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## AC-DC Based Optimal Spot Pricing of Electricity in Restructured Electricity Market

Miss. Snehal N. Saonerkar<sup>1</sup>, Dr. S. B. Warkad<sup>2</sup>

<sup>1</sup> Department of Electrical Engg.,
P.R. Pote (Patil) College of Engg., & Mgmt, Amravati (India)

<sup>2</sup> Professor, Department of Electrical Engg.,
P.R. Pote (Patil) College of Engg., & Mgmt, Amravati (India)

#### **ABSTRACT**

In developing countries, recent trend is to adopt High Voltage Direct Current (HVDC) transmission in the existing AC transmission system to gain its techno-economical benefits. In restructured electricity market, accurate prediction of electricity spot prices have become an important activity to address the system operations and price volatility in the marketplace. Electricity pricing i.e. Spot pricing is a market-pricing approach used to manage the efficient use of the transmission system when congestion occurs on the bulk power grid. Most of the methodology is either on AC or DC system and its implementation. Since AC real power system is not mainly revamped with DC system because of advantages of HVDC system, it developed suitable AC-DC based OPF methodology and its implementation on real power system. The aim of this paper is to model AC-DC OPF based electricity spot pricing and its implementation on standard IEEE 57 Bus System and Real power system. The results are simulated both for standard IEEE-57 Bus system and also for real network 400 kV MSETCL. Finally the results obtained are compared at several possible conditions like addition of 765 kV AC transmission lines, impact on Bus voltages and optimal electricity spot prices.

Keywords—Electricity market, HVDC, Short run marginal costing, Spot price, optimal power flow

#### **I.INTRODUCTION**

In recent years, the electricity industry has undergone drastic changes due to a worldwide deregulation/privatization process that has significantly affected power system management and energy markets. In a deregulated system, operators' goals are balancing consumer power demand using the available generation and ensuring that economical and technical constraints are respected. The prime economical aspect is the social benefit, i.e. power suppliers should obtain maximum prices for their produced energy, while consumers should pay the lowest prices for the purchased electric power. Prices have to be defined in a free market economy and restricted only by power exchange rules.

In restructured electricity market, the desired objective is to achieve a more efficient power system facilitated by competition. A optimal and sustainable pricing scheme becomes a key issue in order to achieve efficient competition. While competition is introduced in generation and retail (or supply) around the world, it is widely

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agreed that transmission is a natural monopoly and should remain centrally controlled. Today transmission and distribution lines provide the critical physical link that makes competition feasible. Thus open access to the network and adequate pricing are essential for the development of competition.

Several electricity transmission pricing methods have been developed in order to meet the various pricing objectives. One of approach of electricity pricing is Spot pricing based on Short Run Marginal Costing (SRMC) principle. It is a market-pricing approach used to manage the efficient use of the transmission system when congestion occurs on the bulk power grid.

Electricity Spot prices have several applications in the competitive electricity market. The two most important applications i.e. first forecasting of electricity prices and secondly to design Financial Transmission Rights (FTRs) to hedge the price risk under transmission congestion.

Optimal Power flow (OPF) is one of the most significant problems for power system planners and operators. The main aim of OPF is to discover new techniques for the optimal settings of a given power system network that improve a selected objective function such as total generation cost, system loss, bus voltage deviation while fulfilling its load flow equations, system protection, and equipment operating limits. The basic objective of OPF problem is to meet the required load demand at minimum production cost, satisfying units' and system's operating constraints, by adjustment of power system control variables. In other words, optimal power flow (OPF) problem deals with finding an optimal operating point of a power system that minimizes an appropriate cost function such as generation cost or transmission loss subject to certain constraints on power and voltage variables.

This study deals with modeling and implementation of AC-DC OPF based electricity spot pricing in restructured electricity market.

