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On Retrial Queue with Collisions and with Fuzzy Parameters

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ABSTRACT

This paper investigates the single server retrial queue with collisions by using the fuzzy set theory. A mathematical programming approach is proposed to develop the membership function of the system performance, where the arrival rate, service rate and retrial rate are fuzzy. Based upon the α-cut approach and Zadeh's extension principle, the fuzzy retrial queue is transformed to a family of crisp retrial queues. The defuzzification of the system characteristics is also provided via Yager ranking index for practical purpose.

Key Words: Fuzzy sets, Membership functions, Non-linear programming, Collisions.

I INTRODUCTION

Retrial queueing systems arise naturally in many telecommunication and computer systems and are characterized by the fact that a customer who finds the server busy upon arrival must leave the service area and joins a retrial group in order to repeat his request after some random time. Between retrials the customers are said to be in the orbit or in the retrial group and are called retrial customers. Retrial queues have been widely used in designing local area networks, data transfer via telephone networks and radio and cellular networks. For a detailed overview, we refer the readers to the surveys by Yang and Templeton [5], Falin [1], the book by Falin and

Templeton [2] and Artalejo [3, 4].

Retrial queues with collisions has wide applications in medium access control protocols for wireless LANs proposed to date. This type of queueing systems has the feature that if an arriving customer finds the server busy, then the arriving customer collides with a customer in service, both customers join the orbit and the server becomes idle immediately, as in the unslotted 1- and pi-persistent Carrier Sense Multiple Access with Collision Detection (CSMA-CD) protocols for a fiber optic bus network with a finite number of stations, each of which has an infinite storage buffer, the collisions occur during the transmission of arbitrary length packets because no slot synchronization is needed. A retrial queueing model with collision arising from the specific communication protocol CSMA/CD has been analyzed by Choi [6]. Some results on the number of collisions in p-persistent CSMA/CD protocols have been obtained by Gomez-Corral [7]. Kim [9] considered a retrial queueing system with collisions and impatience. Recently, Krishna Kumar [8] studied a Markovian single server retrial queue with collisions.

In the literature described above, the inter arrival times, retrial times and service times of customers are determined by certain probability distributions with fixed parameters. However, in many real world applications of retrial queues, the parameter distributions may only be characterized subjectively, rather than with complete probability distribution. In other words, system parameters are more possibilistic than probabilistic in many practical situations. Thus, by extending the usual crisp retrial queues to fuzzy retrial queues in the context, these retrial queueing models become much more useful and realistic than the commony used crisp retrial queues.

Li and Lee [10] analyzed the analytical results for fuzzy queues using Zadeh's extension principle, the possibility concept and fuzzy Markov chaons. Negi and Lee [11] proposed a procedure to analyse fuzzy queues using α-cuts and two variable simulation. Kao et al. [13] constructed the membership functions of the system characteristics using parametric programming for fuzzy queues and applied them to four simple fuzzy queues: $M/F/1/\infty$, $F/M/1/\infty$, $F/F/1/\infty$ and $FM/FM/1/\infty$, where F represents fuzzy time and FM represents fuzzified exponential distributions. Chuan et al. Ke and Lin [19] and Lin et al. [20] analyzed the FM/FM/(FM,FM)/1, $FM^x/FM/1/FV$ and $FM^x/FM/1/FSET$ fuzzy queues, respectively, where FV represents the fuzzified exponential vacation rate and FSET represents the fuzzified exponential setup rate. Similarly, Chen [16, 17] developed FM/FM/1/L and $FM/FM^{[K]}/1/\infty$ fuzzy queues. Recently Ke et al. [21] obtained the membership functions of system characteristics of a retrial queueing model with fuzzy arrival, retrial and service rates.

In this paper, a mathematical programming approach is provided that derives the membership functions of the system characteristics for the considered retrial queue with three fuzzy variables: fuzzified exponential arrival, retrial and service rates. The fuzzy retyrial queues are transformed to a family of crisp retrial queues by applying the α-cuts and Zadeh's extension principle. As α varies, the family of crisp retrial queues is described and solved by using parametric non-linear programming (NLP). The NLP solutions yield the membership functions of the system characteristics.

The rest of the paper is organised as follows. In the next Section, the system characteristics of standard and fuzzy retrial queueing systems with collisions is presented. In Section 3, a mathematical programming approach is developed to derive the membership functions of these system characteristics. In Section 4, a realstic numerical example is described and solved to demonstrate the validity of proposed approach. Conclusions are drawn in last Section 5. For notational convenience, our model in this paper is here after denoted as FM/FM/1-(FR), where FR represents the fuzzified exponential retrial rate and 1 represents the single server.

