

Extended Kalman Filter Based Sensorless Speed Control of Permanent Magnet Synchronous Motor Drive

Jallu Hareesh kumar¹, P Balaraju², Patina Padma³

¹Assistant Professor, Dept. Of EEE, SVCET, Srikakulam, Andhra Pradesh, India-532410

²PG Scholar, Dept. Of EEE, SSCE, Srikakulam, Andhra Pradesh, India-532402

³PG Scholar, Dept. Of EEE, SVCET, Srikakulam, Andhra Pradesh, India-532410

ABSTRACT

This paper proposes the Extended Kalman Filter based sensorless speed control of permanent magnet synchronous motor. The EKF is used to achieve a precise estimation of the rotor speed and dq-axes currents from noisy measurement. The EKF estimated parameters are greatly influenced by error covariance matrices Q & R. The accurate estimated speed is obtained by selecting optimum values of Q & R and which is fed back to PI controller. By selecting the optimum values of K_p & K_i to minimize the speed error and also improves the settling time, overall reliability of the system. The simulation results show that the covariance matrices Q & R improve the convergence of estimation process and quality of the system.

Keywords: *Extended Kalman Filter, Gradient matrix, Permanent Magnet Synchronous Motor, PI controller.*

I. INTRODUCTION

PMSM has wide range of applications compared to remaining industrial drives due to their compactness, superior power density, high torque density, high efficiency, high power factor and their low maintenance cost. Rotor Losses are eliminated due to the absence of slip rings for field excitation which make them ideal for robotic and automatic production systems. The position sensing which is required for the vector control of PMSM drive increases the cost of the system along with its space. Speed/position sensorless control [7][8] of these motor driven systems reduces the system complexity, weight and cost and improves the overall system reliability and dynamic performance. In order to obtain sensorless control various control algorithms like reduced order & full order observers [11][12], sliding mode observers [13], back emf estimation [14], Extended Kalman Filter [14][15][16], Model Reference Adaptive System [17], Artificial Neural Network [18], Fuzzy logic [19] are proposed.

Among the proposed algorithms EKF is one of the promising observers, which offers best possible filtering of the noise in measurement and of the system if the noise covariances are known. EKF is a recursive predictive filter, which provides accurate and quick estimation of variables from all the available measurements regardless of their precision with rapid convergence and also improves overall reliability of the system. If rotor speed

considered as an extended state and is incorporated in the dynamic model of a PMSM, Then the EKF can be used to relinearize the nonlinear for each new value of estimate. As a result, EKF is the best method for estimation of rotor speed, whose performance is influenced by the values of system parameters and error covariance matrices Q & R . The investigation shows that the EKF is capable of tracking the actual (d & q) stator currents and rotor speed, is provided that the elements of the covariance matrices are properly selected.

II. MATHEMATICAL MODEL OF PERMANENT MAGNET SYNCHRONOUS MOTOR

The voltage equations for a PMSM in the rotor reference frame [1][2] can be expressed as [16].

$$V_d = R_s i_d + L_d p i_d - \omega_e L_q i_q \quad (1)$$

$$V_q = R_s i_q + L_q p i_q + \omega_e L_d i_d + \omega_e \psi_f \quad (2)$$

Where

V_d is d-axis stator voltage, V_q is q-axis stator voltage, i_d is d-axis stator current, i_q is q-axis stator current, R_s is stator phase resistance, L_d is d-axis stator inductance, L_q is q-axis stator inductance, ψ_f is Rotor flux, ω_e is Electrical speed in rad/sec.

The electromagnetic torque of PMSM is described as:

$$T_e = \left[\frac{3}{2} P_n i_q (\psi_f - (L_q - L_d) i_d) \right] \quad (3)$$

Where P_n = number of pole pairs

The motion equation is expressed as follows as

$$J \frac{d\omega_r}{dt} + B\omega_r + T_l = T_e \quad (4)$$

Where J is moment of inertia, B is friction coefficient, T_l is load torque.