#### **II.PRESENT STUDY**

The electricity spot price has been modelled following different statistical approaches, mainly based on mean reverting diffusion and auto-regressive models. The specifications of these models require, or at least benefit from, the assumption of normality of the underlying stochastic process. In the past, electric utilities in developing countries have performed poorly. Today most of the electricity reform processes are based on the market-orientation approaches to meet variety of objectives i.e. efficiency (techno-economic), competition, privatization, and new regulatory structure. While reform programmes for the electricity sector have been built around these elements, the details varied to reflect local circumstances [5]. F. C. Schweppe et al., introduces the concept of spot price into power system and provided the foundation and starting point for most successive research [6]. M. Rivier et al., develops spot price model by describing the meaning and numerical properties of the generation and transmission components of spot price based on "slack bus" and "system lambda"[7]. D. Finney et al., described an OPF based decomposition of spot prices to perform the operation of Poolco model [8]. Results are derived using the decomposition of the lagrangian multipliers corresponding to power balance equations into components that represented the sum of generation, losses and system congestion. P. Wijayatunga et al., explained an alternative method which includes transmission security in the pricing mechanism [9]. The main feature of this method was that system security can be separated from the capacity cost and the cost of

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transmission losses in final transmission price, thereby helping the network operator or planner to identify those areas requiring attention. This technique allows the rerouting of power flows with adjusted price while recovering the required revenue. Kai Xie et al., presents an integrated spot pricing model by modifying existing Newton OPF by Interior Point algorithms [10].

#### II.SHORT-RUN MARGINAL COSTING BASED ELECTRICITY SPOT PRICING

The marginal cost in general is the change in total cost (variable) that arises when the quantity produced changes by one unit. Mathematically, the marginal cost function is expressed as the first (order) derivative of the total cost function with respect to quantity (Q). So at each level of production, the marginal cost is the cost of the next unit produced referring to the basic volume. In general terms, marginal cost at each level of production includes any additional cost required to produce the next unit.

In electric power system, the short-run marginal cost is the generation cost of transporting one additional MW across the network when transmission capacity is fixed. SRMC methods are based on location specific generation costs and therefore transmission investment costs are not considered. SRMC methods are also referred as Electricity spot pricing.

#### III.ELECTRICITY SPOT PRICING

When there is no transmission congestion and losses present during the transmission of the electricity, the cheapest power producer will be selected to serve the loads at all locations and therefore, electricity price will be same across the grid. This price is often called the market clearing price. When congestion occurs, one or more transmission line reaches its thermal limit and unable to carry additional power, a more expensive generation unit will be scheduled to serve the load since the cheaper generators could not reach the load location due to congestion. Consequently, electricity prices at this location will increase, since it is served by the more expensive power producers. In addition to transmission congestion, power transmission losses also contribute to the varying prices at different locations.

Spot pricing determines the price of electricity at each bus or location on the grid for each dispatch interval by calculating the cost of serving increment of load on the basis of incremental cost of next merit order generation and transmission constraints. It thus recognizes that this marginal price may vary at different times and locations based on transmission congestion.

"Locational marginal pricing (LMP) is the electricity spot pricing model that serves as the benchmark for market design". A trading arrangement based on LMP takes all relevant generation and transmission costs appropriately into account and hence supports optimal investments." (International Energy Agency, 2007)

In restructured electricity market, several short-term, long-term and real-time benefits of Spot Price are described below-

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- ➤ In short-term, Spot price improves the efficiency of wholesale electricity market by ensuring that the cost of congestion is reflected in electricity prices and ensures that the least-cost supply of electricity is delivered while respecting the physical limitation of the transmission network.
- ➤ In long-term, Spot price helps to relieve congestion by promoting efficient investment decisions. Because Spot price creates price signals that reflect the locational value of electricity, participants can readily determine areas of congestion.
- ➤ Appropriately located generation addition, transmission and demand response will increase the competitiveness of electricity market. Greater access to a larger number of competing suppliers helps to enforce market discipline without resorting to administratively applied market power remedies.
- ➤ Increased access to energy from lower-cost generators or imported power will ensure robust, competitive prices. And increased competition from strategically located low-cost units and demand response will benefit the utility, as the transmission grid is utilized more efficiently.

Ultimately, increased competition should result in a more efficient wholesale energy market with lower costs.

