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The theoretical modeling of decay instability of upper hybrid wave in a dusty plasma cylinder

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ABSTRACT

The effect of dust charge fluctuations (DCF) on the decay instability of an upper hybrid wave into an upper hybrid sideband and ion-cyclotron wave is examined in presence of positively charged dust grains in a magnetized plasma cylinder. The growth rate and mode frequencies of ion cyclotron mode calculated on existing dusty plasma parameters decreases with $\delta_u (= n_{io}/n_{eo}$*, where* n_{i0} *is the ion plasma density and* n_{e0} *is the electron plasma density) both in the presence and absence of dust charge fluctuations (DCF). However, the growth rate decreases sharply in absence of DCF i.e., positively charged dust grains increases the instability.*

Keywords: Dust grains, upper hybrid wave, growth rate & ion cyclotron wave.

I.INTRODUCTION

Upper-hybrid waves are the high frequency waves which have been studied both experimentally [1] and theoretically [2-5] in a wide variety of situation. The first experimental observations of upper hybrid waves in a laboratory plasma were reported by Idehara *et al*. [1]. The parametric instabilities involving large amplitude electrostatic [6-8] and electromagnetic waves [9] have been field of interesting research over the years.

Electrostatic waves in complex plasmas have been explored over the years [10-15]. Chow *et al*. [13-14] have studied the effect of dust charged fluctuations on the collisionless EIC instability using Vlasov theory. Sharma and Ajay [15] have studied the effect of dust charge fluctuations on the excitation of upper hybrid wave in a magnetized plasma cylinder. The dust has also been noted to influence a three-wave parametric process in unmagnetised plasmas [16-18] and magnetized plasma [19]. Konar *et al.* [20] have studied decay instability of an upper hybrid wave in a plasma cylinder without dust grains. In this paper, we study the effect of DCF on decay instability of an upper hybrid wave into an upper hybrid sideband wave and low frequency ion-cyclotron wave in a magnetized plasma cylinder.

II.DISPERSION RELATION

Consider a cylindrical dusty plasma column of radius r_0 immersed in a static magnetic field B_s parallel to z-axis. In the equilibrium the plasma column consists of electrons, ions and dust grains having densities, charge, mass and temperature (n_{e0}, -e, m_e, T_e), (n_{i0}, e, m_i, T_i) and (n_{d0},-Q_{d0}, m_d,T_d), respectively.

We assume the potentials of the three waves of the form

$$\phi_0 = \phi_0(r)\exp\left[-i\left(\omega_0 t - k_{0z}z - l\theta\right)\right],$$

$$\phi_1 = \phi_1(r)\exp[-i(\omega_1 t - k_{1z}z - l\theta)],$$

$$\phi = \phi(r)\exp[-i(\omega t - k_z z - l\theta)],$$

where $\omega_1 = \omega - \omega_0$, $k_{1z} = k_z - k_{0z}$ and 'l' is the azimuthal mode number.

The perturbed densities of electrons, ion and dust are given by

$$n_{e1} = \frac{n_{e0}e\left(\phi + \phi_p\right)}{T_e}, \quad \text{where} \tag{1}$$

$$\phi_p \approx -\frac{e}{2m_e\left(\omega_0^2 - \omega_{ce}^2\right)}\left(\nabla_\perp\phi_0.\nabla_\perp\phi_1 - \frac{i\left(\nabla\phi_0 \times \omega_{ce}\right).\nabla\phi_1}{\omega_0} - \frac{\left(\omega_0^2 - \omega_{ce}^2\right)}{\omega_0^2}k_{0z}k_{1z}\phi_0\phi_1\right). \tag{2}$$

$$n_{i1} = \frac{n_{i0}e}{m_i}\left[-\frac{\nabla_\perp^2\phi}{\omega^2 - \omega_{ci}^2} + \frac{k_z^2\phi}{\omega^2}\right], \tag{3}$$

where $\omega_{ci}\left(= \frac{eB_s}{m_i c}\right)$ is the ion cyclotron frequency.

$$n_{d1} = -\frac{n_{d0}Q_{d0}k^2\phi}{m_d\omega^2}. \tag{4}$$

We obtain dust charge fluctuations by following Jana *et al*. [11] as

$$Q_{d1} = \frac{\left|I_{e0}\right|e}{i(\omega + i\eta)}\left\{\frac{1}{m_i}\left[\frac{-\nabla_\perp^2\phi}{\left(\omega^2 - \omega_{ci}^2\right)} + \frac{k_z^2\phi}{\omega^2}\right] - \frac{\left(\phi + \phi_p\right)}{T_e}\right\}. \text{ where} \tag{5}$$

$\eta = 0.79a\left(\frac{\omega_{pi}}{\lambda_{Di}}\right)\left(\frac{1}{\delta_u}\right)\left(\frac{m_i}{m_e}\frac{T_i}{T_e}\right)^{\frac{1}{2}} \sim 10^{-2}\omega_{pe}\left(\frac{a}{\lambda_{De}}\right)\frac{1}{\delta_u}$ is the dust charging rate and $\delta_u = n_{i0}/n_{e0}$.

