International Journal of Advance Research in Science and Engineering
Volume No.06, Issue No. 12, December 2017
www.ijarse.com
IJARSE
ISSN: 2319-8354

Optimizing Inventory Policy for Seasonal Products with Trade Credit and Price Discounts for Shelf Life items and Backorders

Rahul Kumar Pandey[1], Bindu Vaish[2]

[1]*Department of Mathematics, IMSEC Ghaziabad (India)*

[2]*Department of Mathematics, D. N. College Meerut (India)*

ABSTRACT

In this paper an inventory model for items with fixed shelf life period and seasonal demand is presented. Items are sold out before shelf life (expiry period) of items offering price discount in decreasing phase of demand. Shortages are assumed with partial backlogging. To reduce the lost sales backordered quantity is enhanced by declaration of offering price discount. Further a credit period is allowed by the supplier to the retailer to earn interest on the accumulated sales revenue till the credit period. The retailer pays interest on the amount not sold till the credit period. Numerical illustrations with tables, graphs and sensitivity analysis of the optimal solution are presented.

Keywords: Shelf life, Partial backlogging, Seasonal demand, credit period, Price discounting.

I. INTRODUCTION

Deterioration of items is a common aspect in any inventory control system. It's happen when the items of inventory become outworn, debase, decay or damaged associating on the type of goods. As an upshot of the deterioration shortages may happen. Hence deterioration portion has to be given so much importance while obtaining the optimal policy for an inventory model. Ghare and Schrader (1963) were the first to include the deterioration factor while determining inventory policy. Weibull distribution deterioration factor was initiating first time in EOQ model by Covert and Philip(1973). Since then many researchers presented their models using deterioration factor in different ways like Datta and Pal (1990b), Goyal and Giri (2001). Pandey and vaish (2017) proposed an optimal inventory policy with demand depending on seasonal factor for deteriorating items.

Many products have finite shelf life like packed food items (biscuits, jam and bread etc.) fashionable items and medicines etc. Because of limited shelf-life or expiry period in order to make the balance in inventory cost at optimum level, a perfect inventory model is required which suits to meet the actual demand in the market. Avinadev (2009) developed an EOQ model with fixed shelf life for declining demand rate. Ukil et al (2015) developed an EPQ model for products having finite shelf life. Muriana (2016) introduced an inventory policy with fixed shelf life for perishable products under stochastic demand.

Dave and Patel (1981) first introduced the linear time dependent demand in inventory formulating model. Deng et al (2007) , Darzanou and Skouri (2011) and Wu et al (2014) developed inventory models assuming ramp type demand. Jaggi(2013) introduced inspection ordering policy for deteriorating items with time-dependent demand under inflationary conditions.

Price discount on unit selling price of goods attracts the customers to buy more and more. Thus price discount is an important factor which enhances the demand which in turn increases the total profit per unit time. There are so many authors who used price discount factor with some other related factors in developing their inventory models like Ardalan (1994) with temporary price discount, Pan and Hsiao (2005) and Lee et al (2007) with controllable lead time ,Sana and Chaudhari (2008) with increasing profit, Hsu and Yu (2009) with imperfect items, Lin (2009) with integrated vendor buyer inventory policy with back order, Cardanas-Barron et al (2010) with one-time discount offer and back orders, Garg, Vaish and Gupta (2012) with ramp type demand and Vaish, B and Agarwal, D (2012) for non-instantaneous deterioration with quadratic demand rate . Heydari (2014) developed supply chain inventory model with time based policy for temporary price discounting. Liying et al (2016) developed a price discount policy for coordinating supplier, retailer and carrier. Pandey and Vaish (2017) developed an inventory policy with seasonal demand and price discounting on back orders.

Normally, the payment for any consignment is done by retailer to the supplier immediately. But in present days, due to tough competition in market and for attracting more customers a credit period has to be allotted to retailer by the suppliers to attain the maximum profit. No interest has to pay during this time credit period but beyond it, the retailer has to be pay an interest on the amount in stock. Before the end of time period of trade credit limit, the retailer can accumulate the revenue and earn so much profit and interest. This is termed as permissible delay in payment. Hence it is more profitable to the retailer to delay his payment till the very last day of the settlement period. First time inventory model with permissible delay in payment was developed by Goyal (1985) using suitable conditions. After that many authors presented inventory models using permissible delay factor like Chand and Ward (1987), Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) ,Jamal et al. (1997) ,Chang and Dye (2001), Ouyang et al. (2005),Tsao and Sheen (2008) and Teng et al (2011). Jaggi (2013a) described an inventory model with effects of inspection on retailer's ordering policy for deteriorating items with time-dependent demand. Jun li (2014) introduced an inventory model with credit period. Khanna (2017) developed an inventory policy for deteriorating imperfect quality items including permissible delay in payment.

In many real situations stock ends before the arrival of new consignment and many customers do not want wait for the next consignment. This is called as partial backlogging. This results in a greater amount of lost sales which in turn has a profit loss. While formulating an inventory model with shortages considering the partial backlogging is necessary. Many authors presented their models with shortages using factor of partial backlogging like Montgomery et al. (1973), Skouri (2003) ,Chang and Dye (1999), Dye et al. (2007) and Pandey et al (2016).