#### IV.AC-DC OPF BASED SPOT PRICING METHODOLOGY

The electricity consumption in developing countries is expected to become more than double. Also these countries are facing the problems of infrastructure investment especially in transmission and distribution segment. To reduce the gap between transmission capacity and electricity demand, trend is now to incorporate HVDC transmission in the existing AC networks to gain techno-economical advantages of the investment. Also it was observed that the energy utilization of many HVDC system when incorporated in an existing AC transmission system were substantially below their maximum continuous capacity. In coming years, there is a great opportunity to utilize the unused capacity of the HVDC systems for the purpose of system security enhancement and economic gain by the utility. The AC-DC Optimal power flow based Electricity Spot Pricing methodology is as follows.

#### V.PROBLEM FORMULATION

6.1 AC System Equations: Let  $P = (p_{1,...},p_n)$  and  $Q = (q_{1,...},q_n)$  for n buses system, where  $p_i$  and  $q_i$  be active and reactive power demands of bus-i, respectively. The variables in power system operation to be  $X = (x_1,...,x_m)$ , i.e. real and imaginary parts of each bus voltage. Then the problem of a power system for given load (P, Q) can be formulated as OPF problem [7-10].

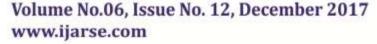
Minimize 
$$f(X, P, Q)$$
 for  $X$  (1)

Subject to 
$$S(X, P, Q) = 0$$
 (2)

$$T(X, P, Q) \le 0 \tag{3}$$

Where  $S(X) = (s_1(X, P, Q), ..., s_{n1}(X, P, Q))^T$  and  $T(X) = (t_1(X, P, Q), ..., t_{n2}(X, P, Q))^T$  have  $n_1$  and  $n_2$  equations, and are column vectors.  $A^T$  is the transpose of vector A.

f(X, P, Q) is a scalar, generator cost function  $f_i(P_{Gi})$  in \$/MWh having cost characteristics represented by,



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$$f = \sum_{i=1}^{NG} a_i \, P_{Gi}^2 + b_i \, P_{Gi} + c_i \tag{4}$$

Where,  $P_{Gi}$  is real power output;  $a_i$ ,  $b_i$  and  $c_i$  is the cost coefficient of the i<sup>th</sup> generator, NG represents the generation buses.

Power system constraints to be satisfied are-

(1) Vector of equality constraint i.e. power flow balance is,

$$P_G = P_D + P_{DC} + P_L; Q_G = Q_D + Q_{DC} + Q_L$$
 (5)

Here suffix D represents the demand, G is the generation, DC represents dc terminal and L is transmission loss.

(2) The inequality constraints is represented as,

$$T(X, P, Q) \le 0 \text{ or} \tag{6}$$

(i) The maximum and minimum real and reactive power outputs of the generators are given by,

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \quad (i \in G_B); \ Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \quad (i \in G_B)$$
 (7)

Where,  $P_{Gi}^{\min}$ ,  $P_{Gi}^{\max}$  and  $Q_{Gi}^{\min}$ ,  $Q_{Gi}^{\max}$  are the minimum and maximum real and reactive power outputs of the generator.  $G_B$  represents generating buses.

(ii) Voltage limits to remain within a narrow limit expressed by the constraints

$$\left|V_i^{\min}\right| \le \left|V_i\right| \le \left|V_i^{\max}\right| \qquad (i=1,...,N_B)$$
(8)

Where, N<sub>B</sub> represents number of buses.

(iii) Transmission line power flow (MVA) limits is expressed by the constraints

$$P_f^{\min} \le P_f \le P_f^{\max} \quad (f=1,...,Noele)$$
(9)

Where, *Noele* represents number of transmission lines connected to grid. Then the operating condition of the combined ac-dc electric power system is the vector

$$X = [\delta, V, x_c, x_d]^t \tag{10}$$

Where,  $\delta$  and V are the vectors of the phases and magnitude of the phasor bus voltages;  $x_c$  is the vector of control variables and  $x_d$  is the vector of dc variables.