II FUZZY RETRIAL QUEUES WITH COLLISIONS

2.1 Some basic definitions of fuzzy set theory

Fuzzy Set: Let X denote a universe of discourse. Then a fuzzy set $\tilde{A}$ in X is characterized by a membership function as follows:

$$\eta_{\tilde{A}} : X \to [0,1]$$

which assigns to each element x in X, and a real number $\eta_{\tilde{A}}(x)$ is in the interval [0,1]. Thus, the function value of $\eta_{\tilde{A}}(x)$ represents the membership of x in $\tilde{A}$.

Convex Fuzzy Set: A fuzzy set $\tilde{A}$ of a set X is convex if

$$\eta_{\tilde{A}}\left(\delta x_1 + (1-\delta)\, x_2\right) \geq \min\left(\eta_{\tilde{A}}\left(x_1\right), \eta_{\tilde{A}}\left(x_2\right)\right); \quad \forall x_1, x_2 \in X \text{ and } \delta \in [0,1].$$

α-Cut Set: $\tilde{A}_\alpha = \{x \,|\eta_{\tilde{A}}\,(x) \geq \alpha, x \in X\} = [l_{\tilde{A}}(\alpha), \psi_{\tilde{A}}(\alpha)]$ is called the α-cut of the fuzzy set $\tilde{A}$ for $\forall\alpha \in [0,1]$. The symbols $l_{\tilde{A}}(\alpha)$ and $\psi_{\tilde{A}}(\alpha)$ represent the lower bound and the upper bound of the α-cut of the fuzzy set $\tilde{A}$, respectively.

Trapezoidal Fuzzy Number: A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by:

$$\eta_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \dfrac{d-x}{d-c}, & c \leq x \leq d \end{cases}$$

2.2 FM/FM/1-(FR) retrial queue

An FM/FM/1-(FR) queueing system is considered. It is assumed that customers arrive at a service facility at rate $\tilde{\lambda}$, where λ is a fuzzy number. An arriving customer enters the service facility and if the server is busy, the arriving customer collides with the customer in service resulting both being shifted to orbit. Retrial customers attempts service after an uncertain amount of time, called retrial time. If the server is free, the arriving customer or the customer from orbit gets served completely and departs the system. The orbit capacity is assumed to be infinite. The successive retrial times are assumed to follow an exponential distribution with fuzzy retrial rate $\tilde{\nu}$. The service time is also exponentially distributed with fuzzy rate $\tilde{\mu}$. The inter arrival times, retrial times and service times are assumed to be mutually independent.

In this queueing model the arrival rate $\tilde{\lambda}$, retrial rate $\tilde{\nu}$ and service rate $\tilde{\mu}$ are approximately known and can be represented by convex fuzzy sets. Let $\eta_{\tilde{\lambda}}(x)$, $\eta_{\tilde{\nu}}(s)$ and $\eta_{\tilde{\mu}}(y)$ denote the membership functions of $\tilde{\lambda}$, $\tilde{\nu}$ and $\tilde{\mu}$, respectively. So, we have the following fuzzy sets:

$$\tilde{\lambda} = \{ (x, \eta_{\tilde{\lambda}}(x)) | \, x \in X \} \tag{1a}$$
$$\tilde{\nu} = \{ (s, \eta_{\tilde{\nu}}(v)) | \, s \in S \} \tag{1b}$$
$$\tilde{\mu} = \{ (y, \eta_{\tilde{\mu}}(y)) | \, y \in Y \} \tag{1c}$$

where X, S and Y are the crisp universal sets of the arrival, retrial and service rates, respectively.

Let $f(x,s,y)$ denote the system characteristics of interest. Since $\tilde{\lambda}$, $\tilde{\nu}$ and $\tilde{\mu}$ are fuzzy numbers, $f(\tilde{\lambda},\tilde{\nu},\tilde{\mu})$ is also a fuzzy number. The membership function of the system characteristic $f(\tilde{\lambda},\tilde{\nu},\tilde{\mu})$ be defined using Zadeh's extension principle([12]) as follows:

$$\eta_{f(\tilde{\lambda},\tilde{\nu},\tilde{\mu})}(z) = \sup_{\Omega} \min \{ \eta_{\tilde{\lambda}}(x), \eta_{\tilde{\nu}}(s), \eta_{\tilde{\mu}}(y) | \, z = f(x,s,y) \} \tag{2}$$

