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A Note on Square Sum Difference Product Prime Labeling

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ABSTRACT

*Square sum difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the sum of the squares of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits square sum difference product prime labeling. Here we investigate, splitting graph of star, double graph of star, triangular book, tensor product of star and path, tadpole graph, friendship graph and helm graph for square sum difference product prime labeling.*

Keywords: Graph Labeling, Greatest Common Incidence Number, Square Sum, Prime Labeling.

I INTRODUCTION

All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5] , we introduced the concept, square sum difference product prime labeling and proved that some path related graphs admit this kind of labeling. In [6] and [7], we extended our study and proved that the result is true for some trees and some cycle related graphs. In this paper we investigated square sum difference product prime labeling of splitting graph of star, double graph of star, triangular book, tensor product of star and path, tadpole graph, friendship graph and helm graph.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than 1, is the greatest common divisor (gcd) of the labels of the incident edges.

II MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection

f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping $f^*_{sqsdppl}$: E(G) → set of natural numbers N by $f^*_{sqsdppl}(uv) = |\{f(u)\}^2 + \{f(v)\}^2 - f(u)f(v)|$.The induced function $f^*_{sqsdppl}$ is said to be square sum difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

Definition 2.2 A graph which admits square sum difference product prime labeling is called a square sum difference product prime graph.

Theorem 2.1 Splitting graph of star $K_{1,n}$ admits square sum difference product prime labeling.

Proof: Let G = $S'(K_{1,n})$ and let $a,b,v_1,v_2,$-------------$,v_n,u_1,u_2,$-------------$,u_n$ are the vertices of G

Here |V(G)| = 2n+2 and |E(G)| = 3n.

Define a function f : V → {0,1,2,3,----------------,2n+1 } by

$$f(v_i) = i+1, \quad i = 1,2,\text{------},n.$$
$$f(u_i) = n+i+1, \quad i = 1,2,\text{------},n.$$
$$f(a) = 0,\ f(b) = 1$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$$f^*_{sqsdppl}(av_i) = (i+1)^2, \quad i = 1,2,\text{------------},n.$$
$$f^*_{sqsdppl}(bv_i) = i^2+i+1, \quad i = 1,2,\text{------------},n.$$
$$f^*_{sqsdppl}(au_i) = (n+i+1)^2, \quad i = 1,2,\text{------------},n.$$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_i) = gcd of $\{f^*_{sqsdppl}(av_i), f^*_{sqsdppl}(bv_i)\}$

= gcd of $\{i^2+2i+1, i^2+i+1\}$

= gcd of $\{i, i^2+i+1\}$

= 1, i = 1,2,------------,n.

gcin of (a) = 1.

gcin of (b) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $S'(K_{1,n})$, admits square sum difference product prime labeling. ■

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Example 2.1 $G = S'(K_{1,4})$

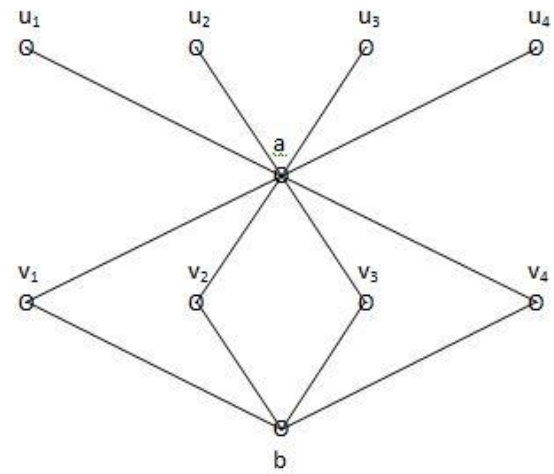

fig – 2.1

Theorem 2.2 Double graph of star $K_{1,n}$ admits square sum difference product prime labeling.