There are items which do not deteriorate slowly but have certain shelf life period (expiry period) from the date of their manufacturing beyond which they become obsolete and cannot be sold. The customer buys these products seeing their expiry date. Thus as the expiry date comes nearer some customers are not willing to purchase these items and thus the demand goes in decreasing phase. Therefore to attract customers and to sell all inventories before expiry date of the product, the inventory manager offers a price discount in the decreasing phase of demand. Further in the case of partial backlogging a declaration of offering a price discount on backordered quantity reduces the lost sales to a great extant. Lastly it is more profitable for the retailer to earn interest on the accumulated sales revenue till the permissible delay payment period. The present paper is prepared considering the above mentioned features.

ASSUMPTIONS and NOTATIONS:

The following assumptions are used to develop the model:

1. The demand rate is $at(T-t)$. a >0

2. It is assumed that product has a certain shelf life period T and after that it becomes obsolete and cannot be sold. So after receiving the inventory at time u (T/2 < u < T) (decreasing phase of demand) a price discount d_1 on selling price of each unit is offered by the retailer to customer to boost up the demand by $\alpha_1(\alpha_1 = (1-d_1)^{-n_1})$ where n_1 is a positive real number. As a result all inventories are sold out before expiry period.

3. Shortages are allowed and are partially backlogged. A declaration of offering price discount d_2 on back ordered quantity increases the demand by $\alpha_2(\alpha_2 = (1-d_2)^{-n_2})$ where n_2 is a positive real number.

c	Purchasing cost per unit
T	Cycle length and shelf life period of item
t_1	The time at which inventory level becomes zero
Q_1	Initial inventory level at the beginning of each cycle and
Q_2	Backordered quantity
Q	Ordered Quantity (Q_1+Q_2)
I_p	Rate of interest paid
Ie	Rate of interest earned
M	The retailer's trade credit period offered by supplier
h	Holding cost per unit per unit time
s	Shortage cost per unit per unit time
l	Lost sale cost per unit per unit time
O	Ordering cost per order
ε	Rate of backlogging $0 < \varepsilon < 1$
SR	Sales revenue per replenishment cycle
I(t)	The inventory level at time t.
$F(t_1)$	Profit per unit time

II. MODEL FORMULATION AND ANALYSIS

The equations governing the inventory level I(t) at any time t during the inventory cycle T are given as follows:

$$\frac{dI(t)}{dt} = -at(T-t) \qquad 0 \le t \le u \qquad \dots (1)$$

$$\frac{dI(t)}{dt} = -\alpha_1 at(T-t) \qquad u \le t \le t_1 \qquad \ldots (2)$$

$$\frac{dI(t)}{dt} = -\alpha_2 \varepsilon at(T-t) \qquad t_1 \le t \le T \qquad \ldots (3)$$

With boundary condition $I(0) = Q_1, I(t_1) = 0$ … (4)

the solutions of above equations are given by:

$$I(t) = -\frac{aTt^2}{2} + \frac{at^3}{3} + Q_1 \qquad 0 \le t \le u \qquad \ldots (5)$$

$$I(t) = \alpha_1[\frac{aT}{2}(t_1^2 - t^2) - \frac{a}{3}(t_1^3 - t^3)] \qquad u \le t \le t_1 \qquad \ldots (6)$$

$$I(t) = \alpha_2 \varepsilon[\frac{aT}{2}(t_1^2 - t^2) - \frac{a}{3}(t_1^3 - t^3)] \qquad t_1 \le t \le T \qquad \ldots (7)$$

Equating the value of I (u) from equation (5) and equation (6) then the value of Q_1 can be obtained as

$$Q_1 = \alpha_1 a(\frac{Tt_1^2}{2} - \frac{t_1^3}{3}) + a(\frac{Tu^2}{2} - \frac{u^3}{3})(1-\alpha_1) \qquad \ldots (8)$$

Q_2 can be obtained as $Q_2 = -I(T)$

$$Q_2 = a\alpha_2 \varepsilon(\frac{T^3}{6} - \frac{Tt_1^2}{2} + \frac{t_1^3}{3}) \qquad \ldots (9)$$

The Purchasing cost *PC* is given by

$$PC = c\{Q_1 + Q_2\}$$

$$PC = ca\{\alpha_1(\frac{Tt_1^2}{2} - \frac{t_1^3}{3}) + (1-\alpha_1)(\frac{Tu^2}{2} - \frac{u^3}{3}) + \alpha_2 \varepsilon(\frac{T^3}{6} - \frac{Tt_1^2}{2} + \frac{t_1^3}{3})\} \qquad \ldots (10)$$

The holding cost *HC* is given by

$$HC = h\{\int_0^u I(t)dt + \int_u^{t_1} I(t)dt + \int_{t_1}^T I(t)dt\}$$

$$HC = ah\{\frac{Tu^3}{3}(1-\alpha_1) + \frac{Tt_1^3}{3}(\alpha_1 - 2\alpha_2 \varepsilon) - \frac{t_1^4}{4}(\alpha_1 - \alpha_2 \varepsilon) - \frac{u^4(1-\alpha_1)}{4} + \frac{\alpha_2 \varepsilon T^2}{12}(6t_1^2 - T^2)\} \qquad \ldots (11)$$