where $\Omega = \{x \in X, s \in S, y \in Y | x > 0, y > 0\}$

Assume that the fuzzy system characteristics of interest are $E[\tilde{W}]$ and $E[\tilde{N}]$, the expected waiting time in the queue and the expected number of customers in the orbit, respectively. Based on the result of the single server retrial queue with collisions [8], if $2x^2/(y-2x^2) < s$ the expected waiting time and the expected number of customers in the system, respectively, are

$$E[W] = \frac{[sy + x(3y - x)]}{(y - x)[s(y - 2x) - 2x^2]} \tag{3a}$$

$$E[N] = \frac{x[sy + x(3y - x)]}{(y - x)[s(y - 2x) - 2x^2]} \tag{3b}$$

Using (2) and (3a), the membership functions of $E[\tilde{W}]$ and $E[\tilde{N}]$, respectively, become

$$\eta_{E[\tilde{W}]}(z) = \sup_{\frac{2x^2}{(y-2x^2)}<s} \min\left\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\nu}}(s), \eta_{\tilde{\mu}}(y) | \, z = \frac{[sy + x(3y - x)]}{(y - x)[s(y - 2x) - 2x^2]}\right\} \tag{4a}$$

$$\eta_{E[\tilde{N}]}(z) = \sup_{\frac{2x^2}{(y-2x^2)}<s} \min\left\{\eta_{\tilde{\lambda}}(x), \eta_{\tilde{\nu}}(s), \eta_{\tilde{\mu}}(y) | \, z = \frac{x[sy + x(3y - x)]}{(y - x)[s(y - 2x) - 2x^2]}\right\} \tag{4b}$$

It is very difficult to imagine the shapes of membership functions associated with $\eta_{E[\tilde{W}]}$ and $\eta_{E[\tilde{N}]}$, since these functions are not expressed in usual forms. In this paper, a mathematical programming technique is used to solve this problem and in the next section parametric NLPs are developed to find the α-cuts of $f(\tilde{\lambda}, \tilde{\nu}, \tilde{\mu})$.

III THE SOLUTION PROCEDURE

As stated previous section, the membership function of $E[\tilde{W}]$ and $E[\tilde{N}]$ are not in usable form. To construct the membership function in usable form, we apply Zadeh's extension principle to derive the α-cuts of $E[\tilde{W}]$ and $E[\tilde{N}]$, respectively.

Denote the α-cuts of $\tilde{\lambda}$, $\tilde{\nu}$ and $\tilde{\mu}$ as crisp interval as follows:

$$\lambda(\alpha) = [x_\alpha^L, x_\alpha^U] = \left[\min_{x\in X}\{x|\,\eta_{\tilde{\lambda}}(x) \geq \alpha\}, \max_{x\in X}\{x|\,\eta_{\tilde{\lambda}}(x) \geq \alpha\}\right] \tag{5a}$$

$$\nu(\alpha) = [s_\alpha^L, s_\alpha^U] = \left[\min_{s\in S}\{s|\,\eta_{\tilde{\nu}}(s) \geq \alpha\}, \max_{s\in S}\{s|\,\eta_{\tilde{\nu}}(s) \geq \alpha\}\right] \tag{5b}$$

$$\mu(\alpha) = [y_\alpha^L, y_\alpha^U] = \left[\min_{y\in Y}\{y|\,\eta_{\tilde{\mu}}(y) \geq \alpha\}, \max_{y\in Y}\{y|\,\eta_{\tilde{\mu}}(y) \geq \alpha\}\right] \tag{5c}$$

From equations (5a)-(5c), it indicates that $\tilde{\lambda}$, $\tilde{\nu}$ and $\tilde{\mu}$ are shown as intervals when the membership functions are not less than a given possibility level for α. By the fundamental property of convexity of fuzzy numbers, the upper and lower bounds of $\tilde{\lambda}$, $\tilde{\nu}$ and $\tilde{\mu}$ can be represented as functions of α as $x_\alpha^L = \min \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $x_\alpha^U = \max \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $s_\alpha^L = \min \eta_{\tilde{\nu}}^{-1}(\alpha)$, $s_\alpha^U = \max \eta_{\tilde{\nu}}^{-1}(\alpha)$, $y_\alpha^L = \min \eta_{\tilde{\mu}}^{-1}(\alpha)$ and $y_\alpha^U = \max \eta_{\tilde{\mu}}^{-1}(\alpha)$. Therefore we can use the α-cut approach to construct the membership functions of $E[\tilde{W}]$ and $E[\tilde{N}]$ since the membership functions defined in (4a) and (4b) are parameterized by α.

