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A mathematical model to predict soil phosphorus status under varying fertilizer practices and its residual effects on soil phosphorus status for another continuous crop

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ABSTRACT

A mathematical model has been proposed to predict the changes in phosphorus level in soil under varying fertilizer practices and its residual effect on soil phosphorus status for another continuous crop. The model also enabled the prediction of the steady state of soil nutrients for specified fertilizer practice.

The model was applied to two years phosphorus availability data of four fertilizer practices in blackgram crop whose residual effect were studied over wheat crop followed in the field experiment entitled "Integrated nutrient management in blackgram " and their residual effect on succeeding wheat was conducted Rajasthan College of Agriculture, Udaipur, Rajasthan, India. The agreement between the predicted soil phosphorus status by the model and the actual was proved by employing reliability index.

Keywords: Mathematical Model, Nutrient Management , Residual Effect, Phosphorus Status.

I. INTRODUCTION

Excessive use of chemical fertilizers is harmful for the biological power of soil, which must be prevented as all nutrient transformations are performed by soil microflora. Organic matter is a big energy source for the soil microflora and organic carbon content is considered to be the soil health index. Nutrient management is a better approach to minimize the use of chemical sources of nutrient along with maximization of their efficiency and economic profit of farmer. Finck[1] discussed that the nutrient supply, the flows and the nutrient added must be managed properly to achieve good amount of yield while minimizing environmental pollution.

Phosphorus (P) is one of the most essential element of modern agro-system. Fertilization of crops comprises the largest proportion of P used in agriculture. Phosphorous use has become increasingly prevalent in last few decades due to its depletion in soils used for crop. Although the benefits of P on agricultural production are evident, but Schröder[2] examined that it can also be a harmful polluting agent of surface waters and can promote eutropication.

So the level of soil P must be managed at that concentration which allows to good crop production, prevents the escape of P to surface water bodies and adequate animal waste disposal. This paper provides the description of a mathematical model for such a prediction of soil p level for a rotational crop system. Sen[3] and Ahmed[4] examined that in rotation, legumes increases the availability of several nutrients for succeeding crops. Legume cultivation leads to increase soil available P probably ascribed to development of P-solubilizing organisms in root zone of legumes. Usherwood[5] found with nitrogen P significantly contributes to optimum crop yield and nitrogen use efficiency. A preliminary verification of data of fertilizer experiment is also presented here.

II. THE MODEL

For predicting the steady state of phosphorus levels in plots which have received the same fertilizer treatments over the years, the following balance equations are taken as basic equation to show the status behavior of phosphorus in soil after fertilization and its residual effect on soil phosphorus status respectively,

$$P_{(i,B)} = P_{(i-1,W)} + F_{i,B} - U_{iB} - F_r + E_B \tag{1}$$

$$P_{(i,W)} = P_{(i,B)} - U_{iW} + E_W \tag{2}$$

Where $P_{(i,B)}$ shows the phosphorus level in soil after the blackgram crop in ith year, $F_{i,B}$ shows the amount of fertilizer applied to ith crop of blackgram only, U_{iB} shows the amount of phosphorus uptake by ith crop of blackgram, $P_{(i,W)}$ shows the phosphorus level in soil after the wheat crop in ith year U_{iW} shows the amount of phosphorus uptake by ith crop of wheat, F_r runoff amount of fertilizer applied, E_B and E_W are the built-up level of phosphorus due to the factor other than considered in basic equations.

We assume that uptake of phosphorus U_{iB} by blackgram crop depends on the phosphorus available in soil after the previous wheat crop $P_{(i-1,W)}$ and the applied fertilizer $F_{i,B}$,

$$U_{iB} = f(P_{(i-1,W)}, F_{i,B})$$

Or it can be written as,

$$U_{iB} = \gamma_B P_{(i-1,W)} + \delta_B F_{iB} + C_B \tag{3}$$

Where constant γ_B shows the expected soil phosphorus nutrient efficiency $(0 \le \gamma_B \le 1)$ for blackgram crop, δ_B shows expected fertilizer nutrient efficiency $(0 \le \delta_B \le 1)$ for blackgram crop and C_B shows the uptake of phosphorus from unaccounted sources by blackgram crop $(C_{\mathbf{B}} \ge 0)$.

