

Applications of Semi groups

Kasam Vijay Kumar¹, Dr. Sonal Bharti²

¹Research Scholar,

Sri Satya Sai University of Technology and Medical Sciences, (India)

ABSTRACT

This section deals with the applications of semi groups in general and regular semi groups in particular. The theory of semi groups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In which we have seen different areas of applications of semi groups. We identified some examples in biology, sociology etc. whose semi group structures are nothing but regular, E-inversive and inverse semi group etc.

I.INTRODUCTION

The concept of a semigroup is relatively young, the first, often fragmentary, studies were carried out early in the twentieth century. Then the necessity of studying general transformations, rather than only invertible transformations (which played a large role in the development of group theory) became clear. During the past few decades connection in the theory of semigroups and the theory of machines became of increasing importance, both theories enriching each other. In association with the study of machines and automata, other areas of applications such as formal languages and the software use the language of modern algebra in terms of Boolean algebra, semigroups and others. But also parts of other areas, such as biology, psychology, biochemistry and sociology make use of semigroups.

The theory of automata has its origins in the work by Turing (Shannon 1948, and Herken 1994.). Turing developed the theoretical concept of what is now called Turing machines, in order to give computability a more concrete and precise meaning. Hannon investigated the analysis and synthesis of electrical contact circuits using switching algebra. The work of McCulloch and Pitts centers on neuron models to explain brain functions and neural networks by using finite automata. Their work was continued by Kleene. The development of technology in the areas of electromechanical and machines and particularly computers had a great influence on automata theory which traces back to the mid-1950s. Many different parts of pure mathematics are used as tools such as abstract algebra, universal algebra, lattice theory, category theory, graph theory, mathematical logic and the theory of algorithms. In turn automata theory can be used in economics, linguistics and learning processes.

The beginning of the study of formal languages can be traced to Chomsky, who introduced the concept of a context-free language in order to model natural languages in 1957. Since then late 1960 there has been considerable activity in the theoretical development of context-free languages both in connection with natural languages and with the programming languages. Chomsky used semi-thue systems to define languages, which can be described as certain subsets of finitely generated free monoids. Chomsky (1957) details a revised approach in the light of experimental evidence and careful consideration of semantic and syntactic structures of sentences. For a common approach to formal languages and the theory of automata we refer to Eilenberg (1974).

Semigroups can be used in biology to describe certain aspects in the crossing of organisms, in genetics and in consideration of metabolisms. The growth of plants can be described algebraically in Hermann and Rosenberg (1975). Further material on this subject is contained in Holcombe (1982). Details on the use of semiautomata in metabolic pathways and the aid of a computer therein, including a theory of scientific experiments, can be found in Krohn, Langer and Rhodes (1976). Rosen (1973) studies ways in which environmental changes can affect the repair capacity of biological systems and considers carcinogenesis and reversibility problems. Language theory is used in cell – development problems, as

introduced by Lindenmayer (1968), Hermann and Rosendal (1975). Suppes (1969) Kiesras (1976) develop a theory of learning in which a subject is instructed to behave like a semiautomaton. The study of kinship goes back to a study by A. Weil in response to an inquiry by the anthropologist C. Levi-Strauss in 1949. White (1963), Kim and Breiger (1979) and Breiger, Boorman and Sable (1975) are also developed the elementary structure of kinship. Ballonoff (1974) presents several fundamental papers on kinship. Carls (1980) gives elementary examples of applications of groups in anthropology and sociology. Rudolf Lidl and Günter Pilz were started with a selected set X of basic relations such that the set of all their relation products yields all remaining kinship relations. In this way they arrive at the concept of a free (hence infinite) semigroups over X . Sociology includes the study of human interactive behavior in group situations, in particular, in underlying structures of societies. Such structures can be revealed by mathematical analysis. This indicates how algebraic techniques may be introduced into studies of this kind.