Now we derive the membership function of the expected waiting time. Using Zadeh's extension principle, $\eta_{E[\tilde{W}]}(z)$ is the minimum of $\eta_{\tilde{\lambda}}(x)$, $\eta_{\tilde{\nu}}(s)$ and $\eta_{\tilde{\mu}}(y)$. To derive the membership function, we need at least one of the three cases of the following to hold such that:

$$z = \frac{[sy + x(3y - x)]}{(y - x)[s(y - 2x) - 2x^2]}$$

to satisfy $\eta_{E[\tilde{W}]}(z) = \alpha$.

Case (i): $(\eta_{\tilde{\lambda}}(x) = \alpha, \eta_{\tilde{\nu}}(s) \geq \alpha, \eta_{\tilde{\mu}}(y) \geq \alpha)$,

Case (ii): $(\eta_{\tilde{\lambda}}(x) \geq \alpha, \eta_{\tilde{\nu}}(s) = \alpha, \eta_{\tilde{\mu}}(y) \geq \alpha)$,

Case (iii): $(\eta_{\tilde{\lambda}}(x) = \alpha, \eta_{\tilde{\nu}}(s) \geq \alpha, \eta_{\tilde{\mu}}(y) = \alpha)$.

$$(E[W])_\alpha^{L_1} = \min_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{6a}$$

$$(E[W])_\alpha^{U_1} = \max_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{6b}$$

similarly for case (ii) are

$$(E[W])_\alpha^{L_2} = \min_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{7a}$$

$$(E[W])_\alpha^{U_2} = \max_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{7b}$$

and for case (iii) are

$$(E[W])_\alpha^{L_3} = \min_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{8a}$$

$$(E[W])_\alpha^{U_3} = \max_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{8b}$$

From the definitions of $\lambda(\alpha)$, $\nu(\alpha)$ and $\mu(\alpha)$ in (5a)-(5c), $x \in \lambda(\alpha)$, $s \in \nu(\alpha)$ and $y \in \nu(\alpha)$ can be replaced by $x \in [x_\alpha^L, x_\alpha^U]$, $s \in [s_\alpha^L, s_\alpha^U]$ and $y \in [y_\alpha^L, y_\alpha^U]$. It is noted that α-cuts of x, s and y forms a nested structure with respect to α ([15, 14]) i.e. for two possibility levels α_1 and α_2, we have $[x_{\alpha_1}^L, x_{\alpha_1}^U] \subseteq [x_{\alpha_2}^L, x_{\alpha_2}^U]$, $[s_{\alpha_1}^L, s_{\alpha_1}^U] \subseteq [s_{\alpha_2}^L, s_{\alpha_2}^U]$ and $[y_{\alpha_1}^L, y_{\alpha_1}^U] \subseteq [y_{\alpha_2}^L, y_{\alpha_2}^U]$, where $0 < \alpha_2 < \alpha_2 \leq 1$. In order to construct the membership function $\eta_{E[\tilde{W}]}$, it suffices to find the left and right shape functions of $\eta_{E[\tilde{W}]}$, which is equivalent to finding the lower and upper bound $[E[W])_\alpha^L$ and $[E[W])_\alpha^U$, respectively, of the α-cuts of $\eta_{E[\tilde{W}]}$, which based on (3a) can be written as:

$$(E[W])_\alpha^{L} = \min_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{9}$$

s.t. $x_\alpha^L \leq x \leq x_\alpha^U$, $s_\alpha^L \leq s \leq s_\alpha^U$ and $y_\alpha^L \leq y \leq y_\alpha^U$,

$$(E[W])_\alpha^{U} = \max_{\frac{2x^2}{(y-2x^2)}<s} \left[\frac{[sy + x(3y - x)]}{(y-x)[s(y-2x)-2x^2]}\right] \tag{10}$$

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s.t. $x_\alpha^L \leq x \leq x_\alpha^U$, $s_\alpha^L \leq s \leq s_\alpha^U$ and $y_\alpha^L \leq y \leq y_\alpha^U$.

