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On w-semi-$\mathfrak{T}$-continuous maps

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ABSTRACT

In this paper we will give introduce w-semi-$\mathfrak{T}$-continuous mappings and give various characterizations of it.

Key Words and phrases: w-semi-$\mathfrak{T}$-open, w-semi-$\mathfrak{T}$-continuous.

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I.INTRODUCTION

In [3], Janković and Hamlett introduced the concept of $\mathfrak{T}$-open sets in topological spaces. In [1] Hatir and Noiri introduced the notion of semi-$\mathfrak{T}$-open sets and semi-$\mathfrak{T}$-continuous functions and in [2] they further investigate the properties of semi-$\mathfrak{T}$-continuous functions. The subject of ideals in topological spaces were introduced by Kuratowski[5] and further studied by Vaidyanathaswamy[6]. Corresponding to an ideal a new topology $\tau^*(\mathfrak{T}, \tau)$ called the *-topology was given which is generally finer than the original topology having the kuratowski closure operator $cl^*(A) = A \cup A^*(\mathfrak{T},\tau)$[7], where $A^*(\mathfrak{T},\tau) = \{x \in X : U \cap A \notin \mathfrak{T}$ for every open subset U of x in X called a local function of A with respect to $\mathfrak{T}$ and τ. We will write τ^* for $\tau^*(\mathfrak{T}, \tau)$.
The following section contains some definitions and results that will be used in our further sections.

Definition 1.1.[5]: Let (X, τ) be a topological space. An ideal $\mathfrak{T}$ on X is a collection of non-empty subsets of X such that (a) $\phi \in \mathfrak{T}$ (b) $A \in \mathfrak{T}$ and $B \in \mathfrak{T}$ implies $A \cup B \in \mathfrak{T}$ (c) $B \in \mathfrak{T}$ and $A \subset B$ implies $A \in \mathfrak{T}$.

Definition 1.2 [2] : Let$(X,\tau,\mathfrak{T})$ be an ideal space and A be any subset of X. Then A is said to be semi-$\mathfrak{T}$-open if

$A \subset cl^*(int(A))$.

Definition 1.3 [1]: Let $(X,\tau,\mathfrak{T})$ and (Y,σ) be two topological spaces. Then a map $f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma)$ is said to be semi-$\mathfrak{T}$-continuous if inverse image of every open set in Y is semi-$\mathfrak{T}$-open in X.

Definition 1.4 [1]: Let $(X,\tau,\mathfrak{T})$ and$(Y,\sigma,\mathcal{J})$ with $\mathcal{J}=f(\mathfrak{T})$ be two ideal topological spaces. Then a map

$f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma,\mathcal{J})$ is said to be semi-$\mathfrak{T}$-irresolute if inverse image of every semi-$\mathcal{J}$-open subset in Y is semi-$\mathfrak{T}$-open in X.

Definition 1.5.[4] : A mapping $f : (X,\tau,\mathfrak{T})\rightarrow(Y,\sigma)$ is said to be pointwise $\mathfrak{T}$-continuous if the inverse image of every open set in Y is τ^*-open in X.

Lemma 1.6.[3] : Let $(X,\tau,\mathfrak{T})$ be an ideal space and Y be subset of X. Then

$\mathfrak{I}_Y = \{I \cap Y \mid I \in \ \}$ is an ideal on Y.

II.RESULTS

Definition 2.1: Let $(X,\tau,\mathfrak{I})$ be an ideal space and A be any subset of X. Then A is said to be w-semi-$\mathfrak{I}$-open if $A \subset cl(int^*(A))$.

Definition 2.2: Let $(X,\tau,\mathfrak{I})$ and (Y,σ) be two topological spaces. Then a map $f : (X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma)$ is said to be w-semi-$\mathfrak{I}$-continuous if inverse image of every open set in Y is w-semi-$\mathfrak{I}$-open in X.

i.e. f is w-semi-$\mathfrak{I}$-continuous if $\forall$ V $\in$ σ, $f^{-1}(V)$ is w-semi-$\mathfrak{I}$-open subset of X.