Substituting perturbed densities in the Poisson's equation $\nabla^2\phi = 4\pi(n_{e1}e - n_{i1}e + n_{d0}Q_{d1} + Q_{d0}n_{d1})$,

we obtain

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \left(p_d^2 - \frac{l^2}{r^2}\right)\phi = \frac{M}{L}\phi_p, \tag{6}$$

$$p_d^2 = \frac{N}{L}, \quad N = \frac{\omega_{pi}^2 k_z^2}{\omega^2} + \frac{i\beta\omega_{pi}^2}{(\omega + i\eta)}\frac{n_{e0}k_z^2}{n_{i0}\omega^2} - \frac{i\beta\omega_{pe}^2}{(\omega + i\eta)v_{te}^2} - \frac{\omega_{pe}^2}{v_{te}^2} - k_z^2,$$

$$M = \frac{\omega_{pe}^2}{v_{te}^2}\left(1 + \frac{i\beta}{\omega + i\eta}\right) \text{ and } L = 1 - \frac{\omega_{pi}^2}{\left(\omega^2 - \omega_{ci}^2\right)} - \frac{i\beta}{\left(\omega + i\eta\right)}\frac{\omega_{pi}^2}{\left(\omega^2 - \omega_{ci}^2\right)}\frac{n_{e0}}{n_{i0}} - \frac{\omega_{pd}^2}{\omega^2}.$$

$\omega_{pe}\left(=\sqrt{4\pi n_{e0}\,e^2/m_e}\right),\omega_{pi}\left(=\sqrt{4\pi n_{i0}\,e^2/m_i}\right)$ and $\omega_{pd}\left(=\sqrt{4\pi n_{d0}\,{Q_{do}}^2/m_d}\right)$ are the electron, ion and dust plasma frequencies, respectively and $\beta=\frac{|I_{e0}|}{e}\left(\frac{n_{d0}}{n_{e0}}\right)=0.397\left(1-\frac{1}{\delta_u}\right)\left(\frac{a}{v_{te}}\right)\omega_{pi}^2\left(\frac{m_i}{m_e}\right)$ is the coupling parameter.

Exactly proceeding in the same way for the sideband, we will obtain

$$\frac{\partial^2\phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_1}{\partial r}+\left(p_{1d}^2-\frac{l^2}{r^2}\right)\phi_1=\frac{\frac{\omega_{pe}^2 eB}{2\left(\omega_0^2-\omega_{ce}^2\right)T_e}\left(1+\frac{i\beta}{\omega+i\eta}\right)}{\left\{1-\frac{\omega_{pe}^2}{\left(\omega_1^2-\omega_{ce}^2\right)}\left[\left(1+\frac{i\beta}{(\omega+i\eta)}\right)\right]-\frac{\omega_{pd}^2}{\omega^2}\right\}} \tag{7}$$

where $B=\left[\phi\nabla_\perp^2\phi_0+\nabla_\perp\phi_0^*.\nabla_\perp\phi+\frac{\left(\omega_0^2-\omega_{ce}^2\right)k_{0z}k_{1z}\phi_0^*\phi}{\omega_0^2}+\frac{\omega_{ce}}{\omega_0}\frac{l}{r}\phi\frac{\partial\phi_0^*}{\partial r}\right]$

and
$$p_{1d}^2=\frac{\left[\frac{\omega_{pe}^2}{\omega_1^2}\left(1+\frac{i\beta}{(\omega+i\eta)}\right)-1\right]k_{1z}^2}{\left\{1-\frac{\omega_{pe}^2}{\left(\omega_1^2-\omega_{ce}^2\right)}\left[\left(1+\frac{i\beta}{(\omega+i\eta)}\right)\right]-\frac{\omega_{pd}^2}{\omega^2}\right\}}$$

In the absence of pump wave, the right hand side of Eqs. (6) and (7) is zero and has well known solutions

$\phi=\phi_q=\tau_q J_l(p_q r)$,

$\phi_1=\phi_{1p}=\tau_{1p}J_l(p_{1p}r)$, where $p_d^2=p_q^2$, $p_{1d}^2=p_{1p}^2$ and τ_q, τ_{1p} are the normalization constants. At r=r_0 , ϕ and ϕ_1 must vanish, hence $J_l(p_q r_0)$= $J_l(p_{1p}r_0)$=0, i.e., $p_q r_0$= $p_{1p}r_0$ =x_n [where x_n are the zeros of the Bessel function $J_l(x)$, n=1, 2, 3,…] .