The equation $\eta_{E[\tilde{W}]}(z) = \alpha$ is true only when at least one of the x, s or y must hit the boundries of their α-cuts. The crisp interval $[(E[W])_\alpha^L, (E[W])_\alpha^U]$ obtained from (9) and (10) represents the α-cuts of $E[\tilde{W}]$. Now, using the results of Zimmermann [14] and Kaufmann [15] and convexity properties to $E[\tilde{W}]$, we have $(E[W])_{\alpha_1}^L \geq (E[W])_{\alpha_2}^L$ and $(E[W])_{\alpha_1}^U \leq (E[W])_{\alpha_2}^U$ for given $0 < \alpha_2 < \alpha_2 \leq 1$. In other words, $(E[W])_\alpha^L$ increases and $(E[W])_\alpha^U$ decreases as α increases. Therefore, the membership function $\eta_{E[\tilde{W}]}(z)$ can be found from (9) and (10).

If both $(E[W])_\alpha^L$ and $(E[W])_\alpha^U$ are invertible with respect to α, then the membership fnction $\eta_{E[\tilde{W}]}(z)$ can be expressed as follows:

$$\eta_{E[\tilde{W}]}(z) = \begin{cases} L(z), & (E[W])_{\alpha=0}^L \leq z \leq (E[W])_{\alpha=1}^L \\ 1, & (E[W])_{\alpha=1}^L \leq z \leq (E[W])_{\alpha=1}^U \\ R(z), & (E[W])_{\alpha=1}^U \leq z \leq (E[W])_{\alpha=0}^U \end{cases} \tag{11}$$

where the left shape function $L(z)$ and the right shape function $R(z)$ are $[(E[W])_\alpha^L]^{-1}$ and $[(E[W])_\alpha^U]^{-1}$, respectively. In most cases, $(E[W])_\alpha^L$ and $(E[W])_\alpha^U$ can not be derived analytically, however, they can be constructed numerically at different possible α levels to approximate the shapes $L(z)$ and $R(z)$. The membership function of the expected number of customers in the system can be derived in a similar manner.

Since the performance measures are described by membership function, the values conserve fuzziness of arrival rate, service rate and retrial rate. To find one crisp value for one of performance measures rather than a fuzzy set, we defuzzify the fuzzy values of performance measures by Yager's ranking index method [18]. Thus suitable values of performance measures are calculated by

$$O(\tilde{\Lambda}) = \int_0^1 \frac{\Lambda_\alpha^L + \Lambda_\alpha^U}{2} d\alpha, \tag{12}$$

where $\tilde{\Lambda}$ is a convex fuzzy number and $(\Lambda_\alpha^L, \Lambda_\alpha^U)$ is the α-cut of $\tilde{\Lambda}$.

IV NUMERICAL EXAMPLE

Consider an Ethernet protocol which uses an access method called CSMA/CD (Carrier Sense Multiple Access/Collision Detection). This is a system where each computer listens to the cable before sending anything through the network. If the network is clear, the computer will transmit the message. If some other node is already

transmitting on the cable, the message comes back to the computer and stored in a buffer to be retransmitted after some time. A situation of collision occurs when sometimes, two computers attempt to transmit the message at the same instant, resulting both messages are lost. The buffer in the computer, the cable and the retransmission policy can be considered as orbit, the server and the retrial policy, respectively, in queueing terminology.

4.1 The fuzzy expected waiting time in the system ($E[\tilde{W}]$)

Suppose the arrival, retrial and service rates are trapezoidal fuzzy numbers represented by $\tilde{\lambda} = [1, 2, 3, 4]$, $\tilde{\nu} = [3, 6, 9, 12]$ and $\tilde{\mu} = [20, 21, 22, 23]$. It is simple to determine $[x^L_\alpha, x^U_\alpha] = [1+\alpha, 4-\alpha]$, $[s^L_\alpha, s^U_\alpha] = [3+3\alpha, 12-3\alpha]$ and $[y^L_\alpha, y^U_\alpha] = [20+\alpha, 23-\alpha]$. Obviously, the expected waiting time in the system attains its minimum value when $x = x^U_\alpha$, $s = s^L_\alpha$ and $y = y^L_\alpha$ and attains its maximum value when $x = x^L_\alpha$, $s = s^U_\alpha$ and $y = y^U_\alpha$. According to Equations (9) and (10), the lower and upper bounds of the α-cut of $E[\tilde{W}]$, respectively, are

$$(E[W])^L_\alpha = \frac{1}{2}\frac{-344 + 17\alpha + \alpha^2}{(-11 + \alpha)(250 - 103\alpha + 7\alpha^2)}$$
$$(E[W])^U_\alpha = \frac{-1}{2}\frac{-284 - 23\alpha + \alpha^2}{(8 + \alpha)(4 + 61\alpha + 7\alpha^2)}$$