Definition 2.3: Let $(X,\tau,\mathfrak{I})$ and $(Y,\sigma, \mathcal{J})$ with $\mathcal{J}=f(\mathfrak{I})$ be two topological spaces. Then a map

$f : (X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma, \mathcal{J})$ is said to be w*-semi-$\mathfrak{I}$-continuous if inverse image of every σ^*-open set in Y is w-semi-$\mathfrak{I}$-open in X.

i.e. f is w*-semi-$\mathfrak{I}$-continuous if $\forall$ V $\in$ σ^*, $f^{-1}(V)$ is w-semi-$\mathfrak{I}$-open subset of X.

Remark 2.4: Since $\sigma \subset \sigma^*$. Therefore, it can be easily seen that every w*-semi-$\mathfrak{I}$-continuous map is w-semi-$\mathfrak{I}$-continuous.

Definition 2.5: Let $(X,\tau,\mathfrak{I})$ and $(Y,\sigma,\mathcal{J})$ with $\mathcal{J}=f(\mathfrak{I})$ be two ideal topological spaces. Then a map

$f : (X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma,\mathcal{J})$ is said to be w-semi-$\mathfrak{I}$-irresolute if inverse image of every w-semi-$\mathcal{J}$-open subset in Y is w-semi-$\mathfrak{I}$-open in X.

i.e. f is w-semi-$\mathfrak{I}$-irresolute if $\forall$ w-semi-$\mathcal{J}$-open subset V in Y, $f^{-1}(V)$ is w-semi-$\mathfrak{I}$-open subset of X.

Remark 2.6: Since, every τ^*-open subset V in an ideal space $(X,\tau,\mathfrak{I})$ is w-semi-$\mathfrak{I}$-open and every open subset of X is τ^*-open. Therefore,

1.) Every pointwise-$\mathfrak{I}$-continuous map is w-semi-$\mathfrak{I}$-continuous.
2.) Every w-semi-$\mathfrak{I}$-irresolute is w-semi-$\mathfrak{I}$-continuous map.

Theorem 2.7: Let $(X,\tau,\mathfrak{I})$ and (Y,σ) be two topological spaces and $f : (X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma)$ be any map. Then prove that the following are equivalent:

a) f is w-semi-$\mathfrak{I}$-continuous.
b) For each $x \in X$ and any open set V containing f(x) in Y, there exists a w-semi-$\mathfrak{I}$-open subset U of x such that $f(U) \subset V$.
c) The inverse image of every closed set in Y is w-semi-$\mathfrak{I}$-closed in X.

Proof: (a) ⇒(b): Let $x \in X$ be any element and V be any open set in Y containing f(x). Then by (1), $f^{-1}(V)$ is w-semi-$\mathfrak{T}$-open subset of X containing x. Let $U = f^{-1}(V)$. Hence there exist w-semi-$\mathfrak{T}$-open subset U of X containing x such that $f(U) = f(f^{-1}(V)) \subset V$.

(b)⇒(a): Let V be an open subset of Y. Then there can be two possibilities:

1.) $f^{-1}(V) = \phi$, then we have nothing to prove.

2.) $f^{-1}(V) \neq \phi$. Let $x \in f^{-1}(V)$ then $f(x) \in V$. Now by (2), there exist w-semi-$\mathfrak{T}$-open subset U of X containing x such that $f(x) \in f(U) \subset V$ and so $x \in f^{-1}(f(U)) \subset f^{-1}(V)$. Therefore, $x \in U \subset f^{-1}(f(U)) \subset f^{-1}(V)$. Hence $\forall x \in f^{-1}(V)$, there exist w-semi-$\mathfrak{T}$-open subset U of X containing x such that $x \in U \subset f^{-1}(V)$. This implies that $f^{-1}(V)$ is the union of w-semi-$\mathfrak{T}$-open sets. But union of w-semi-$\mathfrak{T}$-open sets is also w-semi-$\mathfrak{T}$-open set. So, $f^{-1}(V)$ is w-semi-$\mathfrak{T}$-open subset of X. Hence f is w-semi-$\mathfrak{T}$-continuous.