Similarly the uptake of phosphorus U_{iW} by wheat crop depends on the phosphorus available in soil after the previous blackgram crop $P_{(i-1,B)}$ only as fertilization practice is not applied on wheat crop,

$$U_{iW} = g(P_{(i,B)})$$

Or it can be written as,

$$U_{iW} = \gamma_W P_{(i,B)} + C_W \tag{4}$$

Where constant γ_W shows the expected soil phosphorus nutrient efficiency $(0 \le \gamma_W \le 1)$ for wheat crop, and C_W shows the uptake of phosphorus from unaccounted sources by wheat crop $(C_W \ge 0)$.

III. SOLUTION OF MODEL

Using (3) in (1), we get

$$P_{(i,B)} = P_{(i-1,W)} + F_{i,B} - (\gamma_B P_{(i-1,W)} + \delta_B F_{i,B} + C_B) - F_r + E_B$$

Or $$P_{(i,B)} = (1-\gamma_B)P_{(i-1,W)} + (1-\delta_B)F_{i,B} - F_r + E_B - C_B \tag{5}$$

Using (4) in (2), we get

$$P_{(i,W)} = P_{(i,B)} - (\gamma_W P_{(i,B)} + C_W) + E_W$$

Or $$P_{(i,W)} = (1-\gamma_W)P_{(i,B)} - C_W + E_W \tag{6}$$

Using (6) in (5), we get

$$P_{(i,B)} = (1-\gamma_B)(1-\gamma_W)P_{(i-1,B)} + (1-\delta_B)F_{i,B} + \{(1-\gamma_B)(E_W - C_W) + E_B - C_B - F_r\} \tag{7}$$

Using iteration in (7), we have

$$P_{(i,B)} = (1-\gamma_B)^2(1-\gamma_W)^2 P_{(i-2,B)} + (1-\delta_B)[(1-\gamma_B)(1-\gamma_W)F_{i-1,B} + F_{i,B}]$$

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$$+\{(1-\gamma_B)(E_W-C_W)+E_B-C_B-F_r\}[(1-\gamma_B)(1-\gamma_W)+1] \tag{8}$$

Iterating the right hand side of the above equation, we get

$$P_{(i,B)}=(1-\gamma_B)^n(1-\gamma_W)^n P_{(i-n,B)}+(1-\delta_B)\sum_{j=0}^{n-1}(1-\gamma_B)^j(1-\gamma_W)^j F_{i-j,B}$$

$$+\{(1-\gamma_B)(E_W-C_W)+E_B-C_B-F_r\}\left[\frac{1-(1-\gamma_B)^n(1-\gamma_W)^n}{1-(1-\gamma_B)(1-\gamma_W)}\right] \tag{9}$$

This equation shows relationship of $P_{(i,B)}$ with the available soil phosphorus status at the end of (i-n) th crop and the amount of fertilizer nutrient applied from n to ith crop.

Equation (9) can be reduced to the equation to predict $P_{(i,B)}$ from initial value like P_0 by taking n=i

$$P_{(i,B)}=(1-\gamma_B)^i(1-\gamma_W)^i P_0+(1-\delta_B)\sum_{j=0}^{i-1}(1-\gamma_B)^j(1-\gamma_W)^j F_{i-j,B}$$

$$+\{(1-\gamma_B)(E_W-C_W)+E_B-C_B-F_r\}\left[\frac{1-(1-\gamma_B)^i(1-\gamma_W)^i}{1-(1-\gamma_B)(1-\gamma_W)}\right] \tag{10}$$