Proof: Let $G = D(K_{1,n})$ and let $a,b,v_1,v_2,$-------------$,v_n,u_1,u_2,$-------------$,u_n$ are the vertices of G

Here $|V(G)| = 2n+2$ and $|E(G)| = 4n$.

Define a function $f : V \rightarrow \{0,1,2,3,$----------------$,2n+1\}$ by

$f(v_i) = i+1$, $i = 1,2,$------$,n.$

$f(u_i) = n+i+1$, $i = 1,2,$------$,n.$

$f(a) = 0$, $f(b) = 1$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(av_i) = (i+1)^2,$ $i = 1,2,$------------$,n.$

$f^*_{sqsdppl}(bv_i) = i^2+i+1,$ $i = 1,2,$------------$,n.$

$f^*_{sqsdppl}(au_i) = (n+i+1)^2,$ $i = 1,2,$------------$,n.$

$f^*_{sqsdppl}(bu_i) = (n+i+1)^2 - (n+i+1) + 1,$ $i = 1,2,$------------$,n.$

Clearly $f^*_{sqsdppl}$ is an injection.,

gcin of (u_i) = gcd of $\{f^*_{sqsdppl}(au_i), f^*_{sqsdppl}(bu_i)\}$

= gcd of $\{(n+i+1)^2, (n+i+1)^2 - (n+i+1) + 1\}$

= gcd of $\{(n+i+1), (n+i+1)^2 - (n+i+1) + 1\}$

= 1, $i = 1,2,$------------$,n.$

gcin of (v_i) = 1, $i = 1,2,$------------$,n.$

gcin of (a) = 1.

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gcin of (b) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $D(K_{1,n})$, admits square sum difference product prime labeling. ∎

Example 2.2 $G = D(K_{1,4})$

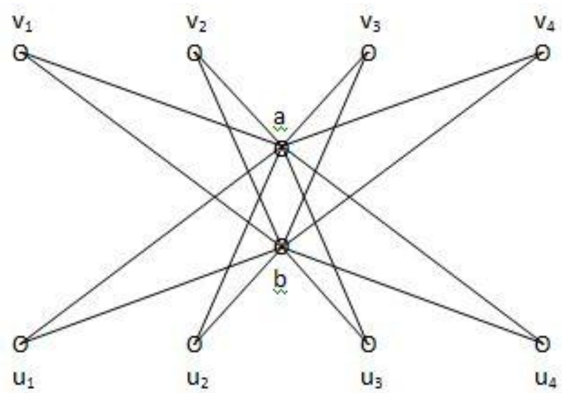

fig – 2.2

Theorem 2.3 Triangular book $P_2 + \overline{K_n}$ admits square sum difference product prime labeling.

Proof: Let G = $P_2 + \overline{K_n}$ and let v_1, v_2,----------------,v_{n+2} are the vertices of G

Here $|V(G)| = n+2$ and $|E(G)| = 2n+1$.

Define a function $f : V \rightarrow \{0,1,2,3,$----------------$,n+1\}$ by

$f(v_i) = i+1$, $i = 1,2,$------$,n$.

$f(a) = 0$, $f(b) = 1$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(av_i)$ = $(i+1)^2$, $i = 1,2,$------------$,n$.

$f^*_{sqsdppl}(bv_i)$ = i^2+i+1, $i = 1,2,$------------$,n$.

$f^*_{sqsdppl}(ab)$ = 1.

Clearly $f^*_{sqsdppl}$ is an injection.,

gcin of (v_i) = 1, $i = 1,2,$------------$,n$.

gcin of (a) = 1. ***gcin*** of (b) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_2 + \overline{K_n}$, admits square sum difference product prime labeling. ∎

Example 2.3 G = $P_2 + \overline{K_4}$

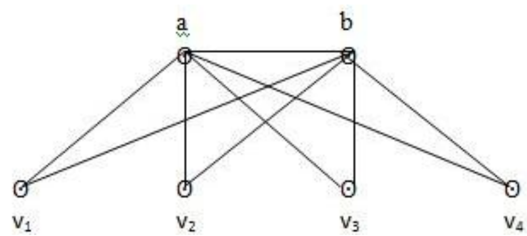

fig – 2.3

Theorem 2.4 Tensor product of star $K_{1,n}$ and path P_2 admits square sum difference product prime labeling.