(a)⇔(c): f is w-semi-$\mathfrak{T}$-continuous if and only if inverse image of every open subset V in Y is w-semi-$\mathfrak{T}$-open in X i.e. if and only if $\forall V \in \sigma$, $f^{-1}(V)$ is w-semi-$\mathfrak{T}$-open subset of X if and only if for every closed set F in Y i.e. Y-F is open in Y, $f^{-1}(Y\text{-}F)$ is w-semi-$\mathfrak{T}$-open in X i.e. if and only if for every closed F in Y, X- $f^{-1}(F)$ is w-semi-$\mathfrak{T}$-open in X i.e. if and only if for every closed F in Y, $f^{-1}(F)$ is w-semi-$\mathfrak{T}$-closed in X if and only if inverse image of every closed set in Y is w-semi-$\mathfrak{T}$-closed in X.

Theorem 2.8: Let $(X,\tau,\mathfrak{T})$ and (Y,σ) be two topological spaces and $f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma)$ be any w-semi-$\mathfrak{T}$-continuous map. If U be any τ-open subset of X then prove that

$$f|U : (U, \tau|U, \mathfrak{T}|U) \rightarrow (Y,\sigma) \text{ is w-semi-}\mathfrak{T}\text{-continuous.}$$

Proof: Let V be any open subset of Y. Then f is w-semi-$\mathfrak{T}$-continuous implies that $f^{-1}(V)$ is w-semi-$\mathfrak{T}$-open in X.

Now, U is τ-open subset of X implies that $f^{-1}(V) \cap U \subset cl(int^*(f^{-1}(V))) \cap int(U) \subset cl(int^*(f^{-1}(V) \cap U))$. Therefore, $f^{-1}(V) \cap U$ is w-semi-$\mathfrak{T}$-open subset of U.

Hence f|U is w-semi-$\mathfrak{T}$-continuous.

Theorem 2.9: Let $f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma,\mathcal{J})$ and $g : (Y,\sigma,\mathcal{J}) \rightarrow (Z,\eta)$ be any two mappings. Then prove that the following hold:

(a) gof is w-semi-$\mathfrak{T}$-continuous if g is continuous and f is w-semi-$\mathfrak{T}$-continuous.
(b) gof is w-semi-$\mathfrak{T}$-continuous if g is w-semi-$\mathfrak{T}$-continuous and f is w-semi-$\mathfrak{T}$-irresolute.

Proof: (a): Let W be any open subset of Z. Then g is continuous implies that $g^{-1}(W)$ is open in Y. Further, f is w-semi-$\mathfrak{T}$-continuous implies that $f^{-1}(g^{-1}(W))$ is w-semi-$\mathfrak{T}$-open subset of X and so $(gof)^{-1}(W)$ is w-semi-$\mathfrak{T}$-open subset of X.

Hence gof is w-semi-$\mathfrak{T}$-continuous.

(b): Let W be any open subset of Z. Then g is w-semi-$\mathfrak{T}$-continuous implies that $g^{-1}(W)$ is w-semi-$\mathcal{J}$-open in Y. Further, f is w-semi-$\mathfrak{T}$-irresolute implies that $f^{-1}(g^{-1}(W))$ is w-semi-$\mathfrak{T}$-open subset of X and so $(gof)^{-1}(W)$ is w-semi-$\mathfrak{T}$-open subset of X.

Hence gof is w-semi-$\mathfrak{T}$-continuous .

Theorem 2.10 : Let $f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma,\mathcal{J})$ be any w*-semi-$\mathfrak{T}$-continuous map such that $f^{-1}(cl(V)) \subset cl(f^{-1}(V))$ for every τ^*-open subset V of Y. Then prove that f is w-semi-$\mathfrak{T}$-irresolute.