6.2 DC System Equations: The average value of the dc voltage of a converter connected to bus 'i' is

$$V_{di} = a_i V_i \cos \alpha_i - r_{ci} I_{di} \tag{11}$$

Here,  $\alpha_i$  is the gating delay angle for rectifier operation/extinction advance angle for inverter operation;  $r_{ci}$  is commutation resistance, and  $a_i$  is converter transformer tap setting. Assuming a lossless converter, the equation of the dc voltage is given by,

$$V_{di} = a_i V_i \cos \varphi_i \tag{12}$$

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Where,  $\varphi_i = \delta_i - \xi_i$ , and  $\varphi$  is the angle by which the fundamental line current lags the line-to-neutral source voltage. The real and reactive power flowing into or out of the dc network at terminal 'i' is

$$P_{di} = V_i I_i \cos \varphi_i \quad ; \quad Q_{di} = V_i I_i \sin \varphi_i \tag{13}$$

The equations (13) could be substituted in equation (5) to form part of the equality constraints. Then the operating condition of the dc system can be described by the vector

$$X_d = [V_d, I_d, a, \cos \alpha, \varphi]^t \tag{14}$$

Equations (1)–(3) are an OPF problem for demand (P, Q).

6.3 Nodal Price: The Lagrangian function of equations (1) - (3) defined as

$$L = \sum_{i=1}^{NG} a_{i} P_{Gi}^{2} + b_{i} P_{Gi} + c_{i} + \sum_{i \in LB} \lambda pi(P_{Di} - P_{Gi} + P_{Li}) + \sum_{i \in LB} \lambda qi(Q_{Di} - Q_{Gi} + Q_{DCi} + Q_{Li})$$

$$+ \sum_{i \in GB} \rho P_{li} (P_{Gi}^{min} - P_{Gi}) + \sum_{i \in GB} \rho P_{ui} (P_{Gi} - P_{Gi}^{max}) + \sum_{i \in GB} \rho q_{li} (Q_{Gi}^{min} - Q_{Gi}) + \sum_{i \in GB} \rho q_{ui} (Q_{Gi} - Q_{Gi}^{max})$$

$$+ \sum_{i=1}^{NB} \rho V_{li} (|V_{i}^{min}| - |V_{i}|) + \sum_{i=1}^{NB} \rho V_{ui} (|V_{i}| - |V_{i}^{max}|) + \sum_{i=1}^{NB} \rho \theta_{li} (\theta_{i}^{min} - \theta_{i}) + \sum_{i=1}^{NB} \rho \theta_{ui} (\theta_{i} - \theta_{i}^{max})$$

$$+ \sum_{i=1}^{Noele} \rho P_{fli} (P_{fi}^{min} - P_{fi}) + \sum_{i=1}^{Noele} \rho P_{fui} (P_{fi} - P_{fi}^{max})$$

$$(15)$$

Where, 'l' and 'u' are lower and upper limits;  $\lambda = (\lambda_1, ..., \lambda_n)$  is the vector of Lagrange multipliers concerning equality constraints;  $\rho = (\rho_1, ..., \rho_n)$  are the Lagrange multipliers concerning inequality constraints. Then at an optimal solution  $(X, \lambda, \rho)$  for a set of given (P, Q), nodal price of real and reactive power for bus are expressed for i = 1, ..., n as,

$$\pi_{p,i} = \frac{\partial L(X,\lambda,\rho,P,Q)}{\partial_{pi}} = \frac{\partial f}{\partial_{pi}} + \lambda \frac{\partial S}{\partial_{pi}} + \rho \frac{\partial T}{\partial_{pi}}$$
(16)
$$\pi_{q,i} = \frac{\partial L(X,\lambda,\rho,P,Q)}{\partial_{qi}} = \frac{\partial f}{\partial_{qi}} + \lambda \frac{\partial S}{\partial_{qi}} + \rho \frac{\partial T}{\partial_{qi}}$$
(17)

Here  $\pi_{p,i}$  and  $\pi_{q,i}$  are active and reactive nodal prices at bus 'i', respectively. The difference  $\pi_{p,i}$  -  $\pi_{p,j}$  represents active transmission charges from bus-j to bus-i. This methodology has been simulated and results are obtained for several conditions and constraints tested over IEEE-57 Bus system and implemented on a Indian power system.

#### VI.PROBLEM SIMULATION AND RESULT

This section presents simulated results and analysis for IEEE standard Bus systems and that for a 400 kV/765 kV MSETCL system.

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In order to ensure universal applicability of the proposed methodology, this AC-DC OPF based electricity nodal pricing methodology is simulated for standard IEEE 57 test bus system. The Bus System is shown in Figure 7.1.

