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Semi Pre Open Sets and Semi Pre Continuity in Sostak Intuitionistic Fuzzy Topological Space

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ABSTRACT

In this paper, we introduce intuitionistic fuzzy semi pre open (closed) sets and intuitionistic fuzzy semi pre continuous mappings in intuitionistic fuzzy topological spaces in Sostak's sense. Also we investigate the characteristic properties of these sets and mappings.

Keywords: Gradation of openness, intuitionistic fuzzy set, intuitionistic gradation of openness, intuitionistic fuzzy topological space.

I. INTRODUCTION

Chang [1] introduced the concept of fuzzy topological spaces using fuzzy sets first introduced by Zadeh [2]. In [3], Sostak generalized the fuzzy toplogical spaces and defined the concepts of gradation of openness. Further Chattopadhyay, Hazra and Samanta (see [4]) rephrased this concept.

Atanassov [5] introduced intuitionistic fuzzy sets and Coker [6] defined intuitionstic fuzzy topological spaces. Several authors have generalized these spaces (see [7], [8]). In 1996, Coker and Dimirci [9] introduced intuitionistic fuzzy topological spaces in Sostak's sense, which is a generalization of fuzzy topological spaces developed by Sostak [3].

In this paper, we introduce intuitionistic fuzzy semi pre open (closed) sets and intuitionstic fuzzy semi pre continuous mappings on intuitionistic fuzzy topological spaces in Sostak's sense (see [10], [11]). Further in these fuzzy topological spaces, the characteristic properties of semi pre open (closed) sets and semi pre continuous mappings have also been investigated.

II. PRELIMINARIES

Let X be a universal set and $I \equiv [0,1]$ be the closed unit interval of real line. Let ξ^X denote the set of all intuitionistic fuzzy sets on X . For the sake of completeness first we define intuitionistic fuzzy sets (see [5]) as follows.

An intuitionistic fuzzy set (IF-set in short) A on X is an object of the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \qquad (1)$$

where the functions $\mu_A : X \to I$ and $\nu_A : X \to I$ define the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) respectively of x in A, with $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. We shall denote for simplicity, IF-set A given in (1) $A = \{< x, \mu_A, \nu_A >\}$ and its complement by $A^c \equiv \{< x, \nu_A, \mu_A >\}$.

The IF-sets $0 \equiv \{< x, 0, 1 >\}$ and $1 \equiv \{< x, 1, 0 >\}$ are called the null IF-set and whole IF-set on X respectively.

An intuitionistic fuzzy topology (IF-topology in short) on a nonempty set X is a family τ of IF-sets in X satisfying the following axioms:

[1.] $0,1 \in \tau$;

[2.] $A \cap B \in \tau$ for any $A, B \in \tau$;

[3.] $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called intuitionistic fuzzy topological space (IF-topological spaces in short) (see [6]).

Let a and b be two real numbers in $[0, 1]$ satisfying the condition $a + b \leq 1$. Then the pair $< a, b >$ is called an intuitionistic fuzzy pair (or IF-pair in short) (see [9]).

An IF-set $\lambda \equiv \{< A, \mu\_\lambda(A), \nu_\lambda(A) >: A \in \xi^X\}$ on $\xi^X$is defines a collection of IF-pairs $< \mu_\lambda(A), \nu_\lambda(A) >$ satisfying $\mu_\lambda(A) + \nu_\lambda(A) \leq 1$ for each $A \in \xi^X$.

Now for each IF-set $A = \{< x, \mu_A(x), \nu_A(x) >\} \in \xi^X$, we define a map $\tau: \xi^X \to I \times I$as follows:

$$\tau(A) = < \mu_\tau(A), \nu_\lambda(A) > = \begin{cases} < 1, 0 >, & if\ A = 0 \\ < \inf_{x \in X} (\mu_A(x)), \sup_{x \in X} (\nu_A(x)) >, & if\ A \neq 0 \end{cases}$$

for each $A \in \xi^X$. Thus τ is an IF-family on X denoted as $\tau(A) = < \mu_\tau(A), \nu_\tau(A) >, A \in \xi^X$. We define the intuitionistic fuzzy topological space in Sostak' sence (see [9]) as follows.

An intuitionistic fuzzy topology in Sostak's sense (So-IF-topology in short) on a non-empty set X is an IF-family τ on X satisfying the following axioms:

(i) $\tau(0) = \tau(1) = 1$;

(ii) $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$ for any $A, B \in \xi^X$

(iii) $\tau(\cup_{i \in J} A) \geq \cap_{i \in J} \tau(A_i)$, for any $\{A_i : i \in J\} \subseteq \xi^X$.

The pair (X, τ) is called an intuitionistic fuzzy topological space in Sostak's sense (So-IF-topological spaces in short). For any $A \in \xi^X$, the number $\mu_\tau(A)$ is called the grade (or degree) of openness and $\nu_\tau(A)$ is called the grade (or degree) of non-openness of IF-set A.