Proof: Let G = $K_{1,n} \otimes P_2$ and let a,b,v_1,v_2,-------------,v_n,u_1,u_2,-------------,u_n are the vertices of G

Here |V(G)| = 2n+2 and |E(G)| = 2n.

Define a function f : V → {0,1,2,3,----------------,2n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n.

$f(u_i) = n+i+1$, i = 1,2,------,n.

f(a) = 0, f(b) = 1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(au_i)$ = $(n+i+1)^2$, i = 1,2,------------,n.

$f^*_{sqsdppl}(bv_i)$ = i^2+i+1, i = 1,2,------------,n.

Clearly $f^*_{sqsdppl}$ is an injection.,

gcin of (a) = 1.

gcin of (b) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $K_{1,n} \otimes P_2$, admits square sum difference product prime labeling. ■

Example 2.4 G = $K_{1,4} \otimes P_2$

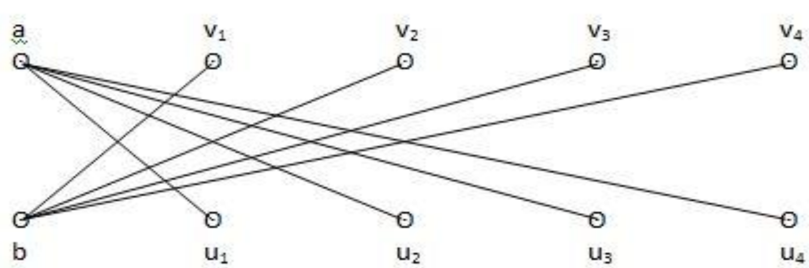

fig – 2.4

Theorem 2.5 Tadpole graph $C_n(P_m)$ admits square sum difference product prime labeling.

Proof: Let G = $C_n(P_m)$ and let v_1, v_2,-------------,v_{n+m-1} are the vertices of G

Here |V(G)| = n+m-1 and |E(G)| = n+m-1.

Define a function f : V → {0,1,2,3,----------------,n+m-2 } by

$f(v_i) = i-1$, i = 1,2,------,n+m-1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1})$ = i^2-i+1, i = 1,2,------------,n+m-2.

$f^*_{sqsdppl}(v_1 v_n)$ = $(n-1)^2$.

Clearly $f^*_{sqsdppl}$ is an injection.,

gcin of (v_1) = gcd of $\{f^*_{sqsdppl}(v_1 v_2), f^*_{sqsdppl}(v_1 v_n)\}$

= gcd of $\{1, (n-1)^2\}$ = 1.

gcin of (v_{i+1}) = 1, i = 1,2,-----------,n+m-3.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n(P_m)$, admits square sum difference product prime labeling. ■

Example 2.5 G = $C_5(P_4)$

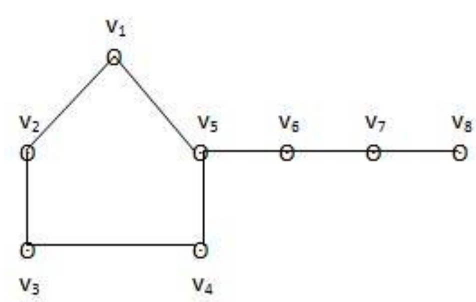

fig – 2.5

Theorem 2.6 Friendship graph F_n admits square sum difference product prime labeling.

Proof: Let G = $F_n$and let v_1, v_2,-------------,v_{2n+1} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 3n.