In the presence of pump wave, the wave function ϕ and ϕ_1 can be expressed in terms of complete orthogonal sets of wave functions ϕ_q and ϕ_{1p} as

$$\phi=\sum_q C_q\phi_q \quad \text{and} \quad \phi_1=\sum_p D_p\phi_{1p}\,. \tag{8}$$

Substituting the value of ϕ and ϕ_1 from Eq. (8) in Eq. (7), multiplying both sides by ϕ_{1n}^*rdr and integrating over r from 0 to r_0 (where r_0 is the plasma radius), we obtain

$$(p_{1d}^2-p_{1n}^2)D_n=\eta_{1d}\sum_q C_q\int_0^{r_0}\left[\phi_q\nabla_\perp^2\phi_0+\nabla_\perp\phi_0^*.\nabla_\perp\phi_q+\frac{\left(\omega_0^2-\omega_{ce}^2\right)k_{0z}k_{1z}\phi_0^*\phi_q}{\omega_0^2}+\frac{\omega_{ce}}{\omega_0}\frac{l}{r}\phi_q\frac{\partial\phi_0^*}{\partial r}\right]\phi_{1n}^*rdr \tag{9}$$

where
$$\eta_{1d}=\frac{\frac{\omega_{pe}^2 e}{2\left(\omega_0^2-\omega_{ce}^2\right)T_e}\left(1+\frac{i\beta}{\omega+i\eta}\right)}{\left\{1-\frac{\omega_{pe}^2}{\left(\omega_1^2-\omega_{ce}^2\right)}\left[\left(1+\frac{i\beta}{(\omega+i\eta)}\right)\right]-\frac{\omega_{pd}^2}{\omega^2}\right\}}\,. \tag{10}$$

Similarly, we get

$$(p_d^2 - p_m^2)C_m = \eta_{2d}\sum_p D_p \int_0^{r_0}\left[\nabla_\perp\phi_0.\nabla_\perp\phi_{1p} - \frac{\left(\omega_0^2-\omega_{ce}^2\right)k_{0z}k_{1z}\phi_0\phi_{1p}}{\omega_0^2} - \frac{\omega_{ce}}{\omega_0}\frac{l}{r}\phi_{1p}\frac{\partial\phi_0}{\partial r}\right]\phi_m^* r dr\ , \tag{11}$$

where

$$\eta_{2d} = -\frac{\dfrac{\omega_{pe}^2 e}{2\left(\omega_0^2-\omega_{ce}^2\right)T_e}\left(1+\dfrac{i\beta}{\omega+i\eta}\right)}{\left\{1-\dfrac{\omega_{pi}^2}{\left(\omega^2-\omega_{ci}^2\right)}\left[\left(1+\dfrac{i\beta n_{e0}}{(\omega+i\eta)n_{i0}}\right)\right]-\dfrac{\omega_{pd}^2}{\omega^2}\right\}}. \tag{12}$$

We assume that only the n[th] mode of the sideband and m[th] mode of the low frequency wave interact resonantly with the zero order radial mode of the pump. With this assumption, we obtain a dispersion relation from Eqs. (9) and (11)

$$(p_{1d}^2 - p_{1n}^2)(p_d^2 - p_m^2) = \eta_{1d}\eta_{2d} I_1 I_2\ , \tag{13}$$

where $I_1 = \int_0^{r_0}\left[\phi_m\nabla_\perp^2\phi_0 + \nabla_\perp\phi_0^*.\nabla_\perp\phi_m + \frac{\left(\omega_0^2-\omega_{ce}^2\right)k_{0z}k_{1z}\phi_0^*\phi_m}{\omega_0^2} + \frac{\omega_{ce}}{\omega_0}\frac{l}{r}\phi_m\frac{\partial\phi_0^*}{\partial r}\right]\phi_{1n}^* r dr,$

$$I_2 = \int_0^{r_0}\left[\nabla_\perp\phi_0.\nabla_\perp\phi_{1n} - \frac{\left(\omega_0^2-\omega_{ce}^2\right)k_{0z}k_{1z}\phi_0\phi_{1n}}{\omega_0^2} - \frac{\omega_{ce}}{\omega_0}\frac{l}{r}\phi_{1n}\frac{\partial\phi_0}{\partial r}\right]\phi_m^* r dr\cdot$$