If a constant amount of fertilizer($F_{i,B}=F_B$) is used every year then the above equation can be reduced in,

$$P_{(i,B)}=(1-\gamma_B)^i(1-\gamma_W)^i P_0+(1-\delta_B)F_B\left[\frac{1-(1-\gamma_B)^i(1-\gamma_W)^i}{1-(1-\gamma_B)(1-\gamma_W)}\right]$$

$$+\{(1-\gamma_B)(E_W-C_W)+E_B-C_B-F_r\}\left[\frac{1-(1-\gamma_B)^i(1-\gamma_W)^i}{1-(1-\gamma_B)(1-\gamma_W)}\right] \tag{11}$$

The phosphorus status in the soil in the long run can also be predicted by taking limit as $i\rightarrow\infty$ in above equation. If $P_{(\infty,B)}$ denotes the steady state of phosphorus status due to constant fertilization, then above equation becomes,

$$P_{(\infty,B)}=\frac{(1-\delta_B)F_B}{1-(1-\gamma_B)(1-\gamma_W)}+\frac{\{(1-\gamma_B)(E_W-C_W)+E_B-C_B-F_r\}}{1-(1-\gamma_B)(1-\gamma_W)} \tag{12}$$

Now solution for residual effects on soil phosphorus status for another continuous crop.

IJARSE
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Using equation(5)in (6), we get

$$P_{(i,W)} = (1-\gamma_W)(1-\gamma_B)P_{(i-1,W)} + (1-\gamma_W)(1-\delta_B)F_{i,B} + (1-\gamma_W)(E_B - C_B - F_r) - C_W + E_W \quad (13)$$

Using iteration in (13), we haves

$$P_{(i,W)} = (1-\gamma_W)^2(1-\gamma_B)^2 P_{(i-2,W)} + (1-\gamma_W)(1-\delta_B)[(1-\gamma_W)(1-\gamma_B)F_{i-1,B} + F_{i,B}]$$

$$+\{(1-\gamma_W)(E_B - C_B - F_r) - C_W + E_W\}\{1+(1-\gamma_w)(1-\gamma_B)\}$$

Iterating the right hand side of the above equation, we get

$$P_{(i,W)} = (1-\gamma_W)^n(1-\gamma_B)^n P_{(i-n,W)} + (1-\gamma_W)(1-\delta_B)\sum_{j=0}^{n-1}(1-\gamma_W)^j(1-\gamma_B)^j F_{i-j,B}$$

$$+\{(1-\gamma_W)(E_B - C_B - F_r) - C_W + E_W\}\left[\frac{1-(1-\gamma_w)^n(1-\gamma_B)^n}{1-(1-\gamma_w)(1-\gamma_B)}\right] \quad (14)$$

by taking n=i this Equation reduced to the equation to predict $P_{(i,B)}$ from initial value like P_0

$$P_{(i,W)} = (1-\gamma_W)^i(1-\gamma_B)^i P_0 + (1-\gamma_W)(1-\delta_B)\sum_{j=0}^{i-1}(1-\gamma_W)^j(1-\gamma_B)^j F_{i-j,B}$$

$$+\{(1-\gamma_W)(E_B - C_B - F_r) - C_W + E_W\}\left[\frac{1-(1-\gamma_w)^i(1-\gamma_B)^i}{1-(1-\gamma_w)(1-\gamma_B)}\right] \quad (15)$$

When constant amount of fertilizer($F_{i,B} = F_B$) is used every year then the above equation can be reduced in,

$$P_{(i,W)} = (1-\gamma_W)^i(1-\gamma_B)^i P_0 + (1-\gamma_W)(1-\delta_B)F_B\left[\frac{1-(1-\gamma_w)^i(1-\gamma_B)^i}{1-(1-\gamma_w)(1-\gamma_B)}\right]$$

$$+\{(1-\gamma_W)(E_B - C_B - F_r) - C_W + E_W\}\left[\frac{1-(1-\gamma_w)^i(1-\gamma_B)^i}{1-(1-\gamma_w)(1-\gamma_B)}\right] \quad (16)$$