Finally the above equations are mentioned in state space form as follows

$$\frac{di_d}{dt} = \frac{1}{L_d} [-R_s i_d + \omega_e L_q i_q] + \frac{V_d}{L_d} \quad (5)$$

$$\frac{di_q}{dt} = \frac{1}{L_q} [-R_s i_q - \omega_e L_d i_d - \omega_e \psi_f] + \frac{V_q}{L_q} \quad (6)$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} \left[\frac{3}{2} P_n i_q (\psi_f - (L_q - L_d) i_d) \right] - \frac{B}{J} \omega_r - \frac{T_l}{J} \quad (7)$$

IV. DESIGN OF EXTENDED KALMAN FILTER

Kalman filter is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms. It estimates the state of a dynamic system. In Kalman filter, effect of noise of both system and surroundings are taken into account using covariance matrices which are updated in each iteration, thus providing estimation and filtering. This dynamic system can be distributed by some noise, mostly assumed as white noise. So that the error covariance of the estimator is minimized. In this sense it is an optimal estimator. This procedure is repeated for each time step with the state of the previous time step as initial value. Therefore the Kalman filter is called a recursive filter. Drawback of Kalman filter is that for non-linear systems, the calculation is time consuming. For the implementation of nonlinear systems, these are functions of the state and consequently change with every time step, iteration and cannot be pre-computed. Shortcomings of this model can be overcome by using Extended Kalman filter (EKF) [19] [20]. EKF is stochastic in nature and is well suited to non-linear systems; the noise in the system gets reduced. Due to its rapid delivery, precise and accurate estimation, it is used in research and applications. The feedback gain used in EKF achieves quick convergence and provides stability for the observer.

$$\dot{x} = f(x, u, w) \quad (9)$$

$$y = h(x, u, v) \quad (10)$$

To derive the discrete time EKF algorithm, to initiate the basic definition of time deviation of a state variable x

$$\dot{x} = \frac{x(k) - x(k-1)}{T_s} \quad (11)$$

$$x(k) = \dot{x}T_s + x(k-1) \quad (12)$$

Replace equation (10) in (13)

$$x(k) = x(k-1) + T_s f(x, u, w) \quad (13)$$

Rearrange the above equation in discrete time system equations

$$x_k = f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \quad (14)$$

Where $f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) = A_d x_{k-1} + B_d u_{k-1} + w_{k-1}$

$$A_d = I + AT_s$$

$$B_d = BT_s$$

$$y_k = h_k(x_k, u_k, v_k) \quad (15)$$

Where $h_k(x_k, u_k, v_k) = C_d x_k + v_k$

$$C_d = C$$

Where x_k is system state vector, u_k is input vector (known), w_k is process noise, v_k is measurement noise, T_s is sampling time.

4.1 EKF Algorithm

Step 1: Initialize the state vector and covariance matrices $x(0)$, P , Q , R

Step 2: compute the Jacobian matrices for f_{k-1} , h_k

$$F = \frac{\partial f_{k-1}}{\partial x} \quad (16)$$

$$H = \frac{\partial h_k}{\partial x} \quad (17)$$

Where $f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) = A_d x_{k-1} + B_d u_{k-1} + w_{k-1}$

$$h_k(x_k, u_k, v_k) = C_d x_k + v_k$$

Step 3: Prediction state (time update)

To perform the time updating of state estimate and estimation of error covariance

$$X = f_{k-1}(x_{k-1}, u_{k-1}, 0)$$

$$X = f_{k-1}(x_{k-1}, u_{k-1}) + x(0) \quad (18)$$

$$P_1 = FPF^T + Q \quad (19)$$

Step 4: Correction state (measurement update)

To perform the measurement updating of state estimate and estimation of error covariance by using Kalman gain

$$\text{Calculation of Kalman gain matrix } K = P_1 H^T / (H P_1 H^T + R) \quad (20)$$

$$\text{Update state prediction } X_1 = X + K(y - y_1) \quad (21)$$

Where $y_1 = Hx_{k-1}$

$$\text{Estimation of error covariance matrix } P = (I - KH)P_1 \quad (22)$$

4.2 EKF estimation for PMSM drive

The dynamic state equations of PMSM are

$$\frac{di_d}{dt} = \frac{1}{L_d} \left[-R_s i_d + P_n \omega_r L_q i_q \right] + \frac{V_d}{L_d}$$

$$\frac{di_q}{dt} = \frac{1}{L_q} \left[-R_s i_q - P_n \omega_r L_d i_d - P_n \omega_r \psi_f \right] + \frac{V_q}{L_q}$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} \left[\frac{3}{2} P_n i_q (\psi_f - (L_q - L_d) i_d) \right] - \frac{B}{J} \omega_r - \frac{T_l}{J}$$

The above equations written in the form of state space

System equation $\dot{x} = Ax + Bu$

Observation equation $y = Cx$

$$\begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \\ \frac{d\omega_r}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L_d} & \frac{P_n \omega_r L_q}{L_d} & 0 \\ \frac{-P_n \omega_r L_d}{L_q} & \frac{-R_s}{L_q} & \frac{-P_n \psi_f}{L_q} \\ \frac{-3P_n (L_q - L_d) i_q}{2J} & \frac{3P_n \psi_f}{2J} & \frac{-B}{J} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