The shortage cost *SC* is given by

$$SC = -s\int_{t_1}^T I(t)dt$$

$$SC = -sa\alpha_2 \varepsilon \int_{t_1}^T [\frac{T}{2}(t_1^2 - t^2) - \frac{1}{3}(t_1^3 - t^3)]dt$$

$$SC = s\alpha_2 a\varepsilon\{\frac{2Tt_1^3}{3} + \frac{T^4}{12} - \frac{t_1^4}{4} - \frac{T^2 t_1^2}{2}\} \qquad \ldots (12)$$

The lost sale *LSC* cost for the system can be calculated as

$$LSC = al\int_{t_1}^{T}\{t(T-t) - \alpha_2 \varepsilon t(T-t)\}dt$$

$$LSC = al(1-\alpha_2\varepsilon)\{\frac{T^3}{6} - \frac{Tt_1^2}{2} + \frac{t_1^3}{3}\} \qquad \dots (13)$$

The Sales Revenue *SR* is given by

$$SR = p\{\int_0^u at(T-t)dt + (1-d_1)\int_u^{t_1}\alpha_1 at(T-t)dt + Q_2(1-d_2)\}$$

$$SR = ap\{\frac{Tu^2}{2} - \frac{u^3}{3} + \frac{(1-d_1)\alpha_1 T(t_1^2-u^2)}{2} - \frac{(1-d_1)\alpha_1(t_1^3-u^3)}{3} + (1-d_2)\alpha_2\varepsilon(\frac{T^3}{6} - \frac{Tt_1^2}{2} + \frac{t_1^3}{3})\} \qquad \dots (14)$$

The Present model also assumes the permissible delay in payments, therefore subjected to the credit period, three possible cases arise for retailer's total profit $M \le u \le t_1$, $u \le M \le t_1$ and $t_1 \le M \le T$

Case-1: $M \le u \le t_1$ in this case the retailer can use sales revenue to earn interest during the period [0, M] and the annual interest will be paid for the amount of inventory not sold till the credit period M. The total interest earned and the total interest paid will be as follows:

Interest Earned

$$IE_1 = pI_e(\int_0^M at(T-t)tdt + (1-d_2)MQ_2)$$

$$IE_1 = apI_e\{\frac{TM^3}{6} - \frac{M^4}{12} + (1-d_2)\alpha_2\varepsilon M(\frac{T^3}{6} - \frac{Tt_1^2}{2} + \frac{t_1^3}{3})\} \qquad \dots (15)$$

Interest Paid

$$IP_1 = cI_p(\int_M^u I(t)dt + \int_u^{t_1} I(t)dt)$$

$$IP_1 = acI_p\{\frac{TM^3}{6} - \frac{M^4}{12} - (1-\alpha_1)(\frac{Tu^3}{6} - \frac{u^4}{12}) + \frac{Tt_1^3}{3} - \frac{t_1^4}{4} + (\frac{Tt_1^2}{2} - \frac{t_1^3}{3})(\alpha_1(u-M)-u)$$
$$+(1-\alpha_1)(u-M)(\frac{Tu^2}{2} - \frac{u^3}{3})\} \qquad \dots (16)$$

The profit function per unit time for this case can be calculated as follows:

$$F(t_1)=\frac{1}{T}[S.R.+IE_1-IP_1-P.C.-H.C.-S.C.-L.S.C.-O.C.]$$

$$F(t_1)=\frac{1}{T}[ap\{\frac{Tu^2}{2}-\frac{u^3}{3}+\frac{(1-d_1)\alpha_1T(t_1^2-u^2)}{2}-\frac{(1-d_1)\alpha_1(t_1^3-u^3)}{3}+(1-d_2)\alpha_2\varepsilon(\frac{T}{2}(T^2-t_1^2)$$
$$-\frac{1}{3}(T^3-t_1^3))\}+apI_e\{\frac{TM^3}{6}-\frac{M^4}{12}+(1-d_2)\alpha_2\varepsilon M(\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3})\}$$
$$-acI_p\{\frac{TM^3}{6}-\frac{M^4}{12}-(1-\alpha_1)(\frac{Tu^3}{6}-\frac{u^4}{12})+\frac{Tt_1^3}{3}-\frac{t_1^4}{4}+(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})(\alpha_1(u-M)-u)+(1-\alpha_1)(u-M)(\frac{Tu^2}{2}-\frac{u^3}{3})\}$$
$$-ca\{\alpha_1(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})+(1-\alpha_1)(\frac{Tu^2}{2}-\frac{u^3}{3})+\alpha_2\varepsilon(\frac{T}{2}(T^2-t_1^2)-\frac{1}{3}(T^3-t_1^3))\}-ah\{\frac{Tu^3}{3}(1-\alpha_1)+$$
$$\frac{Tt_1^3}{3}(\alpha_1-2\alpha_2\varepsilon)-\frac{t_1^4}{4}(\alpha_1-\alpha_2\varepsilon)-\frac{u^4(1-\alpha_1)}{4}+\frac{\alpha_2\varepsilon T^2}{12}(6t_1^2-T^2)\}-sa\alpha_2\varepsilon\{\frac{2Tt_1^3}{3}+\frac{T^4}{12}-\frac{t_1^4}{4}-\frac{T^2t_1^2}{2}\}$$
$$-la(1-\alpha_2\varepsilon)\{\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3}\}-o]$$