As $i \to \infty$, $P_{(\infty,W)}$ denotes the steady state of phosphorus status due to residual effect of constant fertilization, then above equation becomes

$$P_{(\infty,W)} = \left[\frac{(1-\gamma_W)(1-\delta_B)F_B}{1-(1-\gamma_w)(1-\gamma_B)}\right] + \left[\frac{(1-\gamma_W)(E_B - C_B - F_r) - C_W + E_W}{1-(1-\gamma_w)(1-\gamma_B)}\right] \quad (17)$$

IV. VALIDITATION OF MODEL

Soil phosphorus levels (observed and predicted from the model) can be tested by computing a reliability index as suggested by Leggett[6]. This index denoted by *k* interpret that the model predictions agree with observations within a factor of *k*. The index is defined using geometric approach and is justified through agreement with another index developed using statistical techniques. These indices denoted, respectively, by k_g and k_s are given by,

$$k_g = \frac{1+\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1-y_i/x_i}{1+y_i/x_i}\right]^2}}{1-\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1-y_i/x_i}{1+y_i/x_i}\right]^2}}$$

and $$k_s = \exp\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\log\frac{y_i}{x_i}\right)^2}$$

where X_i and Y_i are the predicted values using model and corresponding observed values respectively. For a perfectly predicting model, $k_g = k_s = 1$.

V. THE SOURCE OF DATA

The above prescribed model was applied on investigation entitled "Integrated nutrient management in blackgram (Phaseolus mungo L.)" was conducted during 2003-04 and 2004-05 by Rathore[7] at RCA, Udaipur. The region lies under typical sub-humid climatic conditions average annual rainfall 637 mm, soil of the experimental field was clay loam in texture. Initially, to ascertain various characteristics of the experimental field, soil samples were taken upto 15 cm depth contained 268.40 kg N ha-1 using Alkaline permanganate method by Subbiah[8], 19.50 kg P ha-1 using Olsen's method by Olsen[9], 370.80 kg K_2O ha^{-1} using Extraction with 1 N neutral ammonium acetate at pH 7.0 and estimated by Flame photometer method by Richards[10]. This experiment was consisted of thirty two treatment combinations, out of these we are using here only four which are

i. No phosphorus
ii. 50% P (20 kg P_2O_5 ha^{-1})

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iii. 75% P (30 kg P_2O_5 ha^{-1})
iv. 100% P (40 kg P_2O_5 ha^{-1})

Phosphorus entirely was drilled through Di-Ammonium-Phosphate (DAP) before sowing of the crop also uniform application of nitrogen @ 15 kg N ha^{-1} was done as basal through DAP + urea.

After harvest of blackgram, every experimental plot was prepared without disturbing original plan of layout and only N was given using urea @ 90 kg N ha^{-1} in two equal splits at sowing and at first irrigation. After harvesting soil sample taken from each plot upto 15 cm depth and analyzed also plant analysis for nitrogen using Nessler's reagent, spectrophotometrically method by Snell[11] and phosphorus using Vanadomolybdate phosphoric acid yellow colour method by Jackson[12] were done.

The average soil nutrient efficiency parameter was estimated using the uptake and soil available phosphorus values of control plots. If these are respectively U_t^0 and P_{t-1}^0, then

$$\gamma = \frac{\sum U_t^0 P_{t-1}^0}{\sum (P_{t-1}^0)^2}$$

The average fertilizer nutrient efficiency parameter of applied phosphorus was calculated by using difference of uptake in treatment and control and unit of applied P fertilizer. If uptake from control plot is U_t^0 and form a plot of some treatment is U_t and applied amount of fertilizer is F unit, then

$$\delta = \frac{U_t - U_t^0}{F}$$

VI. RESULT AND DISCUSSION

The estimation of various parameter Υ, δ, C and E for each crop and for each treatment is presented in table 1. Soil phosphorus efficiency (Υ) is very high (0.9215304) for wheat than blackgram. Fertilizer phosphorus efficiency (δ) are approximately same for 20 and 30 kg P ha^{-1}, while for 40 kg P ha^{-1} is approx 18% higher. Uptake from unaccounted sources is very less (0.2243)for blackgram than wheat which is very high(4.0349). Table 1 shows that maximum depletion took place about 100% P treatment for blackgram crop than 50% P and 75% P treatment if built up in soil for blackgram assumed constant as in control then fertilizer run off (F_r) calculated maximum 87% for 100% P treatment and approx 86% for 50% P and 75% P treatment. Maximum built up; as the residual effect of 100% P treatment applied for blackgram seen for wheat crop. While built up about 50% P and 75% P treatment are moderate as a residual effect over wheat crop.