Equation (13), when equated to zero, gives linear dispersion relation of two daughter waves. Let the value of ω, which makes these factors simultaneously zero be ω_r. Then, in the presence of the pump wave, we can expand ω as $\omega = \omega_r + i\gamma$ and $\omega_1 = \omega_{1r} + i\gamma$. Writing p_d^2 and p_{1d}^2 as $p_d^2 = p_m^2 + \frac{2\omega_r}{C_s^2}i\gamma$, $p_{1d}^2 = p_{1n}^2 + i\gamma Y_P$ where

$$C_s^2 = \frac{T_e}{m_i},\ Y_P = \frac{Hk_{1z}^2}{\omega_r},\ H = \sqrt{M_1^2 + M_2^2},\ M_1 = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2},\ M_2 = \frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}$$

$$X_1 = \frac{\omega_{pe}^2}{\omega_1^2}\left[1 + \frac{\beta\eta}{\left(\omega_r^2 + \eta^2\right)}\right] - 1,\ X_2 = 1 - \frac{\omega_{pe}^2}{(\omega_1^2 - \omega_{ce}^2)} - \frac{\omega_{pd}^2}{\omega_r^2} - \frac{\omega_{pe}^2\beta\eta}{(\omega_1^2 - \omega_{ce}^2)\left(\omega_r^2 + \eta^2\right)},$$

$$Y_1 = \frac{\omega_{pe}^2\beta\omega_r}{\omega_1^2\left(\omega_r^2 + \eta^2\right)},\ Y_2 = \frac{-\omega_{pe}^2\beta\omega_r}{(\omega_1^2 - \omega_{ce}^2)\left(\omega_r^2 + \eta^2\right)}.$$

Equation (13), gives the growth rate for the resonant decay into upper hybrid and ion-cyclotron wave as

$$\gamma = \left[\frac{C_s^2\eta_{1d}\eta_{3d}I_1I_2}{2\omega_r Y_P}\right]^{1/2},\ \text{where}\ \eta_{3d} = -\eta_{2d}\ , \tag{14}$$

$$\omega_r = \left\{\omega_{ci}^2 + C_s^2\left[p_d^2 + \frac{\left(p_{1d} + p_0\right)^2}{A_d^2}\right]\right\}^{1/2}, \tag{15}$$

$A_d^2 = \sqrt{N_1^2 + N_2^2}$, $N_1 = \dfrac{S_1S_2 + T_1T_2}{S_2^2 + T_2^2}$, $N_2 = \dfrac{S_2T_1 - S_1T_2}{S_2^2 + T_2^2}$, $S_1 = \dfrac{\omega_{pe}^2}{\omega_1^2}\left[1 + \dfrac{\beta}{2\eta}\right] - 1$,

$$S_2 = 1 - \frac{\omega_{pe}^2}{(\omega_1^2 - \omega_{ce}^2)} - \frac{\omega_{pd}^2}{\eta^2} - \frac{\omega_{pe}^2\beta}{(\omega_1^2 - \omega_{ce}^2)2\eta},\ T_1 = \frac{\omega_{pe}^2\beta}{2\omega_1^2\eta},\ T_2 = -\frac{\omega_{pe}^2\beta}{2\eta(\omega_1^2 - \omega_{ce}^2)}.$$

Now, we consider two cases of interest

Case I: In the presence of dust charge fluctuations, i.e., dust charging rate η is finite.

Case II: In the absence of dust charge fluctuations, i.e., $Q_{d_1} = 0$ when dust charging rate $\eta \to \infty$.

In the absence of dust grains i.e., δ_u=1and β=0, we recover expressions of Konar etal [20] (cf. page 4613and 4614). Dust grain is negatively charged for $\delta_u > 1$ and positively charged for $\delta_u < 1$.

III. RESULTS AND DISCUSSIONS

The numerical values of the real frequency and growth rate of the unstable ion-cyclotron wave are estimated using the following typical dusty plasma parameters: : n_{i0}= 10^{11} cm^{-3}, Te=Ti=0.2eV , B_s=1.0x10^3G, n_{d0}= 10^4 cm^{-3}, $\omega_0 \approx \omega_1 = 10^{10} rad./\sec$, plasma radius r_0=1.0cm, $m_i/m_e \approx 7.16x10^4$ (Potassium), a=1μm, mode number n=1, i.e., the first zero of the Bessel function (x_1=3.8). We vary δ_u from 0.2 to 0.99 for positively charged dust grains.