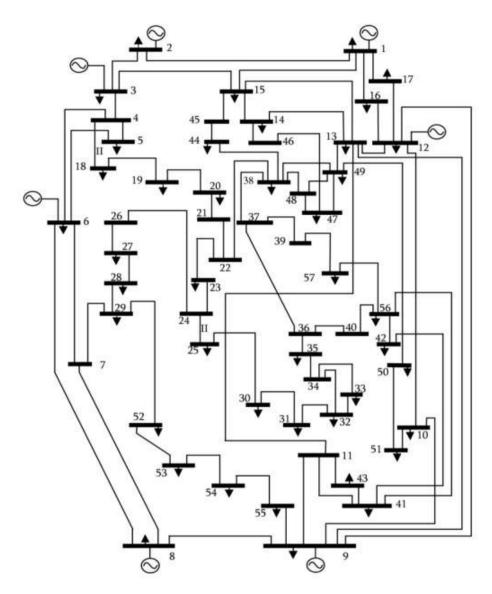
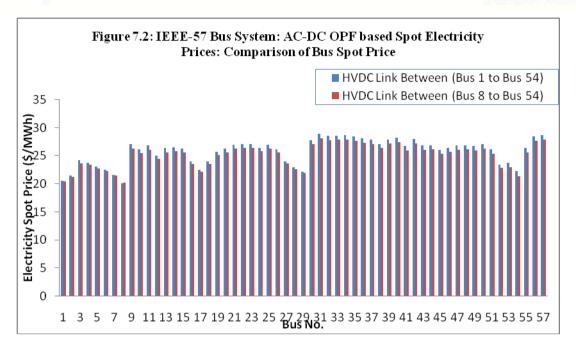


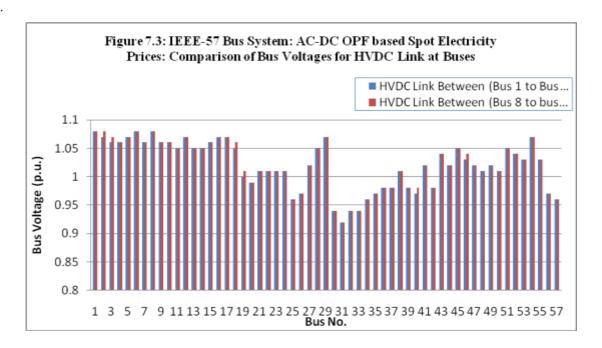
Figure 7.1: Standard IEEE-57 Bus System

The electricity spot prices of real power and reactive power with and without HVDC link have been simulated and compared. Tables shows the tabulated results for voltage behaviour and real nodal prices obtained at buses. Figure 7.2 shows the comparison of electricity spot prices obtained with and without HVDC link at buses and Figure 7.3 shows the comparison of voltages obtained with and without HVDC link at buses. The result indicates that with the incorporation of HVDC link in existing AC transmission system, real electricity nodal prices obtained are lower at few buses and voltages at several buses have varied within narrow range again due to decrease in power flow and voltage drop across few transmission lines.

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The result indicates that with the incorporation of HVDC link in existing AC transmission system, real electricity nodal prices obtained are lower at few buses and voltages at several buses have varied within narrow range again due to decrease in power flow and voltage drop across few transmission lines.

In order to ensure practical applicability of the proposed methodology, the proposed methodology is implemented for 400 kV/765 kV Maharashtra State Electricity transmission Company Limited (MSETCL). The System is shown in Figure 7.4. The Results are shown in Table 7.1. Bus voltage and Spot price is compared as shown in Figure 7.5 and Figure 7.6 respectively.

