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# Big Social Networks: Practical Analysis and Modeling of the ActivityDilemmas

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#### **ABSTRACT**

The communication in social networking is not limited to person to person communication, but extends to a wider range involving thing to thing communication and person to thing communication. Thus, it also called big social networking services .in order to motivate users of online social networks to share information and communicate with each other frequently, we first analyzed the activity statuses of users in one of famous social networks, Weibo, and then proposed a lurker game model for accumulating big data.

In addition to the features of the public goods game, this model also introduces the factor of individualincentive depending on his degree. We found that the individual strategy to be chosen was not relevant to the user's degree, but to an incentive constant of the entire network. The simulation results showed that individual strategies asymptotically followed three different behaviors according to the dynamic organization of the individuals. Active users will emerge during the evolutionary process with an incentive. Without anincentive, active central users can hardly affect the states of their neighbors and may even become lurkersdue to the large number of lurking neighbors. Large noise decreases the influence of the high motivation and causes the chaos of networks. active users will gradually lose interest and leave the network If the continuous chaos exists.

Keywordsbig data analytics, common goods game,complex network, evolutionary game,Social network

#### **I.INTRODUCTION**

The applications of big data, such as big data analytics in the Internet of Things, are more important. The social networking service, as one of these applications, has received extensive attention. Social networking services are not limited to person-to-person communication, but extend to a wider range involving person-to-thing communication and thing-to-thing communication. Therefore, it is also called big social networking services. In the social networks, cooperation plays a key role in the evolution process of species from cellular organisms to vertebrates. However, understanding the emergence of cooperation in the evolution theory remains a challenge to date [1]. Moreover, the evolutionary game theory has been considered as an important researchframework to characterize and understand the cooperation mechanism in the systems consisting of selfish individuals [2]. A lot of attention is being paid to the analysis of evolutionary dynamics of pairwise interactions, such as the prisoner's dilemma game [3], the snowdrift game [4], and the stag-hunt game [5]. In these models, being a defector is always better than being a cooperator. However, both players as cooperators are always better than

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both players as defectors. These game structures reflect the situation of interest in reality [6]. The game model, the network topology, and the evolutionary rule are regarded as three key factors of an evolutionary game in a network. In recent years, these game models in typical networks, such as small-world networks and scale-free networks, havebeen intensively studied [7]. Under some circumstances, social dilemmas involve larger groups of interactional individuals rather than pairs. Public goods game was introduced to explain these phenomena [8]. Although selfishness and competitiveness are inherent characters of human nature, humans are willing to cooperate if proper conditions are available. Cooperation failure results in the exploitation of public goods, such as environmental resources or social benefits, by defectors, who reap benefits on the expense of cooperators. The ``tragedy of the commons' succinctly describes such a situation [9]. An aspiration-induced reconnection mechanism into the spatial public goods game has been introduced [10]. A player will

reconnect to a randomly chosen player if its payoff acquired from the group centered on the neighbor does not exceed the aspiration level.

Social Networking Service (SNS) [11], as the product of Web 2.0, is a new medium, in which `interaction" is regarded as the core. Strictly speaking, SNS typically refers to social networking websites, including its derived products. Those users, who are not only information receivers but also producers, releasers, and disseminators of information, become an essential part of the process of formation and evolution of public opinion in networks. Nowadays, online social networks have attracted millions of users. In social networks,

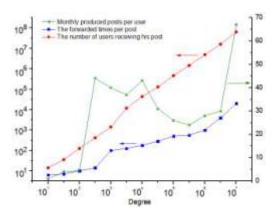
Users interact with others, build relationships, publish posts or replies, and discuss topics. Therefore, the growth of social networks is promoted by users' actions. As a kind of public resource in social networks, information is only produced by a fraction of users and shared among users in a larger scope. Defectors get benefit without any cost, and eventually drive the network into a dull atmosphere. Administrators and operators of social networks fell into a dilemma: how to promote users to share information and communicate with each other frequently. To solve this problem, in this paper, we proposed a lurker game model. The lurker game is a type of the evolutionary public goods game. In addition to the features of the public goods game, this model also introduced the factor of individual incentive depending on his degree. We find that the individual strategy to be chosen is not relevant to the user's degree, but to an incentive constant of the entire network. The simulation results showed that individual strategies asymptotically followed three different behaviors according to the dynamicorganization of the individuals. Active users emerged during the evolutionary process with an incentive. Without anincentive, active central users could hardly affect the statesof their neighbors and might even become lurkers due to thegreat number of lurking neighbors. Large noise decreases theinfluence of the high incentive and causes the chaos of networks. If the chaos is continuous, active users will graduallylose interest and leave the network.

