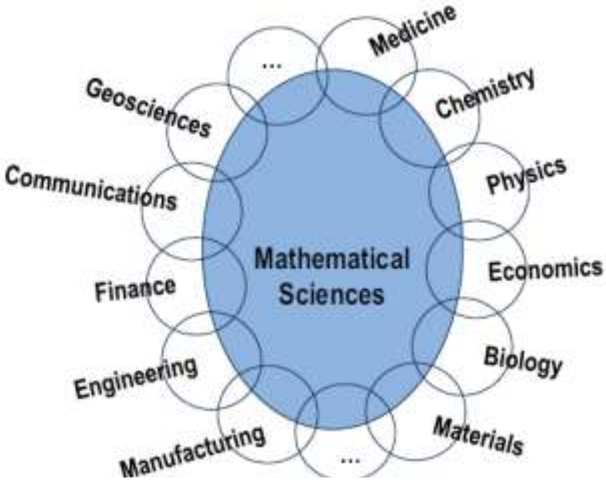

Figure 1: The mathematical sciences and their interfaces (Figure from reference [1])

III. RELATIONSHIP BETWEEN MATHEMATICS AND THE OTHER FIELDS

The relationship between mathematics and the other fields of basic and applied science is especially strong. This is so for several reasons, including the following:

- The alliance between science and mathematics has a long history, dating back many centuries. Science provides mathematics with interesting problems to investigate, and mathematics provides science with powerful tools to use in analyzing data. Often, abstract patterns that have been studied for their own sake by mathematicians have turned out much later to be very useful in science. Science and mathematics are both trying to discover general patterns and relationships, and in this sense they are part of the same endeavor.

- Mathematics is the chief language of science. The symbolic language of mathematics has turned out to be extremely valuable for expressing scientific ideas unambiguously. The statement that $a=F/m$ is not simply a shorthand way of saying that the acceleration of an object depends on the force applied to it and its mass; rather, it is a precise statement of the quantitative relationship among those variables. More important, mathematics provides the grammar of science the rules for analyzing scientific ideas and data rigorously.
- Mathematics and science have many features in common. These include a belief in understandable order; an interplay of imagination and rigorous logic; ideals of honesty and openness; the critical importance of peer criticism; the value placed on being the first to make a key discovery; being international in scope; and even, with the development of powerful electronic computers, being able to use technology to open up new fields of investigation.
- Mathematics and technology have also developed a fruitful relationship with each other. The mathematics of connections and logical chains, for example, has contributed greatly to the design of computer hardware and programming techniques. Mathematics also contributes more generally to engineering, as in describing complex systems whose behavior can then be simulated by computer. In those simulations, design features and operating conditions can be varied as a means of finding optimum designs. For its part, computer technology has opened up whole new areas in mathematics, even in the very nature of proof, and it also continues to help solve previously daunting problems. [2]

IV. INCREASING IMPORTANCE OF CONNECTIONS FOR MATHEMATICAL SCIENCES RESEARCH

Based on testimony received at its meetings, conference calls with leading researchers and the experiences of its members, the committee concludes that the importance of connections among areas of research has been growing over the past two decades or more. This trend has been accelerating over the past 10-15 years, and all indications are that connections will continue to be very important in the coming years. Connections are of two types:

• The discipline itself research that is internally motivated is growing more strongly interconnected, with an increasing need for research to tap into two or more fields of the mathematical sciences;

• Research that is motivated by, or applied to, another field of science, engineering, business, or medicine is expanding, in terms of Suggested Citation:" Important Trends in the Mathematical Sciences." National Research Council(NRC). 2013.Bottom of Form both the number of fields that now have overlaps with the mathematical sciences and the number of opportunities in each area of overlap.

This elaborates on the trend toward greater connectivity on the part of the mathematical sciences themselves. Internally driven research within the mathematical sciences is showing an increasing amount of collaboration and research that involves two or more fields within the discipline. Some of the most exciting advances have built on fields of study for example, probability and combinatory that had not often been brought together in the past. This change is nontrivial because large bodies of knowledge must be internalized by the investigator.

The increased interconnectivity of the mathematical sciences community has led, as one would expect, to an increase in joint work. To cite just one statistic, the average number of authors per paper in the Annals of

Mathematics has risen steadily, from 1.2 in the 1960s to 1.8 in the 2000s. While this increase is modest compared to the multi author traditions in many fields, it is significant because it shows that the core mathematics covered by this leading journal is trending away from the solitary researcher model that is embedded in the folklore of mathematics. It also suggests what has long been the experience of leading mathematicians: that the various subfields in mathematics depend on one another in ways that are unpredictable but almost inevitable, and so more individuals need to collaborate in order to bring all the necessary expertise to bear on today's problems.