Proof: Let W be any w-semi-$\mathcal{J}$-open subset of Y. Then there exist τ^*-open subset G of Y such that

$G \subset W \subset cl(G)$ and so $f^{-1}(G) \subset f^{-1}(W) \subset f^{-1}(cl(G)) \subset cl(f^{-1}(G))$. Now f is w*-semi-$\mathfrak{T}$-continuous implies that

$f^{-1}(G)$ is w-semi-$\mathfrak{T}$-open subset of X. Therefore, $f^{-1}(W)$ is such that there exist w-semi-$\mathfrak{T}$-open subset $f^{-1}(G)$ of X such that $f^{-1}(G) \subset f^{-1}(W) \subset cl(f^{-1}(G))$. Hence $f^{-1}(W)$ is w-semi-$\mathfrak{T}$-open subset of X.

Hence f is w-semi-$\mathfrak{T}$-irresolute.

Next we introduce w-semi-$\mathfrak{T}$-compact spaces.

Definition 2.11: An ideal space $(X,\tau,\mathfrak{T})$ is said to be w-semi-$\mathfrak{T}$-compact if for every w-semi-$\mathfrak{T}$-open cover

$\{V_\alpha | \alpha \in \Delta\}$ of X, there is a finite subset Δ_0 of Δ such that $X - \cup\{V_\alpha | \alpha \in \Delta_0\} \in \mathfrak{T}$.

Theorem 2.12: Let $(X,\tau,\mathfrak{T})$ and (Y,σ) be two topological spaces and $f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma, \mathcal{J})$ be any surjective w-semi-$\mathfrak{T}$-continuous mapping. If X is w-semi-$\mathfrak{T}$-compact space then prove that Y is also $\mathcal{J}$-compact space.

Proof: Let $\{U_\alpha | \alpha \in \Delta\}$ be an open cover of Y i.e. $Y = \bigcup_\alpha U_\alpha$. Then f is w-semi-$\mathfrak{T}$-continuous implies that $f^{-1}(U_\alpha)$ is w-semi-$\mathfrak{T}$-open subset of X $\forall\ \alpha \in \Delta$. Now $Y = \bigcup_\alpha U_\alpha$ implies that $f^{-1}(Y) = f^{-1}(\bigcup_\alpha U_\alpha) = \bigcup_\alpha f^{-1}(U\alpha)$ and so $X = \bigcup_\alpha f^{-1}(U\alpha)$. But X is w-semi-$\mathfrak{T}$-compact implies that there exist a finite subset Δ_0 of Δ such that

$X - \cup\{ f^{-1}(U\alpha) \mid \alpha \in \Delta_0\} \in \mathfrak{T}$. Therefore, $f(X - \bigcup_{\alpha\in\Delta 0} f^{-1}(U\alpha)) \in f(\mathfrak{T})$ and so $Y - f(\bigcup_{\alpha\in\Delta 0} f^{-1}(U\alpha)) \in f(\mathfrak{T})$ and so $Y - \bigcup_{\alpha\in\Delta 0} f(f^{-1}(U\alpha)) \in f(\mathfrak{T})$. Now, since f is surjective so $Y - \bigcup_{\alpha\in\Delta 0} U_\alpha \in f(\mathfrak{T})$ i.e. $Y - \bigcup_{\alpha\in\Delta 0} U_\alpha \in \mathcal{J}$. Hence Y is $\mathcal{J}$-compact space.

REFERENCES

[1]E. Hatir and T.Noiri, *On decompositions of continuity via idealization* , Acta Math. Hunger., 96 (2002), 341-349.

[2]E. Hatir and T.Noiri, *On semi-I-open sets and semi-I-continuous functions*, Acta Math. Hunger., 107 (2005), 345-353.

[3]D.Jankovic and T.R. Hamlett, *New topologies from old via ideals*, The American Mathematical Monthly, 97, No. 4 (1990), 295-310.

[4] J. Kanicwski and Z. Piotrowski, *Concerning* continuity a*part from a meager set,* Proc. Amer. Math. Soc., 98(1986), 324-328.

[5] K. Kuratowski, *Topology*, volume I, Academic Press,New York, 1966.

[6] R. Vaidyanathaswamy, *The localisation Theory in Set Topology*, Proc. Indian Acad. Sci.,20(1945),51-61.

[7] -------, *Set Topology*, Chelsea Publishing Company, New York,1946.