#### II.PRACTICALANALYSIS

Weibo, a Chinese microblogging service, has become one of the most popular social media on the Internet and its registered users and monthly active users in 2015 are respectively 503 million and 212 million. Its influence expands from original domains such as news and entertainment to fresh domains such as finance, sports, and travel. Weibo is always the source of popular topics with public opinions towards them. Therefore, studying the

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fluctuation process of user's activity behaviors of Weibo helps us to understand the influencing factors of the activity levels in online social networks. In this paper, a lot of data were extracted with our web spider, including information about users and their related posts. After half an hour of collection from Weibo, 422 171 user profiles and approximately 2.5 million posts from May 2015 to July 2015 were downloaded. The average degree was 26.4 and the clustering coefficient was 0.15. The degree distribution obeyed a power law, a typical property of scale-free networks [12]. Then we observed the activity status of each user every other hour in this website.



**FIGURE 1.**User's influence versus user's degree. A user's influenceconsists of three aspects: The monthly produced posts per user (activitylevel), the forwarded times per post (transmissibility) and the number of users receiving his post directly and indirectly (coverage).

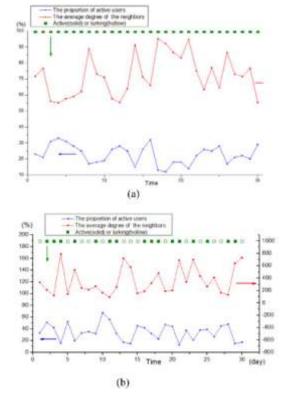
Due to the social networks' features of high clustering and high connectivity, hot topics can spread rapidly in a wide range. Users in the central position are usually willing to share new resource because their obtained influence is much greater than their original influence derived from their previously provided resources. The influence of a user consists of three aspects. The first one is the activity level, which is measured by the amount of the informative content produced by a user; the second one is the transmissibility, the average forwarded times per post of a user; the last one is the coverage, which is usually measured by the number of users receiving his post, including his direct fans and indirect users receiving his post forwarded by other users. Fig. 1 shows the variations in the three aspects with users' degrees. A user with a huge degree, 107, is willing to publish several posts every day because he/she may be a celebrity, especially a pop star or a film star. Each of his/her actions arouses the public interest. Based on his popularity, the number of forwarded times per post and the number of users receiving his post is far larger than other users in the network. People focusing on these stars are just for entertainment and likely to share the stars' statuses among their friends. A user with asmaller degree, 106 or 105, may be a celebrity in specialized fields such as finance and sports. He/she usually publishesone post every day to maintain his position in the field. An active user in a small community, with a degree of 500 to 104, is more motivated than the former to publishposts because he wants to promote his fame. An active useroften publishes more than one post every day to keep intouch with his friends more frequently. An inactive user witha degree less than 100 has no motive to publish any post. He always views other people's posts and sometimes forwards one of them. In this paper, a new term, lurkers, isgiven to this type of users. A post from a lurker cannot arouse attention and is rarely forwarded by other users. Inaddition, the number of users receiving a user's post is