While some collaborative work involves mathematical scientists of similar backgrounds joining forces to attack a problem of common interest, in other cases the collaborators bring complementary backgrounds. In such cases, the increased collaboration in recent years has led to greater cross-fertilization of fields to ideas from one field being used in another to make significant advances. A few recent examples of this interplay among fields are given here. This list is certainly not exhaustive, but it indicates the vitality of cross-disciplinary work and its importance in modern mathematical sciences.[2]

V. USE OF MATHEMATICAL SOFTWARE FOR TEACHING AND LEARNING MATHEMATICS

The computer algebra systems (CAS) such as Mathematica, Maple, MuPAD, MathCAD, Derive, Maxima have potential to facilitate an active approach to learning, to allow students to become involved in discovery and to consolidate their own knowledge, thus developing conceptual and geometrical understanding and a deeper approach to learning. Emergence of such mathematical tools and its ability to deal with most of the undergraduate mathematics cannot be ignored by mathematics educators.

Few working in mathematics education today would be unaware of the growth in recent years of computer technologies for teaching, learning and research in mathematics. Calculating technology in mathematics has evolved from four function calculators to scientific calculators to graphing calculators and now to computers with computer algebra system software. The use of CAS in education is still relatively rare but the growing body of research and the interest suggests that its extended use is imminent.

With the traditional undergraduate curriculum, students do not often regard themselves as active participants in mathematical exploration. Rather they are passive recipients of a body of knowledge, comprising definitions, rules and algorithms. Computers offer a number of didactic advantages that can be exploited to promote a more active approach to learning. Students can become involved in the discovery and understanding process, no longer viewing mathematics as simply receiving and remembering algorithms and formulae. The power of computer algebra goes beyond routine computation. It has the potential to facilitate an active approach to learning, allowing students to become involved in discovery and constructing their own knowledge, thus developing conceptual understanding and a deeper approach to learning.-

Example 1. Use the method of Lagrange Multipliers to maximize/minimize

$y - x^2$ subjected to $y^2 + x^2 = 2$

For convenience let $f(x,y) = y - x^2$ and *g*(*x*,*y*)= $2x^2 + y^2 - 2.$ Geometrically, the maximum/ minimum of the above problem occur where ever the gradient of $f(x,y) = y - x^2$ and gradient of

$g(x,y) = 2x^2 + y^2 - 2$ are parallel. This is same as, the level curves of *f* and *b* have common tangents at these points. Using MuPAD animation, we can show that there are four points on the ellipse $g(x,y) = 0$ at which this happens.

```
>>f:=(x,y)-> y-x^2//to define the function f;
>>g:=(x,y)->2*x^2+y^2-2//to define the function g
>>pf:=plot::Implicit2d(f(x,y)=c,x=-5..5,y=-5..5,
c=-3..3,Color=RGB::Red,Frames=100,LineWidth=0.5)
>>pg:=plot::Implicit2d(g(x,y)=0,x=-3..3,y=-3..3,Color=RGB::Blue,Line
Width=0.75)
>>plot(pf,pg,Scaling=Constrained);
```

The output is shown in the figure below

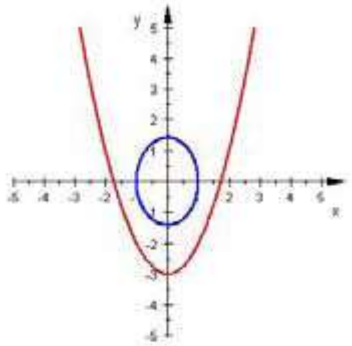

Figure 2: Output using CAS (Figure from reference [3])

When we animate the graph we see that there are four points at which the level curves of *f* and *g* have common tangents.[3]

VI. INTER-DISCIPLINE MATHEMATICS

Currently, efforts are being undertaken to facilitate collaborative research across traditional academic fields and to help train a new generation of interdisciplinary mathematicians and scientists. Also similar efforts are slowly being introduced in undergraduate and postgraduate mathematics curricula and pedagogy. Disciplines that hitherto hardly used mathematics in their curricula are now demanding substantial doses of knowledge of and skills in mathematics. For example the pre-requisites for mathematical knowledge and skills for entry in into biological and other life sciences as well as the mathematics content in the university curricula of these programmes is becoming quite substantial. Curricula for the social sciences programmes now include sophisticated mathematics over and above the traditional descriptive statistics. Curricula of some universities in the developed countries have inter-disciplinary programmes where mathematics students and students from other sciences (including social sciences) work jointly on projects. [4]

VII. CONCLUSION

From the above review, we can say that Math is utilized in a variety of ways within many fields of engineering, business, computer science, and industry. The most crucial aspects of math within this field involve cost–benefit–risk analysis and the use of mathematical models. "Mathematical models may include a set of rules and instructions that specifies precisely a series of steps to be taken, whether the steps are arithmetic, logical, or geometric.

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