Where $A = \begin{bmatrix} \frac{-R_s}{L_d} & \frac{P_n \omega_r L_q}{L_d} & 0 \\ \frac{-P_n \omega_r L_d}{L_q} & \frac{-R_s}{L_q} & \frac{-P_n \psi_f}{L_q} \\ \frac{-3P_n (L_q - L_d) i_q}{2J} & \frac{3P_n \psi_f}{2J} & \frac{-B}{J} \end{bmatrix}$, $B = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix}$, $x = \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix}$ & $u = \begin{bmatrix} V_d \\ V_q \end{bmatrix}$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix}$$

Where $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

The above equations written in the form of discretization

System equation $x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$

Observation equation $y_k = Cx_k + v_k$

$$x_k = f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1})$$

$$f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) = A_d x_{k-1} + B_d u_{k-1} + w_{k-1}$$

$$A_d = I + AT_s$$

$$B_d = BT_s$$

$$f(x_{k-1}, u_{k-1}, w_k) = \begin{bmatrix} 1 - \frac{R_s}{L_d} T_s & \frac{P_n \omega_r L_q T_s}{L_d} & 0 \\ \frac{-P_n \omega_r L_d T_s}{L_q} & 1 - \frac{R_s}{L_q} T_s & \frac{-P_n \psi_f T_s}{L_q} \\ \frac{-3P_n (L_q - L_d) i_q}{2J} T_s & \frac{3P_n \psi_f T_s}{2J} & 1 - \frac{B}{J} T_s \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} T_s & 0 \\ 0 & \frac{1}{L_q} T_s \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

Gradient matrix $F = \frac{\partial f_{K-1}}{\partial x}$

$$F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$$

$$F_{11} = \frac{\partial f_{K-1}}{\partial x_1} = 1 - \frac{R_s}{L_d} T_s, \quad F_{12} = \frac{\partial f_{K-1}}{\partial x_2} = \frac{P_n \omega_r L_q T_s}{L_d}, \quad F_{21} = \frac{\partial f_{K-1}}{\partial x_1} = \frac{-P_n \omega_r L_d T_s}{L_q}$$

$$F_{22} = \frac{\partial f_{K-1}}{\partial x_2} = 1 - \frac{R_s}{L_q} T_s, \quad F_{23} = \frac{\partial f_{K-1}}{\partial x_3} = \frac{-P_n \psi_f T_s}{L_q}, \quad F_{31} = \frac{\partial f_{K-1}}{\partial x_1} = \frac{-3P_n (L_q - L_d) i_q}{2J} T_s,$$

$$F_{32} = \frac{\partial f_{K-1}}{\partial x_2} = \frac{3P_n \psi_f T_s}{2J}, \quad F_{33} = \frac{\partial f_{K-1}}{\partial x_3} = 1 - \frac{B}{J} T_s.$$

$$F = \begin{bmatrix} 1 - \frac{R_s}{L_d} T_s & \frac{P_n \omega_r L_q T_s}{L_d} & 0 \\ \frac{-P_n \omega_r L_d T_s}{L_q} & 1 - \frac{R_s}{L_q} T_s & \frac{-P_n \psi_f T_s}{L_q} \\ \frac{-3P_n (L_q - L_d) i_q}{2J} T_s & \frac{3P_n \psi_f T_s}{2J} & 1 - \frac{B}{J} T_s \end{bmatrix}$$

$$y_k = h_k(x_k, u_k, v_k)$$

$$h_k(x_k, u_k, v_k) = C_d x_k + v_k$$

Where $C_d = C$

$$H = \frac{\partial h_K}{\partial x}, \quad H_{11} = \frac{\partial h_K}{\partial x_1} = 1, \quad H_{21} = \frac{\partial h_K}{\partial x_2} = 1$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