Table 1. Estimates of Υ, δ, C and E-F_r for phosphorus for the crops in the sequence

Parameter	Crop	
	Blackgram	Wheat
Υ	278322	0.9215304
δ		
50% P (20 kg P_2O_5 ha^{-1})	0.052	
75% P (30 kg P_2O_5 ha^{-1})	0.055167	
100% P (40 kg P_2O_5 ha^{-1})	0.06175	
C(kg P ha^{-1})	0.22433	4.0349
E-Fr(kg P ha^{-1})		
Control	7.905	16.2
50% P (20 kg P_2O_5 ha^{-1})	-9.41	17.52
75% P (30 kg P_2O_5 ha^{-1})	-17.93	18.65
100% P (40 kg P_2O_5 ha^{-1})	-27.065	20.58

Predicted soil phosphorus status of long term practices about various treatment is shown in table2. Accumulation of phosphorus in soil is maximum for both crop about 100% P. Comparison between predicted and observed phosphorus soil level is presented in table 3 and 4 for both crops respectively. The reliability indices show observed and predicted data from model are approximately same and for the treatment 100% P are closely agreed.

Table 2. Predicted steady state soil phosphorus levels for the crops in the sequence

Treatment	Crop	
	Blackgram	Wheat
Control	17.4480334	13.5342402
50% P (20 kg P_2O_5 ha^{-1})	20.20158118	15.07030999
75% P (30 kg P_2O_5 ha^{-1})	21.9829562	16.34009378
100% P (40 kg P_2O_5 ha^{-1})	23.51240722	18.39010919

Table 3. Observed and predicted soil phosphorus status after the harvest of each crop of Blackgram

Treatment	year 2003-04		year 2004-05	
	Predicted	Observed	Predicted	Observed
Control	17.39434647	18.4	16.51286942	20.56
50% P (20 kg P_2O_5 ha^{-1})	19.99196143	21.7	20.18971046	22.35
75% P (30 kg P_2O_5 ha^{-1})	21.67245757	22.83	21.96537273	23.33
100% P (40 kg P_2O_5 ha^{-1})	23.11529611	23.77	23.4899189	23.43

Table 4. Observed and predicted soil phosphorus after the harvest of each crop of Wheat

Treatment	year 2003-04		year 2004-05	
	Predicted	Observed	Predicted	Observed
Control	13.70219054	16.8	13.54375119	18.89
50% P (20 kg P_2O_5 ha^{-1})	15.15127303	18.6	15.07489491	19.18
75% P (30 kg P_2O_5 ha^{-1})	16.34914923	18.98	16.34060659	19.41
100% P (40 kg P_2O_5 ha^{-1})	18.28307271	19.92	18.38404774	19.52

Table 5. Reliability indices for the proposed model

Index	Treatment			
	Control	50% P (20 kg P_2O_5 ha^{-1})	75% P (30 kg P_2O_5 ha^{-1})	100% P (40 kg P_2O_5 ha^{-1})
k_g	1.252297931	1.18635304	1.128133841	1.055642337
k_s	1.252982397	1.18662048	1.128240143	1.055655801

VII. CONCLUSION

Thus the theoretical basis provided by this proposed mathematical model is valid since it enables the prediction of soil phosphorus level within permissible limit of variation. It is also useful in establishing estimates on the steady state of soil phosphorus status for a specific fertilizer treatment apply on one and not other in a continuous cropping system.

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