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nearlyin proportion to his degree. Taking all the three aspects into account as a whole, a user's influence is consistent withhis degree. A large degree always results in a significant influence. We exacted two users from the huge number of accumulated users. Both users use the service more or less, do notcompletely lurk or leave the service in June 2015. One ofthem has a large degree (1,794,702), the other has a relativelysmall degree (533). As shown in Fig. 2, both users are greatly influenced by the activity statuses of their neighbors. Theuser in Fig. 2(a), who has a large number of neighbors, is acentral user. Meanwhile, the user in Fig. 2(b) is an ordinaryuser. Most of the neighbors of the central user may just viewthe post from celebrities and do not publish any post bythemselves. Therefore, the activity levels of his neighbors are lower than those in Fig. 2(b). When active neighbors becomeless, the rest of them are central users, who frequently publishposts to maintain their popularity in the communities. Thegreen squares in the upper part of Fig. 2(a) and Fig. 2(b) indicate the user's activity statuses during the month. The centraluser is always active without any interruption because of hiscentral position in the whole network. His fans are willingto know his daily life and working situation. High profit foreach of his posts motivates him to keep active. In contrast, the ordinary user changes his activity status according to hisneighbors' previous statuses. If the lurkers of his neighborsare more than actives ones, nowthe user may probably chooseto lurk.

#### **III.MODEL**

The emergence of online social networking services overthe past decade has revolutionized the way that socialscientists study the structure of human relationships. In its simplest form, a social network contains individuals as vertices and edges as relationships among vertices. An active cluster of a social network consists of active users who share opinions and communicate with othersof the cluster frequently. The content and resource of the



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**FIGURE 2.**Two typical users are greatly influenced by the activity statuses of their neighbors. The user (a) has a large number of neighbors, theother user (b) is an ordinary user.

network stem from all users. The users to choose the strategy of sharing contents gain payoff from public information and popularity; while the lurkers to share no resourcewill also take the advantage of public information. The strategy selection of next moment between *activity* (A) and *lurking* (L) is a kind of public goods game. Activity and lurking are the two strategies equivalent to the strategies of cooperation and defection in public goods game. The processis named as a lurker game and the game model can be described as:

1) The game involving many people and two strategies. It is supposed that there are two types of strategies (si) foruser i: activity (A) or lurking (L). An active user brings profitb to all of his neighbors. The active status of a user canonly influence directly linked neighbors and has no effecton neighbors' neighbors. nidenotes the number of user i'sneighbors (including user i itself) with the strategy A, and ki is the degree of user i. In a step of the game, the profit of useri is described as:

$$\frac{nib}{ki+1}(1)$$

The active strategy brings much more profit, such as promoting the reputation, to a user than lurking strategy. Continuously active users always have more significant influences in the network, and become central users more easily. The influence of user i is mainly associated with the number of his neighbors, i.e. his degree. User i brings "kitimes of profit to the network. "is a constant called gain coefficient. The high connectivity of social networks means high gain coefficient, so  $\square > 1$ . The profit of user i with the active strategy can be denoted as:

$$\frac{\varepsilon ki \cdot b}{ki + 1} \tag{2}$$

In each step, each user needs the same cost c to keep in theactive status. The payoff of each user obtained in the game is defined as Pi. When user i chooses the active strategy,

$$Pi = \frac{nib}{ki+1} + \frac{\varepsilon ki \cdot b}{ki+1} - c = \frac{(\varepsilon ki + ni)b}{ki+1dx} - c$$
 (3)

When user *i* chooses the lurking strategy,

$$Pi = \frac{nib}{ki+1} \tag{4}$$

2) Strategy changes. At every evolutionary step, user i chooses a neighbor j randomly and compares his payoffwith user j's payoff. User i will change his strategy to userj's strategy with certain probability in the next evolutionarystep.

$$W(Si \leftarrow Sj) = \frac{1}{1 + \exp((Pi - Pj)/k)}$$
 (5)

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Where si and Pi represent the strategy and payoff of useriat this step, respectively. The fermifunction of statistical physics is adopted in Eq. (5), which reveals the following mechanism: if user i obtains the lower payoff than user j, user i intends to adopt user j's strategy. However, if user i obtains the higher payoff than user j, user i may adopt userj's strategy with tiny probability. K indicates the environmentnoise and describes the uncertainty that a user changes hisstrategy. When  $K \to 0$ , the influence of the uncertainty iszero. If the payoff of his friend is higher than his, he will adoptit; otherwise he will persist in his strategy. When  $K \to 1$ , theuser is surrounded by noise. He is incapable to make a rational decision and can only update his strategy randomly.