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(v_i) = i-1$, i = 1,2,------,2n+1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^{*}_{sqsdppl}(v_{2i-1}v_{2i})$ = $4i^2$-2i+1, i = 1,2,------------,n.

$f^{*}_{sqsdppl}(v_1v_{2i})$ = $(2i)^2$, i = 1,2,------------,n.

$f^{*}_{sqsdppl}(v_1v_{2i-1})$ = $(2i-1)^2$, i = 1,2,------------,n.

Clearly $f^{*}_{sqsdppl}$ is an injection.

gcin of (v_1) = 1

gcin of (v_{2i-1}) = gcd of {$f^{*}_{sqsdppl}(v_1v_{2i-1})$, $f^{*}_{sqsdppl}(v_{2i-1}v_{2i})$ }

= gcd of { $(2i-1)^2$, $4i^2$-2i+1}

= gcd of {2i-1 , 2i(2i-1)+1}

=1, i = 1,2,------------,n.

gcin of (v_{2i}) = gcd of {$f^{*}_{sqsdppl}(v_1v_{2i})$, $f^{*}_{sqsdppl}(v_{2i-1}v_{2i})$ }

= gcd of { $(2i)^2$, $4i^2$-2i+1}

= gcd of { 2i, 2i(2i-1)+1} =1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence F_n, admits square sum difference product prime labeling. ■

Example 2.6 G = F_3.

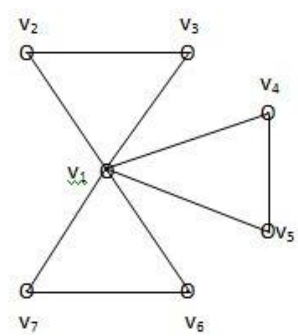

fig – 2.6

Theorem 2.7 Helm graph H_n admits square sum difference product prime labeling.

Proof: Let G = H_n and let a,v_1,v_2,-------------,v_{2n} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 3n.

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(v_i)$ = i , i = 1,2,------,2n.

f(a) = 0.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^{*}_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_{2i-1}v_{2i})$ = $4i^2-2i+1$, i = 1,2,------------,n.

$f^*_{sqsdppl}(v_{2i-1}v_{2i+1})$ = $4i^2+3$, i = 1,2,------------,n-1.

$f^*_{sqsdppl}(av_{2i-1})$ = $(2i-1)^2$, i = 1,2,------------,n.

$f^*_{sqsdppl}(v_1v_{2n-1})$ = $4n^2-6n+3$.

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (a) = 1

gcin of (v_{2i-1}) = gcd of {$f^*_{sqsdppl}(av_{2i-1})$, $f^*_{sqsdppl}(v_{2i-1}v_{2i})$ }

= gcd of { $(2i-1)^2$, $4i^2-2i+1$}

= gcd of {2i-1 , 2i(2i-1)+1}

=1, i = 1,2,------------,n.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence H_n, admits square sum difference product prime labeling. ∎

Example 2.7 G = H_4.

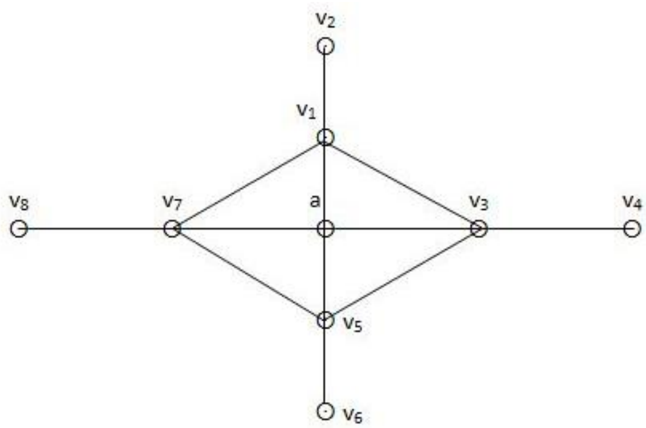

fig – 2.7

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