V RESULTS AND DISCUSSIONS

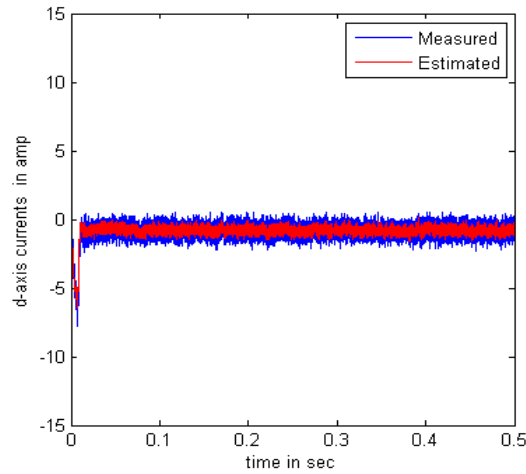


Fig 2: Measured & estimated waveforms of d-axis current i_d

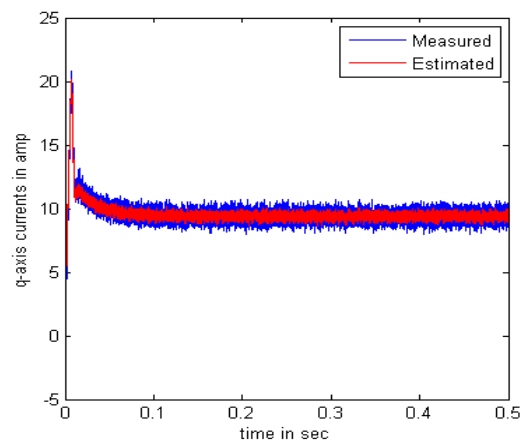


Fig 3: Measured & Estimated waveforms of q-axis current i_q

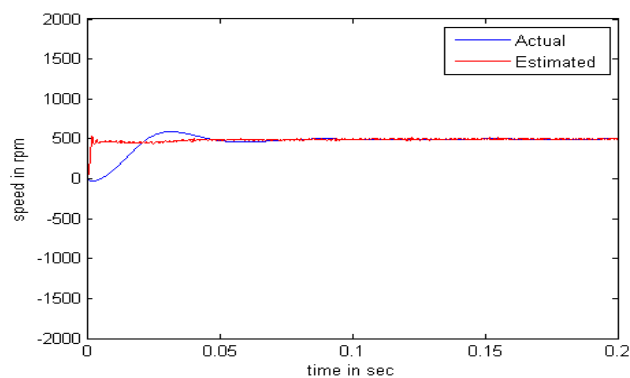


Fig 4: Actual & Estimated waveforms of rotor speed

In the simulation i_d, i_q, v_d, v_q are input variables of EKF algorithm and $\hat{i}_d, \hat{i}_q, \omega_r$ are the estimated state variables. In order to imitate the condition of real system Gaussian white noises are added to feedback values of $\hat{i}_d, \hat{i}_q$. The simulation model has the same noise in the current as the real system, if the power is set to 3×10^{-6} and sample time of the white noise block is set to 2×10^{-5} sec. The choice of elements of the covariance matrices P, Q and R is the important step in the design of Kalman filter because of its effect on the performance, convergence and stability of the system. The initial state error covariance matrix (P) is a diagonal matrix, which will cause the initial disturbances due to choosing of random values. But when the algorithm converges, it disappears the effect of matrix P. The High model noise or parameter uncertainties indicates the higher value of Q which tends in the increase of Kalman gain, resulting in faster filter dynamics but, it leads to poorer steady-state performance. Measurement noise depends upon the matrix R. whenever there is a increase in the value of the elements of R, Current measurements are more effected by noise and thus less reliable. This results in the decrease of filter gain, yielding poorer transient response.

MATLAB Simulated values of error covariance matrices Q and R are given by:

$$Q = \text{diagonal of } [5.108 \quad 8.753 \quad 8.553]$$

$$R = \text{diagonal of } [7.963 \quad 8.491]$$

The measured and estimated waveforms of i_d and i_q are shown in figure 2 & 3. The estimated values of currents having large deviations due to convergence problem of state error covariance matrix at the beginning. After 0.05 sec both the state variables are converge to actual values.

The performance of EKF becomes a part of speed control, while the estimated speed as feedback. Therefore EKF runs in closed loop condition, but the parameters of speed controller remain unchanged. In figure 4 the actual and estimated speeds are compared and the reference speed is given as 500 rpm. The estimated speed is tracks the actual speed at 0.001 sec only, so estimated speed is quickly converges due to precise values of matrices Q & R. When estimated speed as feedback, speed error is given to the PI controller which is reduce the error & settling time and also improves the steady state performance of the system.