Now we observe that if (X, τ) is a So-IF-topological space, then for a given pair (ρ, σ) such that $\rho \in I_0 = (0, 1]$, $\sigma \in I_1 = [0, 1)$ and $\rho + \sigma \leq 1$, the family $\tau_{\rho,\sigma}$ defined as $\tau_{\rho,\sigma} \equiv \{A \in \xi^X : \tau(A) \geq < \rho, \sigma >\}$ is actually an IF-topological space in sense of Coker [9] and is called the (ρ, σ)-level IF-topology on X. In this case IF-sets belonging to $\tau_{\rho,\sigma}$ are called IF-(ρ, σ)-open sets and their complements are called IF-(ρ, σ)-closed sets.

Let (X, τ) be a So-IF-topological spaces and $A \in \xi^X$ be an IF-set on X. Then the interior and closure of A with respect to $\tau_{\rho,\sigma}$ are denoted as $Int_{\rho,\sigma}(A)$ and $Cl_{\rho,\sigma}(A)$ respectively. Thus

$$Int_{\rho,\sigma}(A) = \cup \{G \in \xi^X : G \subseteq A, G \in \tau_{\rho,\sigma}\}$$

$$Cl_{\rho,\sigma}(A) = \cap \{K \in \xi^X : A \subseteq K, K^c \in \tau_{\rho,\sigma}\}$$

where $\rho \in I_0 = (0, 1], \sigma \in I_1 = [0, 1)$ such that $\rho + \sigma \leq 1$ (see [5]).

III. INTUITIONISTIC FUZZY-(ρ, σ)-SEMI PRE OPEN (CLOSED) SETS

In this section, we introduce IF-(ρ, σ)-semi pre open set and IF-(ρ, σ)-semi pre closed set in sostak intuitionistic topological space. We also study their significant properties.

Definition 3.1: Let (X, τ) be a So-IF-topological space and A be an IF-set on X. Then for a given $\rho \in I_0$ and $\sigma \in I_1$ with $\rho + \sigma \leq 1$, IF-set A is called an

[1.] IF-(ρ, σ)-semi open set if $A \subseteq Cl_{\rho,\sigma}\left(Int_{\rho,\sigma}(A)\right)$

[2.] IF-(ρ, σ)-alpha open set if $A \subseteq Int_{\rho,\sigma}\left(Cl_{\rho,\sigma}\left(Int_{\rho,\sigma}(A)\right)\right)$

[3.] IF-(ρ, σ)-pre open set if $A \subseteq Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A))$.

Remark 3.1: For a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$ (see [10], [11])

[1.] Every IF-(ρ, σ)-open (resp. IF-(ρ, σ)-closed) set is IF-(ρ, σ)-alpha open (resp. IF-(ρ, σ)-alpha closed) set.

[2.] Every IF-(ρ, σ)-alpha open (resp. IF-(ρ, σ)-alpha closed) set is IF-(ρ, σ)-semi open (resp. IF-(ρ, σ)-semi closed) set.

[3.] Every IF-(ρ, σ)-alpha open (resp. IF-(ρ, σ)-alpha closed) set is IF-(ρ, σ)-pre open (resp. IF-(ρ, σ)-pre closed) set.

But converse of (1), (2), (3) may not be true in general.

Now we will define intuitionistic fuzzy-(ρ, σ)-semi pre open (closed) sets in Sostak's sense as follows.

Definition 3.2: Let (X, τ) be a So-IF-topological space and $A \in \xi^X$ be an IF-set. Then for a given $\rho \in I_0$ and $\sigma \in I_1$ such that $\rho + \sigma \leq 1$, IF-set A is said to be an

[1.] IF-(ρ, σ)-semi pre open set if there exists an IF-(ρ, σ)-pre open set B such that $B \subseteq A \subseteq Cl_{(\rho,\sigma)}(B)$.

[2.] IF-(ρ, σ)-semi pre closed set if there exists an IF-(ρ, σ)-pre closed set B such that $Int_{(\rho,\sigma)}(B) \subseteq A \subseteq B$.

Example 3.1: Let $X = \{a, b\}$ and $A, B \in \xi^X$ be IF-sets defined as

$$A = \{< a, 0.3, 0.5 >, < b, 0.1, 0.6 >\}$$

$$B = \{< a, 0.4, 0.3 >, < b, 0.3, 0.6 >\}$$

We define an IF-topology $\tau: \xi^X \to I \times I$ as follows:

$$\tau(F) = \begin{cases} < 1, 0 >, & if\ F = 0, 1 \\ < 0.1, 0.6 >, & if\ F = A \\ < 0.3, 0.6 >, & if\ F = B \\ < 0, 1 >, & otherwise \end{cases}$$

Let $\rho = 0.2$ and $\sigma = 0.8$. Consider IF-set $E = \{< a, 0.5, 0.2 >, < b, 0.4, 0.3 >\} \in \xi^X$. Then we see that an IF-(ρ, σ)-pre open set B is such that $B \subseteq E \subseteq Cl_{\rho,\sigma}(B) = 1$. Thus E is an IF-(ρ, σ)-semi pre open set in X.