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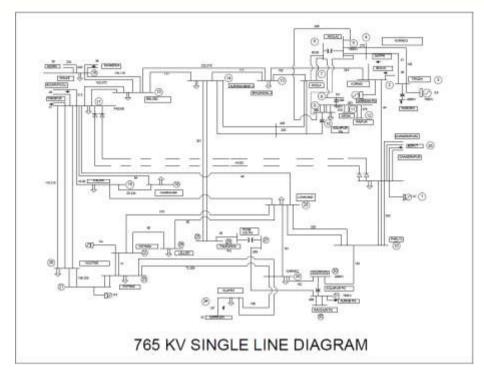


Figure 7.4: A 400 kV / 765 kV MSETCL system

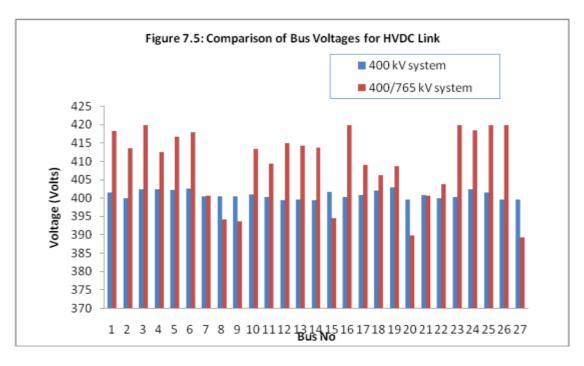
Table 7.1: Result of 400kV/765 kV Maharashtra (MSETCL)

Bus No.	Voltage	Real Power	Reactive Power	Angle	Spot Price
	(Volts)	(p.u.)	(p.u.)		(Rs./kWh)
1	418.38	2.58	1.97	0.14	1.87
2	413.65	2.65	0.54	0.26	1.86
3	420.00	2.53	0.00	0.00	1.90
4	412.56	2.49	0.00	0.00	1.92
5	416.71	2.54	0.00	0.00	1.91
6	417.91	2.65	0.00	0.00	1.88
7	400.72	1.84	0.00	0.00	2.23
8	394.12	1.83	0.00	0.00	2.28
9	393.69	1.82	0.00	0.00	2.28
10	413.43	1.87	0.00	0.00	2.12
11	409.39	1.85	0.00	0.00	2.15
12	415.07	1.93	0.80	0.12	2.13
13	414.33	1.92	0.00	0.00	2.13
14	413.72	1.92	0.10	0.17	2.13
15	394.60	1.88	0.00	0.00	2.30
16	420.00	1.81	0.00	0.00	2.13
17	409.14	1.93	0.00	0.00	2.12

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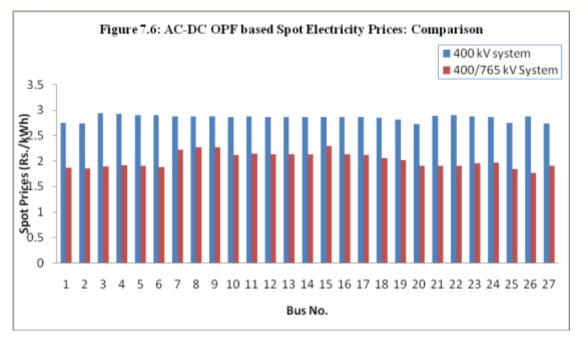


18	406.21	2.08	0.00	0.00	2.06
19	408.75	2.18	0.00	0.00	2.02
20	389.81	2.59	0.40	0.41	1.91
21	400.69	2.56	0.00	0.00	1.91
22	403.75	2.53	0.00	0.00	1.91
23	420.00	2.37	0.00	0.00	1.96
24	418.58	2.33	0.00	0.00	1.97
25	420.00	2.59	0.39	0.11	1.85
26	420.00	2.91	0.50	0.02	1.77
27	389.24	2.54	0.12	0.50	1.91
28	420.00	3.00	0.84	0.13	1.78
29	409.12	2.68	0.10	0.24	1.88
30	401.54	1.90	0.19	0.15	2.21
31	401.37	1.86	0.19	0.01	2.23



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#### VII.CONCLUSION

The study presented optimal electricity pricing methodology suitable for real power systems. The methodology has been tested on IEEE 57-Bus system and implemented on a real power system of MSETCL, Maharashtra. The optimal electricity prices are simulated for with and without 765 kV HVDC link. Numerical results are compared as shown in result Tables. This study concludes that optimal electricity prices at several buses are lower with incorporation of 765 kV HVDC link in an existing AC transmission system. This study can be useful to the Transmission System Operator (TSO) in understanding Spot electricity market in restructured electricity market.

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