With the help of MATLAB® 7.4.0, the inverse functions of $(E[W])^L_\alpha$ and $(E[W])^U_\alpha$ exist, yielding the membership function

$$\eta_{E[\tilde{W}]}(z) = \begin{cases} L(z), & \frac{86}{1375} \leq z \leq \frac{163}{1540} \\ 1, & \frac{163}{1540} \leq z \leq \frac{17}{72} \\ R(z), & \frac{17}{72} \leq z \leq \frac{71}{16} \end{cases}$$

where

$$\begin{aligned} L(z) &= 1/(14z-1)(56440z^3+76404z^2-34710z-1025+168(-4812z^4+107076z^3 \\ &-61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3-61815z^2 \\ &-260550z-13875)^{(1/2)})^{(1/3)}+(1492z^2+836z+145)/(14z-1)/(56440z^3 \\ &+76404z^2-34710z-1025+168(-4812z^4+107076z^3-61815z^2-260550z \\ &-13875)^{(1/2)}z-12(-4812z^4+107076z^3-61815z^2-260550z \\ &-13875)^{(1/2)})^{(1/3)}-3(26z+3)/(14z-1) \end{aligned}$$

and

$$\begin{aligned} R(z) &= -1/2/(14z-1)(-56440z^3-76404z^2+34710z+1025+168(-4812z^4+ \\ &107076z^3-61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3 \\ &-61815z^2-260550z-13875)^{(1/2)})^{(1/3)}-1/2(1492z^2+836z+145)/ \\ &(14z-1)/(-56440z^3-76404z^2+34710z+1025+168(-4812z^4+107076z^3 \\ &-61815z^2-260550z-13875)^{(1/2)}z-(1492z^2+836z+145)/(14z-1)/ \\ &(-56440z^3-76404z^2+34710z-12(-4812z^4+107076z^3-61815z^2 \\ &-260550z-13875)^{(1/2)})^{(1/3)}+6(20z+1)/(14z-1)+1025+168(-4812z^4 \\ &+107076z^3-61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3 \\ &-61815z^2-260550z-13875)^{(1/2)})^{(1/3)} \end{aligned}$$

as shown in Figure 1. The membership functions $L(z)$ and $R(z)$ have complex values with their imaginary parts approaching zero when $\frac{86}{1375} \leq z \leq \frac{163}{1540}$ for $L(z)$ and $\frac{17}{72} \leq z \leq \frac{71}{16}$ for $R(z)$. Hence, the imaginary parts of these two functions can be disregarded.

Now find the expected waiting time in the system by applying the Yager ranking index method stated (12) as follows:

$$O(E[\tilde{W}]) = \int_0^1 \frac{1}{4}\left[\frac{-344+17\alpha+\alpha^2}{(-11+\alpha)(250-103\alpha+7\alpha^2)} - \frac{-284-23\alpha+\alpha^2}{(8+\alpha)(4+61\alpha+7\alpha^2)}\right] d\alpha$$

$$= 0.4303$$

The α-cuts of arrival, retrial and service rates and fuzzy expected waiting time in the system at eleven distinct α values are presented in Table 1. The range of fuzzy expected waiting time in the system at $\alpha = 0$ is $[0.0625, 4.4375]$, indicating that the value of expected waiting time in the system will never exceed 0.0625 or fall below 4.4375. Moreover, the fuzzy expected waiting time in the system is most likely to fall between 0.1058 and 0.2361 when $\alpha = 1$.

4.2 The fuzzy expected number of customers in the system $(E[\tilde{N}])$

Similar to $(E[\tilde{W}])$, the α-cuts of fuzzy expected number of customers in the system $(E[\tilde{N}])$ are

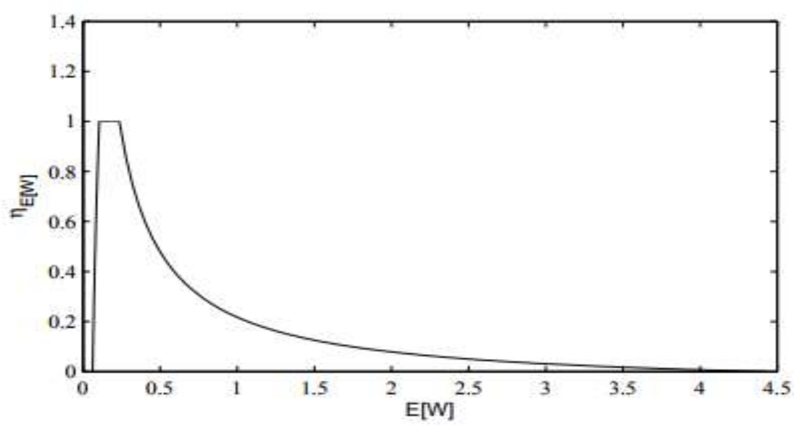

Figure 1: The membership function for fuzzy expected waiting time in the system.