…(17)

Case-2: $u\le M\le t_1$ in this case the retailer can use sales revenue to earn interest during the period [0, *M*] and the annual interest will be paid for the amount of inventory not sold till the credit period M. The total interest earned and the total interest paid will be as follows:

Interest Earned

$$IE_2=pI_e\{\int_0^u at(T-t)tdt+(M-u)\int_0^u at(T-t)dt+(1-d_1)\int_u^M a\alpha_1t(T-t)tdt+MQ_2\}$$

$$IE_2=apI_e\{\frac{u^4}{4}-\frac{(T+M)u^3}{3}+\frac{MTu^2}{2}+(1-d_1)^{-n_1}(1-d_1)(\frac{TM^3}{6}-\frac{M^4}{12}-\frac{u^4}{4}+\frac{(T+M)u^3}{3}-\frac{MTu^2}{2})$$
$$+(1-d_2)\alpha_2\varepsilon M(\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3})\} \quad \ldots(18)$$

Interest Paid

$$IP_2=cI_p\int_m^{t_1}I(t)dt$$

$$IP_2=acI_p\alpha_1\{\frac{TM^3}{6}-\frac{M^4}{12}+\frac{Tt_1^3}{3}-\frac{t_1^4}{4}-\frac{MTt_1^2}{2}+\frac{Mt_1^3}{3}\} \quad \ldots(19)$$

The profit function per unit time for this case can be calculated as follows:

$$F(t_1)=\frac{1}{T}[S.R.+IE_2-IP_2-P.C.-H.C.-S.C.-L.S.C.-O.C.]$$

$$F(t_1)=\frac{1}{T}[ap\{\frac{Tu^2}{2}-\frac{u^3}{3}+\frac{(1-d_1)\alpha_1 T(t_1^2-u^2)}{2}-\frac{(1-d_1)\alpha_1(t_1^3-u^3)}{3}+(1-d_2)\alpha_2\varepsilon(\frac{T}{2}(T^2-t_1^2)$$
$$-\frac{1}{3}(T^3-t_1^3))\}+apI_e\{\frac{u^4}{4}-\frac{(T+M)u^3}{3}+\frac{MTu^2}{2}+(1-d_1)^{-n_1}(1-d_1)(\frac{TM^3}{6}-\frac{M^4}{12}-\frac{u^4}{4}+\frac{(T+M)u^3}{3}-\frac{MTu^2}{2})$$
$$+(1-d_1)^{-n}(1-d_1)(\frac{1}{4}(t_1^4-u^4)-\frac{1}{3}(T+M)(t_1^3-u^3)+\frac{MT}{2}(t_1^2-u^2))+(1-d_2)\alpha_2\varepsilon M(\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3})\}$$
$$-acI_p\{\frac{TM^3}{6}-\frac{M^4}{12}-(1-\alpha_1)(\frac{Tu^3}{6}-\frac{u^4}{12})+\frac{Tt_1^3}{3}-\frac{t_1^4}{4}+(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})(\alpha_1(u-M)-u)+(1-\alpha_1)(u-M)(\frac{Tu^2}{2}-\frac{u^3}{3})\}$$
$$-ca\{\alpha_1(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})+(1-\alpha_1)(\frac{Tu^2}{2}-\frac{u^3}{3})+\alpha_2\varepsilon(\frac{T}{2}(T^2-t_1^2)-\frac{1}{3}(T^3-t_1^3))\}-ah\{\frac{Tu^3}{3}(1-\alpha_1)$$
$$+\frac{Tt_1^3}{3}(\alpha_1-2\alpha_2\varepsilon)-\frac{t_1^4}{4}(\alpha_1-\alpha_2\varepsilon)-\frac{u^4(1-\alpha_1)}{4}+\frac{\alpha_2\varepsilon T^2}{12}(6t_1^2-T^2)\}-sa\alpha_2\varepsilon\{\frac{2Tt_1^3}{3}+\frac{T^4}{12}-\frac{t_1^4}{4}-\frac{T^2t_1^2}{2}\}$$
$$-la(1-\alpha_2\varepsilon)\{\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3}\}-o] \qquad (20)$$

Case-3: $u\le t_1\le M\le T$ in this in this case the retailer can use sales revenue to earn interest during the period [0, M] and as there is no inventory on hand beyond the credit period M so the retailer has to pay no interest. The total interest earned will be as follows:

Interest Earned

$$IE_3=pI_e\{\int_0^u at(T-t)tdt+(M-u)\int_0^u at(T-t)dt+(1-d_1)\int_u^{t_1} a\alpha_1 t(T-t)tdt+(M-t_1)\int_u^{t_1} a\alpha_1 t(T-t)dt+$$
$$MQ_2(1-d_2)\}$$
$$IE_3=apI_e\{\frac{u^4}{4}-\frac{(T+M)u^3}{3}+\frac{MTu^2}{2}+(1-d_1)^{-n}(1-d_1)(\frac{1}{4}(t_1^4-u^4)-\frac{1}{3}(T+M)(t_1^3-u^3)+\frac{MT}{2}(t_1^2-u^2))+(1-d_2)\alpha_2\varepsilon M(\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3})\} \quad (21)$$

The profit function per unit time for this case can be calculated as follows:

$$F(t_1)=\frac{1}{T}[S.R.+I_e-I_p-P.C.-H.C.-S.C.-L.S.C.-O.C.]$$

$$F(t_1)=\frac{1}{T}[ap\{\frac{Tu^2}{2}-\frac{u^3}{3}+\frac{(1-d_1)\alpha_1 T(t_1^2-u^2)}{2}-\frac{(1-d_1)\alpha_1(t_1^3-u^3)}{3}+(1-d_2)\alpha_2\varepsilon(\frac{T}{2}(T^2-t_1^2)$$
$$-\frac{1}{3}(T^3-t_1^3))\}+apI_e\{\frac{TM^3}{6}-\frac{M^4}{12}+(1-d_2)\alpha_2\varepsilon M(\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3})\}$$
$$-acI_p\{\frac{Tm^3}{6}-\frac{m^4}{12}-(1-\alpha_1)(\frac{Tu^3}{6}-\frac{u^4}{12})+\frac{Tt_1^3}{3}-\frac{t_1^4}{4}+(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})(\alpha_1(u-m)-u)+(1-\alpha_1)(u-m)(\frac{Tu^2}{2}-\frac{u^3}{3})\}$$
$$-ca\{\alpha_1(\frac{Tt_1^2}{2}-\frac{t_1^3}{3})+(1-\alpha_1)(\frac{Tu^2}{2}-\frac{u^3}{3})+\alpha_2\varepsilon(\frac{T}{2}(T^2-t_1^2)-\frac{1}{3}(T^3-t_1^3))\}-ah\{\frac{Tu^3}{3}(1-\alpha_1)+$$
$$\frac{Tt_1^3}{3}(\alpha_1-2\alpha_2\varepsilon)-\frac{t_1^4}{4}(\alpha_1-\alpha_2\varepsilon)-\frac{u^4(1-\alpha_1)}{4}+\frac{\alpha_2\varepsilon T^2}{12}(6t_1^2-T^2)\}-sa\alpha_2\varepsilon\{\frac{2Tt_1^3}{3}+\frac{T^4}{12}-\frac{t_1^4}{4}-\frac{T^2t_1^2}{2}\}$$
$$-la(1-\alpha_2\varepsilon)\{\frac{T^3}{6}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3}\}-o]$$

… (22)

International Journal of Advance Research in Science and Engineering
Volume No.06, Issue No. 12, December 2017
www.ijarse.com
IJARSE
ISSN: 2319-8354

To find optimal values for profit function F (t_1)

The optimal value of t_1 is obtained by solving the equation

$$\frac{dF(t_1)}{dt_1} = 0 \qquad \text{… (23)}$$

Provided $\frac{d^2F(t_1)}{dt_1^2} < 0$ (24)

Numerical Illustration for case 1: $M \le u \le t_1$

T=**1** year, u=0.55, M=0.50units, p = 45rs, s=0.2rs/unit/time, l=0.2rs/unit/time, a=50000, h=0.8 rs/unit/time, n_1=5, ε= 0.6, c=40rs, α_2= $(1\text{-}0.20)^{-2}$, o =200rs/order, I_e=0.05, I_p=0.07, b=0.8, d_1=0.12, d_2=0.20, Applying the solution procedure described above the optimal values obtained are as follows:

t_1*= 0.765436 days, $F^*(t_1)$ =12875.8rs, Q^*= 10393.7, $\frac{d^2F^*(t_1)}{dt_1^2} = -33695.1$

Numerical Illustration for case 2: $u \le M \le t_1$

T=**1** year, u=0.55, M=0.75units, p = 45rs, s=0.2rs/unit/time, l=0.2rs/unit/time, a=50000, h=0.8 rs/unit/time, n_1=5, n_2= 2, ε= 0.6, c=40rs, o =200rs/order, I_e=0.05, I_p=0.07, d_1=0.12, d_2=0.20, Applying the solution procedure described above the optimal values obtained are as follows:

t_1*= 0.823708 days, $F^*(t_1)$ = 210435 rs, Q^*= 108482, $\frac{d^2F^*(t_1)}{dt_1^2} = -454455$

Numerical Illustration for case 3: $t_1 \le M \le T$

T=**1** year, u=0.55, M=0.95units, p = 45rs, s=0.2rs/unit/time, l=0.2rs/unit/time, a=50000, h=0.8 rs/unit/time, n_1=5, ε= 0.6, c=40rs, α_2= $(1\text{-}0.20)^{-2}$, o =200rs/order, I_e=0.05,I_p=0.07, b=0.8, d_1=0.12, d_2=0.20, Applying the solution procedure described above the optimal values obtained are as follows:

t_1*= 0.935795 days, $F^*(t_1)$ = 257845 rs, Q^*= 114102, $\frac{d^2F^*(t_1)}{dt_1^2} = -141353$