Using Eq. (15) we have plotted in Fig.1 the real frequency ω_r of the unstable drift waves as a function of $\delta_u = n_{io}/n_{eo}$. In Fig.1 it is seen that the wave frequency ω_r increases by a factor ~1.85 when δ_u changes from 0.2 to 0.8 if dust charge fluctuations are taken into account, and by a factor ~2.0 in the absence of dust charge fluctuations under the plasma parameters listed above. Barkan *et al.* [8] have found that the wave frequency was about 10-20% larger than the ion-cyclotron frequency in the presence of negatively charged dust grains. Chow and Rosenberg [9] have shown, in their kinetic analysis on the effect of negatively charged dust grains on the collisionless electrostatic ion cyclotron instability, the wave frequency ω_r/ω_{ci} increases about 11% when δ_u is changed from 1 to 4 under similar conditions. Thus the impact of positively charged grains is opposite of negatively charged dust grains in plasma.

In Fig.2, we have plotted the growth rate γ obtained from Eq. (14) as a function of δ_u for the same parameters as those used in Fig.1. From Fig.2 it can be seen that the growth rate γ decreases with δ_u in both cases. However, the decrease is much sharper in the absence of dust charge fluctuations. Thus dust charge fluctuations reduces the damping effect and make a wave more unstable in presence of positively charged dust grains in plasma in this three wave interaction process.

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FIGURE CAPTIONS

FIG.1.Real part of the frequency ω_r (in rad/sec) of the unstable ion cyclotron wave as a function of δ_u [with and without dust charge fluctuations].

FIG.2.Growth rate γ (in rad/sec) of the unstable ion cyclotron wave as a function of δ_u [with and without dust charge fluctuations].

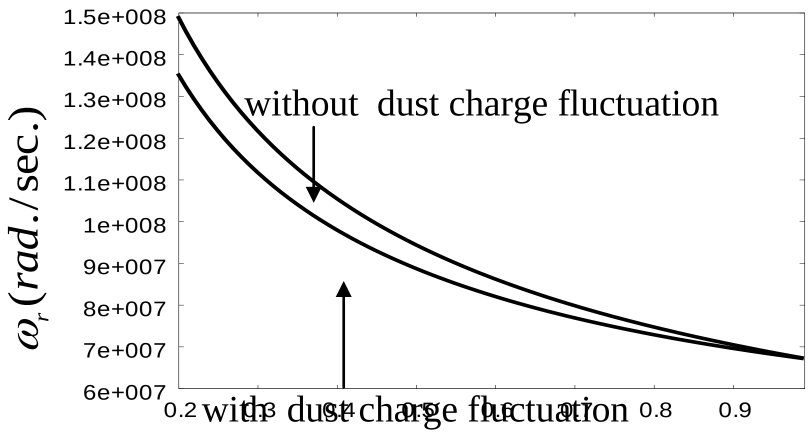

FIG.1

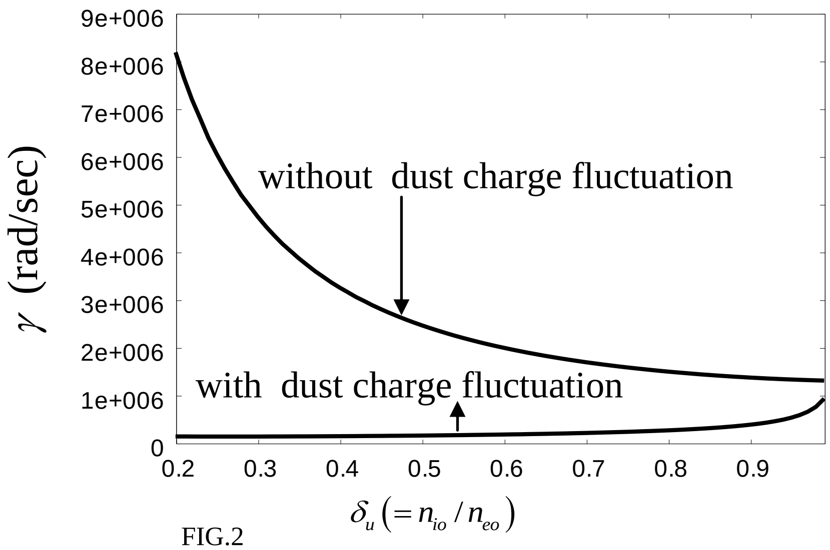
9e+006
8e+006
7e+006
6e+006
5e+006
4e+006
3e+006
2e+006
1e+006
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
γ (rad/sec)
without dust charge fluctuation
with dust charge fluctuation
$\delta_u \left(= n_{io} / n_{eo}\right)$

FIG.2