Table 1: α-cuts of arrival, retrial and service rates and expected waiting time in the system

α	x_α^L	x_α^U	s_α^L	s_α^U	y_α^L	y_α^U	$E[W]_\alpha^L$	$E[W]_\alpha^U$
0.0	1.00	4.00	20.00	23.00	3.00	12.00	0.0625	4.4375
0.1	1.10	3.90	20.10	22.90	3.30	11.70	0.0655	1.7377
0.2	1.20	3.80	20.20	22.80	3.60	11.40	0.0686	1.0677
0.3	1.30	3.70	20.30	22.70	3.90	11.10	0.0721	0.7640
0.4	1.40	3.60	20.40	22.60	4.20	10.80	0.0757	0.5909
0.5	1.50	3.50	20.50	22.50	4.50	10.50	0.0797	0.4791
0.6	1.60	3.40	20.60	22.40	4.80	10.20	0.0841	0.4010
0.7	1.70	3.30	20.70	22.30	5.10	9.90	0.0888	0.3435
0.8	1.80	3.20	20.80	22.20	5.40	9.60	0.0939	0.2993
0.9	1.90	3.10	20.90	22.10	5.70	9.30	0.0996	0.2644
1.0	2.00	3.00	21.00	22.00	6.00	9.00	0.1058	0.2361

$$(E[N])^L_\alpha = \frac{1}{2}\frac{(1+\alpha)(-344+17\alpha+\alpha^2)}{(-11+\alpha)(250-103\alpha+7\alpha^2)}$$
$$(E[N])^U_\alpha = \frac{1}{2}\frac{(-4+\alpha)(-284-23\alpha+\alpha^2)}{(8+\alpha)(4+61\alpha+7\alpha^2)}$$

The membership function of $(E[\tilde{N}])$ is represented as:

$$\eta_{E[\tilde{N}]}(z) = \begin{cases} L(z), & \frac{86}{1375} \leq z \leq \frac{163}{770} \\ 1, & \frac{163}{770} \leq z \leq \frac{17}{24} \\ R(z), & \frac{17}{24} \leq z \leq \frac{71}{4} \end{cases}$$

where

$$\begin{aligned} L(z) \;=\; & -1/2/(14z-1)(-56440z^3-76404z^2+34710z+1025+168(-4812z^4 \\ & +107076z^3-61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3 \\ & -61815z^2-260550z-13875)^{(1/2)})^{(1/3)}-1/2(1492z^2+836z+145)/ \\ & (14z-1)/(-56440z^3-76404z^2+34710z+1025+168(-4812z^4+107076z^3 \\ & -61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3-61815z^2 \\ & -260550z-13875)^{(1/2)})^{(1/3)}+6(20z+1)/(14z-1)-(1492z^2+836z+145) \\ & /(14z-1)/(-56440z^3-76404z^2+34710z+1025+168(-4812z^4+107076z^3 \\ & -61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3-61815z^2 \\ & -260550z-13875)^{(1/2)})^{(1/3)}) \end{aligned}$$

and

$$\begin{aligned} R(z) \;=\; & 1/(14z-1)(56440z^3+76404z^2-34710z-1025+168(-4812z^4+107076z^3 \\ & -61815z^2-260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3-61815z^2 \\ & -260550z-13875)^{(1/2)})^{(1/3)}+(1492z^2+836z+145)/(14z-1) \\ & (56440z^3+76404z^2-34710z-1025+168(-4812z^4+107076z^3-61815z^2 \\ & -260550z-13875)^{(1/2)}z-12(-4812z^4+107076z^3-61815z^2-260550z \\ & -13875)^{(1/2)})^{(1/3)}-3(26z+3)/(14z-1) \end{aligned}$$

as shown in Figure 2. Here also the membership functions $L(z)$ and $R(z)$ have complex values with their imaginary parts approaching zero when $\frac{86}{1375} \leq z \leq \frac{163}{770}$

