

A Problem in Game Theory with Fibonacci Numbers

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ABSTRACT

The paper is aimed to discuss a row and column both dominance game with Fibonacci Numbers by Brown's Algorithm. A 6 x 6 game is constituted on dominance strategy. Some results are established by doing maximum iterations. Lower bounds and/or Upper bounds of this game are calculated at different levels.

Keywords: Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound

AMS Classification: 91A05,91A18,91A43, 91A90

I. INTRODUCTION

Rapoport [3], Dresher [4], Raiffa [5] & McKinsey [6] etc studied game theory and its application. They derived many useful identities in Game theory. Billy E. Gillett [1] invented algorithms in 1979 to resolve many situations in game theory in an effective manner. The theory of games and related areas of applications were investigated by Levin and Desjardins [2] in 1970.

II. BASIC FORMATION OF 6 X 6 GAME:

The game is framed with 6 rows and 6 columns according to player A and Player B. A consists of ten possible actions of A i.e A1, A2, A3, A4, A5, A6 which will effect on the other six possible actions of player B i.e B1, B2, B3, B4, B5, B6. The components are filled by fibonacci numbers for analysing the nature of the Game.

The pay off matrix of constituted game having the size 6x6 is given below.

0	1	1	2	3	5
1	8	13	21	34	55
1	13	89	144	233	377
2	21	144	610	987	1597
3	34	238	987	2584	4181
5	55	377	1597	4181	6765

III MATERIAL AND METHODS

The author applied Brown's algorithm to solve this special case of 6x6 game in which row and columns both dominated. Brown's Algorithm:

Step 1: Player A chooses one of the possible actions(A_{i_1}) from A1-A6 to play, and Player B then plays with the possible action B_{j_1} corresponding to the smallest element in the selected action A_{i_1} .

Step 2: Player A then picks out the possible action (A_{i_2}) from A1 – A6 to play corresponding to the largest element in the possible action (B_{j_1}) selected by Player B in step 1.

Step 3: Player B sums the actions of Player A who has played thus far, and plays with the possible action of B_{j_2} corresponding to a smallest sum element.

Step 4: Player A sums the actions of Player B who has played thus far, and plays the possible action (A_{i_3}) corresponding to a largest sum element. After the required iterations are computed, then go to step 5; otherwise, come back to step 3.

Step 5: Compute an upper and lower bound $\underline{\gamma}$ and $\bar{\gamma}$ respectively.

$$\bar{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}} \quad \text{and} \quad \underline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

Step 6: let X_i be the portion of the time Player A played row i with $i=1,2,\dots,m$ and let Y_j be the proportion of the time Player B played column j with $j=1,2,\dots,n$. These strategies approximate the optimal min max strategies. Upper and Lower bounds of the value of the game where $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ are calculated in step 5. The Process completes.

IV. RESULTS

The influences of player B on Player A are given in the tabular forms from Table (1) to Table (10) at each iteration by Brown's Algorithm for obtaining required best strategies from 50th iteration to 500th iterations by Java program.

Table-1: Player A vs Player B at 50th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	245	246	246	247	248	250
50	2696	2703	2708	2716	2729	2750
50	18474	18486	18562	18617	18711	18850
100	78255	78274	78397	78863	79240	79850
150	204872	204903	205102	205856	207453	209050
250	331490	331540	331862	333082	335666	338250

Table-2: Player A vs Player B at 100th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	495	496	496	497	498	500
100	5446	5453	5458	5466	5479	5500
100	37324	37336	37412	37467	37561	37700
200	158105	158124	158247	158713	159090	159700
300	413922	413953	414152	414906	416503	418100
500	669740	669790	670112	671332	673916	676500

Table-3: Player A vs Player B at 150th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	745	746	746	747	748	750
150	8196	8203	8208	8216	8229	8250
150	56174	56186	56262	56317	56411	56550
300	237955	237974	238097	238563	238940	239550
450	622972	623003	623202	623956	625553	627150
750	1007990	1008040	1008362	1009582	1012166	1014750

Table-4: Player A vs Player B at 200th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	995	996	996	997	998	1000
200	10946	10953	10958	10966	10979	11000
200	75024	75036	75112	75167	75261	75400
400	317805	317824	317947	318413	318790	319400
600	832022	832053	832252	833006	834603	836200
1000	1346240	1346290	1346612	1347832	1350416	1353000