#### IV.DISCUSSION

The payoff of each user is different under different conditions:

- 1) When most of neighbors are active, to get some qualitative conclusions, it is supposed that  $ni \rightarrow ki$ . Meanwhile, the degree of a user is much larger than one, i.e.ki>>1. A user with the active strategy gains the payoff $Pi = \Box$  b+bc, and a user with the lurking strategygains the payoff Pi = b.
- 2) When most of neighbors are lurking, nis much less than ki, i.e. ni < < ki. In the extreme case, ni = 0. A userwith the active strategy gains the payoff  $bi = \Box b \ll c$ , and a user with the lurking strategy gains the payoff Pi = 0. The payoff matrix is expressed as:

$$A\begin{pmatrix} A & L \\ \varepsilon b + b - c & \Box b - c \\ b & 0 \end{pmatrix}$$
 (6)

Wherethe strategy A or L chosen by neighbors indicates a taus that the majority of them choose the strategy A or L.In the lurker game, what is the optimal choice to a rationaluser? An obvious conclusion can be drawn from the matrix follows: the individual strategy to be chosen depends on the value of  $\Box b - c$  other than the value of ki. When  $\Box b$ -c>0, mostusers adopt the active strategy to gain more payoff; while

 $\Box b$ -c<0, the lurking strategy is the prior one. A central user obtains high payoff as his large degree. His any movement receives high attention. Without the gain coefficient ( $\Box ki=1$ ), the payoff obtained by the active useri in the game is different. When user i chooses the active strategy,

$$Pi = \frac{nib}{ki+1} + \frac{b}{ki+1} - c = \frac{(ni+1)b}{ki+1} - c$$
 (7)

When user i chooses the lurking strategy,

$$Pi = \frac{nib}{ki+1} \tag{8}$$

Then, the payoff matrix is:

$$A\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$$
 (9)

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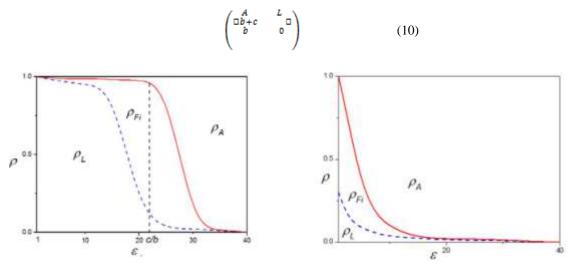
Apparently a user with the lurking strategy obtains morepayoff than a user with the active strategy. Lack of incentive results in a dull atmosphere and hampers the long-termdevelopment of an online social network. High incentive isessential to a social network. Especially the incentive bringsmuch more payoff to central users than ordinary users. Central vertices have high betweenness and play a critical rolein the process of information transmission. Central vertices are more willing to participate in this process for the highincentive. As shown in Table 1, there are different Nashequilibria in the opposite cases.

TABLE 1. Nash equilibria of the lurking game in opposite cases.

With incentive	Without incentive
Active (A)	Lurking (L)

#### **V.RESULTS**

Individual strategies asymptotically follow three different behaviors according to the dynamic organization of the individuals. *Pure active users* are those individuals who are always active throughout the evolution process. Conversely, *pure defectors*, are those individuals who keep lurking. A third class is constituted by fl*uctuating individuals* who alternatively act as active users or lurkers. The dynamic organization of individuals participating in the lurker gameof a social network is shown in Fig. 3. All individuals are supposed to be rational (K = 0.1). According to the Matrix (5), when  $\Box < c = b$ , active usersalways gain the less payoff than lurkers. Friends' strategieshave no effect on the active users' decisions. The active userswill vanish in the network. The influence of users gets more significant with the increase in the value of ". More usersare willing to share information and result in the increasing fraction of pure active users. When " is large enough, all thevertices in the network are induced to become pure activeusers by the considerable payoff. Here, we discussed a special circumstance. An onlinesocial network is a large community, in which users canobtain emotional comfort. It meets the interpersonal communication demands and makes users feel close to their friends. When a user reads the comments of his/her post, he/she feels self-fulfilled. If publishing a post gives a user emotional comfort and will become one of a user's habits. In that case, the cost is nearly zero for him, i.e.  $c \rightarrow 0$  or c < b. Then, thepayoff matrix is:



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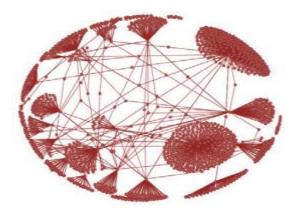


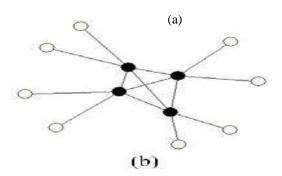
**FIGURE 3.**Fractions (referred to the number of specific individuals versusthe population) of pure and fluctuating strategies as a function of  $\Box$ . Theborder lines separates the pure and the underlying. when  $\Box \to 1$ , theactive uses will vanish in the network. Note that all individuals are supposed to be rational (K=0.1).

Apparently, users are willing to choose the active strategy for the non-zero  $\Box$ . The value c/b becomes the original point of  $\Box$ . As shown in Fig. 4, most of users will become pure active users. However, to develop the users' habit is still a great challenge for all the social networking websites. A great number of central users keep active in a mature social network. These pure active users constitute a central cluster with high connectivity. The central cluster plays a significant role in the operation of the whole network. Fig. 5(a) shows the topology of the network containing the data acquired from Weibo. The central vertices are connected with each other to form a central cluster. Meanwhile, each of them is connected with other marginal vertices. Fig. 5(b)shows a schematic diagram, in which four central black vertices are connected with each other to form a central cluster.

**FIGURE 4.**Fractions (referred to the number of specific individuals versus the population) of pure and fluctuating strategies as a function of  $\Box$ . When  $c \to 0$  or c << b, users are willing to choose the active strategy for the non-zero ". Note that all individuals are supposed to be rational (k=0.1).

Meanwhile, each of them is connected with another two white marginal vertices. Central vertices usually stay in the activestate. Due to their high inner connectivity, the states of themcannot be changed by the marginal lurkers. In the wholenetwork, a small number of lurkers have no effect on theactive users. These lurkers will be affected by their activeneighbors and change their states.





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**FIGURE 5.**(a) Topology of the network whose data are acquired from Weibo. The central vertices are connected with each other to form acentral cluster. Meanwhile, each of them is connected with other marginal vertices. (b) Schematic diagram of the central cluster (blackvertices) and the marginal vertices (white vertices).

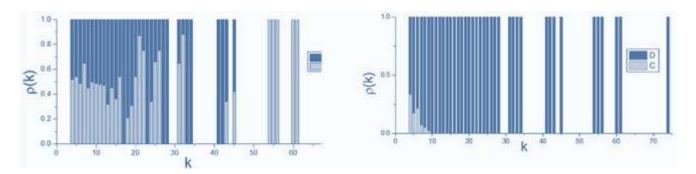
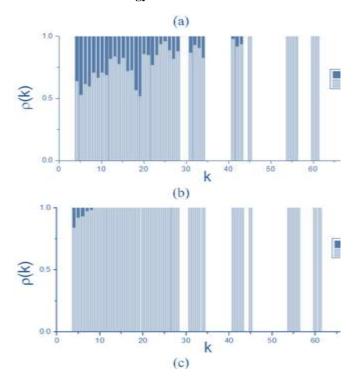


FIGURE 7.Strategy distribution in the stable state without incentive.



**FIGURE 6.**Strategy distribution under different evolutionary states.(a), (b) and (c) respectively represent the initial state, the medium state and the stable state with a incentive. Active users and lurkers are denoted by green bars and red bars, respectively. All the simulations were obtained from Weibo data with 4221 vertices.