For the case 2 the effects of various parameters on total profit per unit are obtained as follows;

Effects of parameter"u" on Total Profit per Unit Time

%change in u	u	t1	F
-15%	0.4675	0.823708	151190
-10%	0.495	0.823708	170748
-5%	0.5225	0.823708	190536
0	0.55	0.823708	210435
5%	0.5775	0.823708	230320
10%	0.605	0.823708	250062
15%	0.6325	0.823708	269531

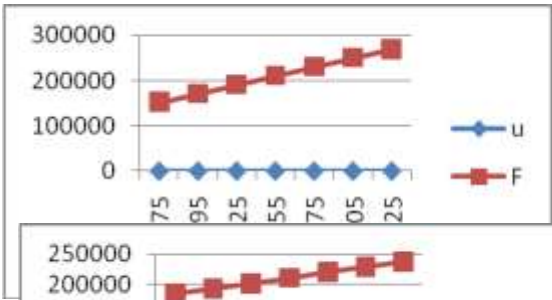
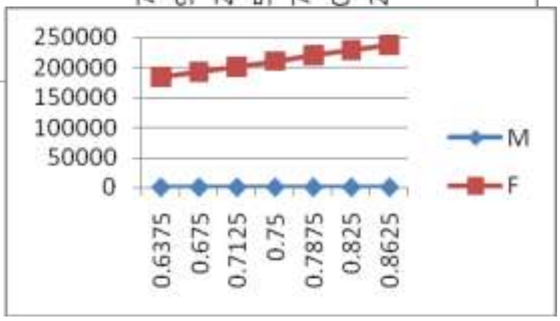


Table-1

Effects of parameter"M" on Total Profit per Unit

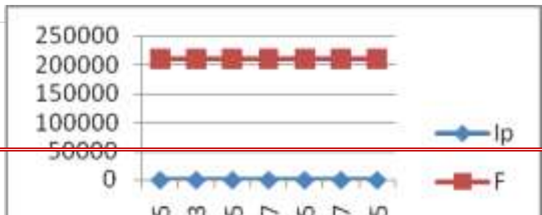


%change in M	M	t1	F
-15%	0.6375	0.758675	183914
-10%	0.675	0.780353	192743
-5%	0.7125	0.80203	201584
0	0.75	0.823708	210435
5%	0.7875	0.845386	219295
10%	0.825	0.867063	228164
15%	0.8625	0.888741	237046

Table-2 **Fig-2**

Effects of parameter"I_p" on Total Profit per Unit Time

%change in I_p	I_p	t1	F



IJARSE
ISSN: 2319-8354

-15%	0.0595	0.835393	210631
-10%	0.063	0.830535	210560
-5%	0.0665	0.82697	210495
0	0.07	0.823708	210435
5%	0.0735	0.820711	210379
10%	0.077	0.817948	210328
15%	0.0805	0.815393	210280

Table-3 **Fig-3**

Effects of parameter"I_e" on Total Profit per Unit Time

%change in I_e	I_e	t1	F
-15%	0.0425	0.854039	201389
-10%	0.045	0.843928	204362
-5%	0.0475	0.833818	207376
0	0.05	0.823708	210435
5%	0.0525	0.813598	213540
10%	0.055	0.803487	216694
15%	0.0575	0.793377	219898

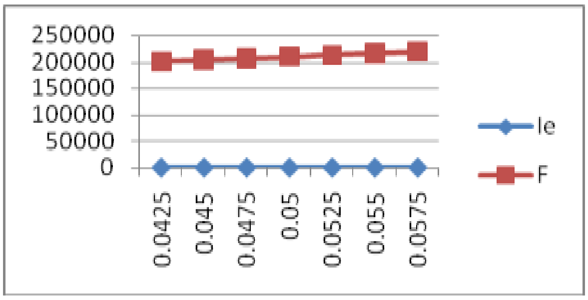


Table-4 **Fig-4**

Effects of parameter"h" on Total Profit per Unit Time

%changein h	h	t1	F
-15%	0.68	0.85742	216932
-10%	0.72	0.846043	214714
-5%	0.76	0.834807	212548
0	0.8	0.823708	210435
5%	0.84	0.812744	208377
10%	0.88	0.801913	206376
15%	0.92	0.791211	204434