II.SEMIGROUPS AND ITS APPLICATIONS

Now a days the theory of semigroups has been expanded greatly due to its applications to computer science and we also find its usage in biological science and sociology. In this section we discuss some applications of semigroups in different areas.

a) Semigroups –Automaton

The algebraic theory of automata, which uses algebraic concepts to formalize and study certain types of finite-state machines. One of the main algebraic tools used to do this is the theory of semigroups. Automaton is an abstract model of computing device. Using this models different types of problems can be solved. We discuss what is common to all automata by describing an abstract model will be amenable to mathematical treatment and see that there is a close relationship between automata and semigroup. We can establish a correspondence between automata and monoids.

The problem may be identifying or adding two integers etc. i.e., we will be encountering automata in several forms such as calculating machines, computers, money changing devices, telephone switch boards and elevator or left switching's. All the above

have one aspect in common namely a "box" which can assume various states. These states can be transformed into other states by outside influence and process "outputs" like results of computations.

Semi automata: A semi automaton is a triple $Y = (Z, A, \delta)$ consisting of two non empty sets Z and A and a function $\delta : Z \times A \rightarrow Z$. Z is called the set of states, A is the set of input alphabet and the “next – state function” of Y .

Automata: An automaton is a quintuple $\check{A} = (Z, A, B, \delta, \omega)$ where (Z, A, δ) is a semi automaton, B is a non empty set called the output alphabet and $\omega : Z \times A \rightarrow B$ is the “output function.”

If $z \in Z$ and $a \in A$ then we interpret $(z, a) \in Z$ as the next state into which z is transformed by the input a . $(z, a) \in B$ is the output of z resulting from the input a . Thus if the automaton is in the stage z and receives input a , then it changes to state (z, a) with out put (z, a) . A(semi)-automaton is finite, if all the sets Z, A and B are finite, finite automata are also called mealy automata.

Description by graphs

We depict $z_1, z_2, \dots, z_k$ as “discs” in the plane and draw an arrow labeled a_i from z_r to z_s , if $(z_r, a_i) = z_s$. In case of an automaton we denote the arrow also by (z_r, a_i) . This graph is called the state graph

Example: (marriage Automation)

Let us consider the following situation in a household . The husband is angry or bored or happy: the wife is quite or shouts or cooks his favorite dish. Silence on her part does not change the husband’s mood, shouting “lowers” it by one “degree “(if he is already angry, then no change), cooking of his favorite dish creates general happiness for him. We try to describe this situation in terms of a semi-automaton $Y = (Z, A, \delta)$. We define $Z = (z_1, z_2, z_3)$ and $A = a_1, a_2, a_3$ with For this situation add the output $B = b_1, b_2$ with the interpretation

$b_1 =$

husband shouts, $b_2 =$ husband quite. Let us assume that the husband is only shouts if he is angry and his wife shouts. Otherwise he is quite even in state z_3 . We shall define the output function by using the following output table.

	a ₁	a ₂	a ₃
z ₁	b ₂	b ₁	b ₂
z ₂	b ₂	b ₂	b ₂

Let us consider the semi automaton given in above marriage automata. First we construct the table for $f, f_{a1}, f_{a2}, f_{a3}$ indicating their actions on the states z_1, z_2, z_3 .

	f	f_{a1}	f_{a2}	f_{a3}
z ₁	z ₁	z ₁	z ₁	z ₃
z ₂	z ₂	z ₂	z ₁	z ₃
z ₃	z ₃	z ₃	z ₂	z ₃

Semigroups in Biology

Semigroup can be used in biology to describe certain aspects in the crossing of organisms, in genetics, and in consideration of metabolisms.

Sociology includes the study of human interactive behavior in group situations, in particular in underlying structures of societies. Such structures can be revealed by mathematical analysis. This indicates how algebraic techniques may be introduced into studies of this kind. So the study of such relations can be elegantly formulated in the language of semigroups. Definition: $(R(M), o)$ is called the relation semigroup on M . The operation o is called the relation product, where M is a monoid.

It is obvious that $RR(M)$ is transitive if and only if $R \circ R \subseteq R$. The set of their relation products yields all remaining kinship relations. In this way they arrive at the concept of a free semigroup over X . But there are only finite people on the earth and some kinship relations like “daughter of a mother” and “daughter of a father” might be considered to be “the same”.

Definition: A kinship system is a semigroup $S = X, R$ where R is a relation on X , which express equality of kinship relationships.

This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc.

In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc. For consider the following

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