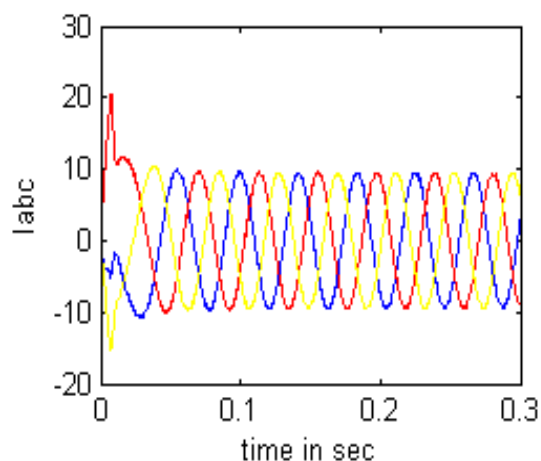


Fig 5: I_{abc} current waveform

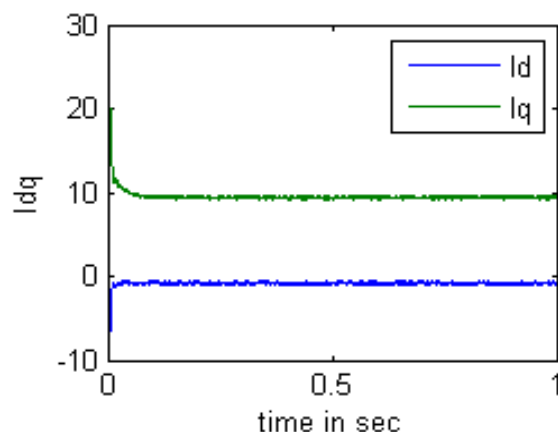


Fig 6: I_{dq} current waveforms

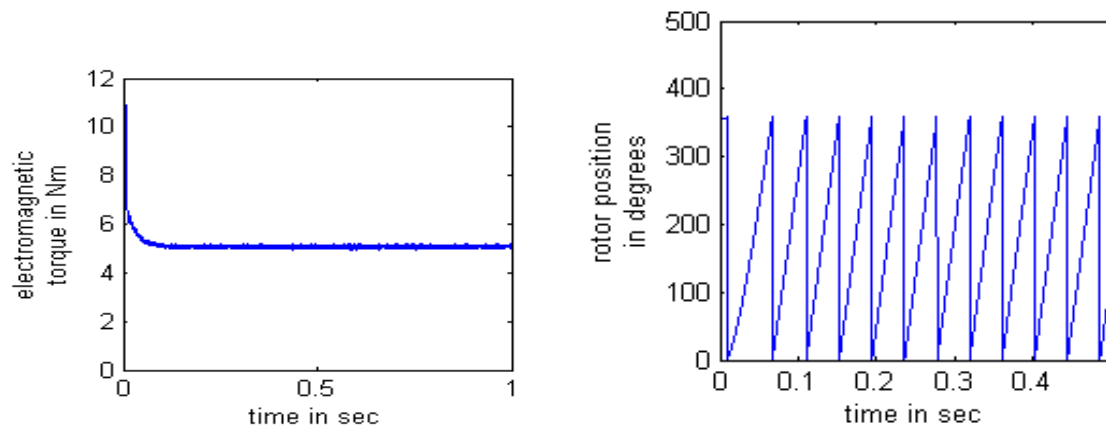


Fig 7. Electro magnetic torque waveform Fig 8: Rotor position in degrees

Figure 5 & 6 shows the stator currents of abc and dq axes respectively. These currents having large ripples at the beginning due to high speed error is given to PI controller. At 0.02 sec these currents are settled. Figure 7 shows the electromagnetic torque of PMSM drive, which settles at 0.1 sec. From Figure 8, it is clear that rotor position starts from 0.01 sec.

V. CONCLUSION

The intent of this paper, EKF based sensorless speed control of PMSM drive has been presented to show the results of estimated values of speed and dq-axes stator currents. The performance of EKF mainly depends on error covariance matrices Q & R , which are suitably selected. These matrices improved the system convergence and quality of estimation. The simulation results show the superior performance in terms of settling time, reduction of noise and overall system stability.

Appendix A:

Simulation Parameters values of PMSM drive:

S.NO	Parameters used in PMSM	Symbol	Numerical value
1	Resistance of stator	R_s	0.675 ohm
2	Direct inductance of stator	L_d	0.0085 Henry
3	Quadrature inductance of stator	L_q	0.0085 Henry
4	Flux linkages	ψ_f	0.12 Weber
5	Inertia of rotor	J	0.0011 Kg/m ²
6	Viscous friction coefficient	B	0.0014 Nm/s ²
7	Pair of poles	P_n	3
8	Rated speed	ω	1000 rpm

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