Table-5: Player A vs Player B at 250 th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	1245	1246	1246	1247	1248	1250
250	13696	13703	13708	13716	13729	13750
250	93874	93886	93962	94017	94111	94250
500	397655	397674	397797	398263	398640	399250
750	1041072	1041103	1041302	1042056	1043653	1045250
1250	1684490	1684540	1684862	1686082	1688666	1691250

Table-6: Player A vs Player B at 300th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	1495	1496	1496	1497	1498	1500
300	16446	16453	16458	16466	16479	16500
300	112724	112736	112812	112867	112961	113100
600	477505	477524	477647	478113	478490	479100
900	1250122	1250153	1250352	1251106	1252703	1254300
1500	2022740	2022790	2023112	2024332	2026916	2029500

Table-7: Player A vs Player B at 350th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	1745	1746	1746	1747	1748	1750
350	19196	19203	19208	19216	19229	19250
350	131574	131586	131662	131717	131811	131950
700	557355	557374	557497	557963	558340	558950
1050	1459172	1459203	1459402	1460156	1461753	1463350
1750	2360990	2361040	2361362	2362582	2365166	2367750

Table-8: Player A vs Player B at 400th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	1995	1996	1996	1997	1998	2000
400	21946	21953	21958	21966	21979	22000
400	150424	150436	150512	150567	150661	150800
800	637205	637224	637347	637813	638190	638800
1200	1668222	1668253	1668452	1669206	1670803	1672400
2000	2699240	2699290	2699612	2700832	2703416	2706000

Table-9: Player A vs Player B at 450th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	2245	2246	2246	2247	2248	2250
450	24696	24703	24708	24716	24729	24750
450	169274	169286	169362	169417	169511	169650
900	717055	717074	717197	717663	718040	718650
1350	1877272	1877303	1877502	1878256	1879853	1881450
2250	3037490	3037540	3037862	3039082	3041666	3044250

Table-10: Player A vs Player B at 500th Iteration

Player A	Player B					
	A1	A2	A3	A4	A5	A6
0	2495	2496	2496	2497	2498	2500
500	27446	27453	27458	27466	27479	27500
500	188124	188136	188212	188267	188361	188500
1000	796905	796924	797047	797513	797890	798500
1500	2086322	2086353	2086552	2087306	2088903	2090500
2500	3375740	3375790	3376112	3377332	3379916	3382500

V. OPTIMUM MIXED STRATEGIES OF PLAYER A AND PLAYER B

The optimum mixed strategies of Player A and Player B are shown in

Table-11.

Table-11

Opitimum Mixted strategies of Player A and Player B (Iteration wise)																			
50		100		150		200		250		300		350		400		450		500	
A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

VI. UPPER BOUNDS AND LOWER BOUNDS AT ALL ITERATIONS

At each play of the game the minimum sum element selected by player B divided by the number of place of the game is known as lower bound.

Similarly At each play of the game the maximum sum element selected by player A divided by the number of place of the game is called as upper bound.

The Values of U.Bs and L.Bs in 6 x 6 game are tabulated in Table-12.

Table-12

U.B	Lower Bounds									
Iterations	50	100	150	200	250	300	350	400	450	500
50-500										
5	4.90	4.95	4.966	4.975	4.980	4.983	4.985	4.987	4.988	4.990
5	4.92	4.96	4.973	4.980	4.984	4.986	4.988	4.990	4.991	4.992
5	4.92	4.96	4.973	4.980	4.984	4.986	4.988	4.990	4.991	4.992
5	4.94	4.97	4.980	4.985	4.988	4.990	4.991	4.992	4.993	4.994
5	4.96	4.98	4.986	4.990	4.992	4.993	4.994	4.995	4.995	4.996
5	5	5	5	5	5	5	5	5	5	5

VII.CONCLUSIONS

- (i).The value of the game is 5.
- (ii).The value of Lower bound is gradually increasing and it will become the value of upper bound.
- (iii).The errors are minimized step by step.
- (iv).The influence of Player B uniformly effects on the possible action of PlayerA in each iteration.
- (v).systematic developments are established.
- (vi).Constant differences between the values of possible actions of player A at any two consequent iterations are determined.
- (vii).Variations in the iterations are negligible.
- (viii).It is a strictly determinable game.

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