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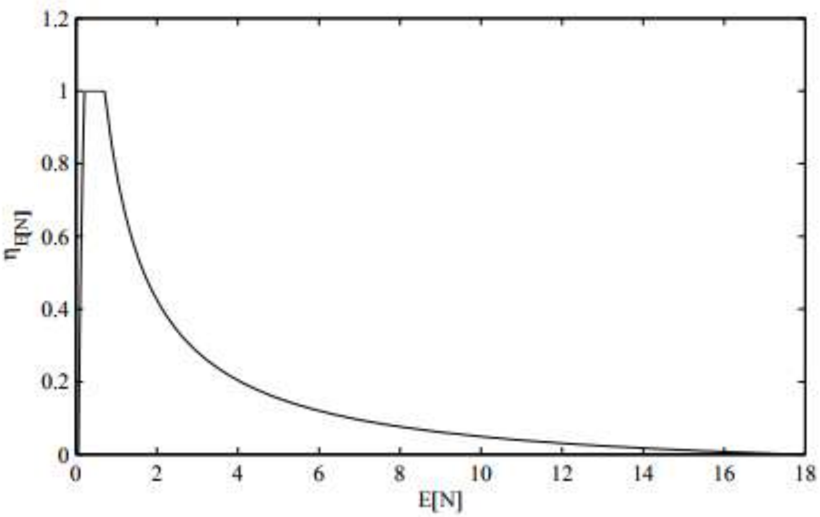

Figure 2: The membership function for fuzzy expected number of customers in the system.

for $L(z)$ and $\frac{17}{24} \leq z \leq \frac{71}{4}$ for $R(z)$. So, the imaginary parts of these two functions can be disregarded.

By the same argument of $(E[\tilde{W}])$, the Yager ranking index of the fuzzy expected number of the customers in the system is as follows:

$$O(E[\tilde{N}]) = \int_0^1 \frac{1}{4}\left[\frac{(1+\alpha)(-344+17\alpha+\alpha^2)}{(-11+\alpha)(250-103\alpha+7\alpha^2)} + \frac{(-4+\alpha)(-284-23\alpha+\alpha^2)}{(8+\alpha)(4+61\alpha+7\alpha^2)}\right] d\alpha$$

$$= 1.5121$$

Table 2 presents the α-cuts of arrival, retrial and service rates and fuzzy expected number of customers in the system at eleven distinct α values. At one extreme level $\alpha = 1$ the range of fuzzy expected number of cutomers in the system is $[0.2117, 0.7083]$, indicating that the fuzzy expected number of customers in the sys-

Table 2: α-cuts of arrival, retrial and service rates and expected number of customers in the system

α	x_α^L	x_α^U	s_α^L	s_α^U	y_α^L	y_α^U	$E[N]_\alpha^L$	$E[N]_\alpha^U$
0.0	1.00	4.00	20.00	23.00	3.00	12.00	0.0625	17.7500
0.1	1.10	3.90	20.10	22.90	3.30	11.70	0.0720	6.7770
0.2	1.20	3.80	20.20	22.80	3.60	11.40	0.0824	4.0571
0.3	1.30	3.70	20.30	22.70	3.90	11.10	0.0937	2.8268
0.4	1.40	3.60	20.40	22.60	4.20	10.80	0.1060	2.1272
0.5	1.50	3.50	20.50	22.50	4.50	10.50	0.1196	1.6769
0.6	1.60	3.40	20.60	22.40	4.80	10.20	0.1345	1.3636
0.7	1.70	3.30	20.70	22.30	5.10	9.90	0.1509	1.1335
0.8	1.80	3.20	20.80	22.20	5.40	9.60	0.1691	0.9578
0.9	1.90	3.10	20.90	22.10	5.70	9.30	0.1892	0.8196
1.0	2.00	3.00	21.00	22.00	6.00	9.00	0.2117	0.7083

tem falls between 0.2117 and 0.7083. Moreover, at the other extreme level $\alpha = 0$, the fuzzy expected number of customers in the system is impossible to fall below 0.0625 or exceed 17.75.

V CONCLUSION

In this paper, a fuzzy retrial queue with collisions is investigated. The concepts of α-cuts and Zadeh's extension principle has been applied to construct membership functions of the expected waiting time and the expected number of customers in the system using paired NLP models. The proposed approach leads to the closed-form expressions for the system characteristics by inverting the α-cuts of the membership functions of system characteristics. Since the system characteristics are described by the membership functions, it would provide more realistic information to system analysts or designers.

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