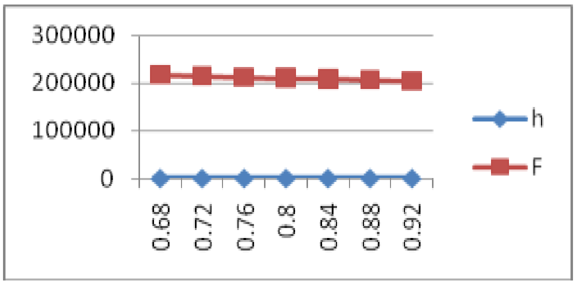


Table-4 **Fig-4**

Sensitivity Analysis

International Journal of Advance Research in Science and Engineering
Volume No.06, Issue No. 12, December 2017
www.ijarse.com
IJARSE
ISSN: 2319-8354

parameter	% change	%change t_1	%change $F(t_1)$
u	-15	0	-0.281535866
	-5	0	-0.094561266
	5	0	0.094494737
	15	0	0.280827809
M	-15	-0.078951522	-0.126029415
	-5	-0.026317579	-0.042060494
	5	0.026317579	0.042103262
	15	0.078951522	0.126457101
Ip	-15	0.014185852	0.000931404
	-5	0.003960141	0.000285124
	5	-0.003638425	-0.000266115
	15	-0.010094597	-0.000736569
Ie	-15	0.036822515	-0.042987146
	-5	0.012273767	-0.014536555
	5	-0.012273767	0.01475515
	15	-0.036822515	0.044968755
h	-15	0.040927125	0.030874142
	-5	0.014974967	0.010041105
	5	-0.013310542	-0.009779742
	15	-0.039452087	-0.028517119

III. OBSERVATIONS

1. Table (1) reveals that as the parameter (u) decreases, profit per unit time of the system increases.
2. From table (2) it is observed parameter (M) increases, profit per unit time of the system also increases.
3. Table (3) shows that as the interest paid (I_p) increases, profit per unit time of the system also decreases.
4. From table (4) it is observed that the interest earn (I_e) increases, the unit time profit of the system increases.
5. Table (5) reveals that the holding cost (h) decreases, the unit time profit of the system also increases.
6. From sensitivity table (6) it has been observed $F(t_1)$ is negligible sensitive to (I_p), moderately sensitive to (I_e) and h and fairly sensitive to M and *u*.

IV. CONCLUSION

In the present paper an inventory model is developed with realistic features of seasonal demand and partial backlogging. The most important feature of the modal is the product shelf life or expiry period of items. These items have no selling value after their date of expiry. So it is necessary to sell all inventory of such type of items before their expiry date. Considering this fact a price discount is offered to the customers in decreasing phase of demand to sell out all inventories before the expiry period and to reduce the loss. Further declaration of price discount at the start of shortage period enhances the backorder quantity. The most common feature of present market is that the suppliers allot the retailers a credit period to settle the account. In this period the retailers earn interest on the accumulated sales revenue and pays interest on the amount not sold till credit period. The present paper is prepared considering the above mentioned real features. Three possible cases for credit period are discussed and the numerical illustrations, tables, graph and sensitivity analysis represents the reality of the model to practical situations of the market. The present study can be further studied for some other situations of demand and partial backlogging useful for inventory problems.

REFERENCES

[1.] Aggarwal, S. P. and Jaggi, C. K. (1995) “Ordering policies of deteriorating items under permissible delay in payments,” Journal of the Operational Research Society, 46(5) 658–662.

[2.] Ardalan A (1994) Optimal prices & Order quantities when temporary price discounts result on increase in demand, European Journal of Operational Research72(1) 52-61.

[3.] Avinadav, T. , & Arponen, T. (2009). An EOQ model for items with a fixed shelf-life and a declining demand rate based on time-to-expiry. Asia-Pacific Journal of Op- erational Research, 26 (6), 759–767 3465

[4.] Cardenas-Barron LE, Smith NR and Goyal SK (2010) Economic production quantity with rework process at a single stage manufacturing system with planned backorders. Applied MathematicalModeling34, 1642,1652.

[5.] Chang, H.-J. and Dye C.-Y. (2001) “An inventory model for deteriorating items with partial backlogging and permissible delay in payments,” International Journal of Systems Science, vol. 32, no. 3, pp. 345–352.

[6.] Cinzia Muriana (2016) An EOQ model for perishable products with fixed shelf life under stochastic demand conditions, Production, Manufacturing and Logistics,1-9.

[7.] Covert, R. P. and Philip, G. C. (1973): An EOQ model for items with Weibull distribution deterioration, AIIE Transactions, 5(4), 323-326.

[8.] Darzanou, G. and Skouri, K. (2011) “An inventory system for deteriorating products with ramp-type demand rate under two-level trade credit financing,” Advances in Decision Sciences, vol. 2011, Article ID 761961, 15 pages, 2011.

[9.] Das, D., Roy, A., Kar, S., (2011) “Optimum payment time for retailer under permitted delay of payment bythe wholesaler with dynamic demand and hybrid number cost parameters”, Opsearch, 48(3) 171-196.

[10.] Datta, T. K. and Pal, A. K., (1990b): Deterministic inventory systems for deteriorating items with inventory-level-dependent demand rate and Shortages, Opsearch, 27, 167–176.

[11.] Dave, U. and Patel, L. K. (1981):(T, S_i) policy inventory model for deteriorating items with time proportional demand. Journal of Operational Research Society, 32, 137-142.

[12.] Deng, P. S., Lin, R. H. J. and Chu, P. (2007): A note on the inventory models for deteriorating items with ramp type demand, European Journal of Operational Research,178, 112-120.