The active vertices will emerge during the evolutionary process with an incentive. As shown in Fig. 6, 4200 vertices have different strategy distributions in different time, where  $\Box = 32$  and K=0.1. Initially, the strategy is randomly distributed and the central vertices also adopt the lurking strategy. After a period, most of central vertices will be changed to the active state. The lurkers mainly exist in the vertices with medium and small degrees. The fraction of active users gets larger with the increase in the degree. When the network is stable, most of vertices become active excepts everal vertices with small degrees, which are connected with each other to form

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some tiny clusters. These clusters are usually constrained in the extremely marginal regions and can hardly communicate with other parts of the network. They have a dull atmosphere and will become ``permanentlurkers". In a network without incentive, all the vertices have insufficient incentive. Active central vertices can hardly affect

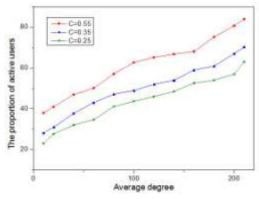
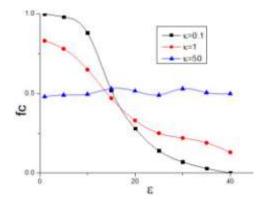


FIGURE 8.Proportion of active users versus the average degree of the network and the clustering coefficient.

The states of their neighbors and may even become lurkersdue to the great number of lurking neighbors. As shown in Fig. 7, only vertices with small degrees can keep active because the small clusters in the marginal areas form acommunity isolated from other vertices. They have their own interests and topics. Although no incentive is provided, high clustering coefficient of these marginal clusters can still maintain a high activity level. A large number of networks show a tendency for the linkage formation between neighboring vertices. That is to say, the network topology deviates from uncorrelated randomnetworks, in which triangles are sparse. This tendency is called clustering and reflects the clustering of edges into tightly connected neighborhoods. The friends of a personare likely to know each other. The clustering around user *i* is quantified by the clustering coefficient *Ci*, which is defined as the number of triangles in which user *i* participates and normalized by the maximum possible number of such triangles [13]:

$$Ci = \frac{2ti}{ki(ki-1)} \tag{11}$$

where *ti* denotes the number of triangles around *i*. Hence *Ci*D0 if none of the neighbors of a user is connected, and *Ci*D 1 if all of the neighbors are connected. To study the intrinsic features of active users, we generate an artificial social network with 42 000 users according to previous results. The hybrid network is a model to



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**FIGURE 9.**Density of active users in the stable state varies with  $\Box$  underdifferent noise conditions. The line with k=0.1 represents the rational situation, in which the density of active users gets smaller. The line with k=50 denotes the noisy situation, in which everyone becomes irrational and does not choose the strategy according to the game rule. All the simulations were obtained from Weibo data with 4221 vertices.

Describe the features of formation and evolution of generalized social networks. The key features such as averagedegree can be adjusted manually. To simplify the analysis, there is only one community in the generated network. Fig. 8shows the proportion of active users versus the average degree of the network under three different clustering coefficients. Obviously, when the clustering coefficient is a definite value, the proportion of active users is increased with the increase in the average degree. It means that the increase of vertices in the network is not influenced by the closeness. More friends and more contents inspire an existing user to be more active. When the average degree of the whole network is a definite value, the proportion of active users is increased with the increase in the clustering coefficient. A user usually shares common interest with his/her friend's friend. If a user isinterested in most of the posts, he/she is inspired to be active and may give comments or forward the posts.

In a real social network, some users are affected by irrational factors, such as individual differences, which result inbehaviors against the game theory. Fig. 9 shows the variation of density of active users with " under different noiseconditions. When individuals are nearly rational (k=0.1), the density of active users gets smaller and the trend is thesame to that shown in Fig. 3. When the noise is large enough(k=50), everyone becomes irrational and does not choosethe strategy according to the game rule. Each individual in thenetwork chooses his strategy independently, thus leading tothe disappearance of ``hot region". Large noise generates the chaos of networks. If the continuous chaos exists, active users are not able to affect their neighbor lurkers, so they graduallylose interest and leave the network.

#### **V.CONCLUSIONS**

In order to motivate users of a social network to share information and communicate with each other frequently, in this paper, we proposed a lurker game model for accumulating big data, which had similar mechanism with the public goods game. Moreover, individual incentive was introduced in the lurker game model. Motivating the activity levels of users is the basis for accumulating data. The study is conducive to further understanding and exploring the internal activity mechanism of social networks. Seeking methods to guide users' behaviors in a social network will be a research focus in the future.

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