[13.] Douglas C. Montgomery, M. S. Bazaraa, Ajit K. Keswani(1973) Inventory models with a mixture of backorders and lost sales, Navel research logistics, 20(2), 255-263.

[14.] Garg, G., Vaish, B. and Gupta, S. (2012) An economic production lot size model with price discounting and non-instantaneous deteriorating items with ramp type production and demand rates Int.J.Contemp.Math.Science7 (11), 531-554.

[15.] Ghare, P. M. and Schrader, G. H. (1963): A model for exponentially decaying inventory system, International Journal of Production Research, 21, 46-49.

[16.] Goyal, S. K. and Giri, B. C. (2001b):A comment on chang and Dye (1999): An EOQ model for deteriorating items with time varying demand and partial backlogging, Journal of Operational Research Society, 52(2), 238-239

[17.] Goyal, S. K. (1985) "Economic order quantity under conditions of permissible delay in payments," Journal of the Operational Research Society, vol. 36, no. 4, pp. 335–338,

[18.] Heydari, J. (2014) Supply chain coordination using time-based temporary price discount, Computers & Industrial Engineering, 75, 96-101.

[19.] Hwang, H. and Shinn, S. W. (1997) "Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments," Computers &Operations Research, vol. 24, no. 6, pp. 539–547, 1997.

[20.] Jaggi, C. K., Mittal, M., & Khanna, A. (2013a) Effects of inspection on retailer's ordering policy for deteriorating items with time-dependent demand under inflationary conditions. International Journal of Systems Science, 44(9), 1774-1782.

[21.] Jaggi, C. K., Goel, S. K., & Mittal, M. (2013b) Credit financing in economic ordering policies for defective items with allowable shortages. Applied Mathematics and Computation, 219(10), 5268-5282.

[22.] Jamal, A. M. M., Sarker, B. R. and Wang, S. (1997) "An ordering policy for deteriorating items with allowable shortage and permissible delay in payment," Journal of the Operational Research Society,vol. 48, no. 8, pp. 826–833.

[23.] Jun Li, Hairong Feng, Yinlian Zeng (2014) Inventory games with permissible delay in payments, European Journal of Operational Research, 234(3) 694-700

[24.] Khanna, A., Gautam, P. and Jaggi, C. K. (2017) Inventory modelling for deteriorating imperfect quality items with selling price dependent demand and shortage backordering under credit financing, International Journal of Mathematical, Engineering and Management Sciences 2(2),110–124, 2017

[25.] Lee WC, Wu JW, and Lei CL (2007)Computational algorithmic procedure for optimal inventory policy involving ordering cost reduction and back-order discounts when lead time demand is controllable, Applied Mathematics and Computation189 (1), 186–200.

[26.] Lin, Yu-Jen (2009) An integrated vendor–buyer inventory model with backorder price discount and effective investment to reduce ordering cost, Computers & Industrial Engineering, 56(4),1597-1606.

[27.] Liying Li, Yong Wang, Wei Dai (2016) Coordinating supplier retailer and carrier with price discount policy, Applied Mathematical Modelling, 40(1), 646-657.

[28.] Ouyang, L.Y., Teng, J.T., Chen, L.H.(2005) Optimum ordering policy for deteriorating items with partial backlogging under permissible delay in payments, Journal of Global Optimization, 34, 245-271

[29.] Pan JCH and Hsiao YC (2005) Integrated inventory models with controllable lead time and backorder discount considerations. International Journal of Production Economics, 94, 387-397.

[30.] Pandey RK et al (2016) An EOQ model with quantity incentive strategy for deteriorating items and partial backlogging, Uncertain Supply Chain Management 5(2),135-142.

[31.] Pandey RK and Vaish B(2017) Optimal inventory policy for deteriorating items with seasonal demand under the effect of price discounting on lost sales, IOSR Journal of Mathematics 13(3), 35-42.

[32.] Papachristos, S. and Skouri, K.(2003) An inventory model with deteriorating items, quantity discount, pricing and time-dependent partial backlogging, International journal of production economics, 83(3), 247-256.

[33.] Sana SS and Chaudheri KS(2008) A deterministic EOQ models with delays in payments and price-discount Offers European Journal of Operational Research 184,509-533.

[34.] Teng, J.-T. , Krommyda, I.-P., Skouri, K. and Lou, K.-R. (2011) “A comprehensive extension of optimal ordering policy for stock dependent demand under progressive payment scheme,” European Journal of Operational Research, 215(1), 97–104.

[35.] Tsao , Y.-C. and Sheen, G.-J. (2008) “Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payments,” Computers & Operations Research, 35(11),3562–3580.

[36.] Ukil, S. I., Islam, M. E. and Uddin, M. S. (2015) A Production Inventory Model of Power Demand and Constant Production Rate Where the Products Have Finite Shelf-Life, Journal of Service Science and Management, 2015, 8, 874-885.

[37.] Within, T. M. (1957): The theory of inventory management, Princeton University Press, Princeton, New Jersey, 62-72.

[38.] Wu, J. , Ouyang, L.-Y. ,Cardenas-Barron, L.E. and Goyal, S. K. (2014) Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing, European Journal of Operational Research, 237(3),898–908.