Remark 3.2: We easily observe that for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$

[1.] Every IF-(ρ, σ)-semi open (resp. IF-(ρ, σ)-semi closed) set is an IF-(ρ, σ)-semi pre open (resp. IF-(ρ, σ)-semi pre closed) set.

[2.] Every IF-(ρ, σ)-pre open (resp. IF-(ρ, σ)-pre closed) set is an IF-(ρ, σ)-semi pre open (resp. IF-(ρ, σ)-semi pre closed) set.

Proof: (1.) Let A be an IF-(ρ,σ)-semi open set in So-IF-topological space (X,τ), then $A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(A))$. Suppose $Int_{\rho,\sigma}(A) = B$. Being $(\rho,\sigma) - interior$ of A, IF-set B is an IF-(ρ,σ)-open set in X and every IF-(ρ,σ)-open set is IF-(ρ,σ)-pre open set. Hence B is an IF-(ρ,σ)-pre open set in X. Since $Int_{\rho,\sigma}(A) \subseteq A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(A))$. It implies $B \subseteq A \subseteq Cl_{\rho,\sigma}(B)$. Thus A is an IF-(ρ,σ)-semi pre open set in X.

(2.) can similarly be proved.

In the following examples we show that the converse of (1) and (2) may not be true in general.

Example 3.2: Considering Example 3.1, we observe that IF-set E is an IF-(ρ,σ)-semi pre open set. But E is not an IF-(ρ,σ)-semi open set because $E \nsubseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(E)) = A^c$.

Example 3.3: Let $X = \{a, b\}$ and $A, B, C \in \xi^X$ be IF-sets defined as

$$A = \{< a, 0.2, 0.8 >, < b, 0.3, 0.6 >\}$$
$$B = \{< a, 0.3, 0.7 >, < b, 0.4, 0.5 >\}$$
$$C = \{< a, 0.6, 0.3 >, < b, 0.5, 0.4 >\}$$

We define a IF-topology $\tau: \xi^X \to I \times I$ as follows:

$$\tau(F) = \begin{cases} < 1,0 >, & if\ F = 0,1 \\ < 0.2, 0.8 >, & if\ F = A \\ < 0.3, 0.7 >, & if\ F = B \\ < 0,1 >, & otherwise \end{cases}$$

Let $\rho = 0.1$ and $\sigma = 0.9$. We see that IF-(ρ,σ)-pre open set B is such that $B \subseteq C \subseteq Cl_{\rho,\sigma}(B) = B^c$. Thus C is an IF-(ρ,σ)-semi pre open set in X. But C is not an IF-(ρ,σ)-pre open set in X because $C \nsubseteq int_{\rho,\sigma}(Cl_{\rho,\sigma}(C))$.

In the following we obtain some interesting results, which describe the characteristic properties of IF-(ρ,σ)-semi pre open (closed) sets.

Theorem 3.1: Let A be an IF-set in a So-IF-topological space (X,τ). Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, following statements are equivalent:

a) A is an IF-(ρ,σ)-semi pre open set;

b) A^c is an IF-(ρ,σ)-semi pre closed set;

c) $A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A)))$;

d) $Int_{\rho,\sigma}\left(Cl_{\rho,\sigma}\left(Int_{\rho,\sigma}(A^c)\right)\right) \subseteq A^c$.

Proof: (a)$\Longrightarrow$(b): Let A be an IF-(ρ,σ)-semi pre open set in X, then there exists an IF-(ρ,σ)-pre open set B such that $B \subseteq A \subseteq Cl_{\rho,\sigma}(B)$. Since B is an IF-(ρ,σ)-pre open set, so that B^c is an IF-(ρ,σ)-pre closed set and also $Int_{\rho,\sigma}(B^c) \subseteq A^c \subseteq B^c$. Hence A^c is an IF-(ρ,σ)-semi pre closed set in X. We can prove (b)$\Rightarrow$(a) by taking complements.

(a)$\Rightarrow$(c): Let A be an IF-(ρ,σ)-semi pre open set in X, then there exists an IF-(ρ,σ)-pre open set B such that

$B \subseteq A \subseteq Cl_{\rho,\sigma}(B)$. Since B is an IF-(ρ,σ)-pre open set in X, therefore $B \subseteq Int_{\rho,\sigma}\left(Cl_{\rho,\sigma}(B)\right) \subseteq Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A))$. It implies $Cl_{\rho,\sigma}(B) \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A)))$. Thus we have $A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A)))$.

(c)⇒(a): If (c) holds, suppose IF-set A is such that $A \subseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A)))$, then (a) follows by taking $B = Int_{\rho,\sigma}(Cl_{\rho,\sigma}(A))$.

(b)⇔(d): can be proved by taking complements in above.

Theorem 3.2: Let (X,τ) be a So-IF-topological space and $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$. Then

[1.] Any union of IF-(ρ,σ)-semi pre open sets is an IF-(ρ,σ)-semi pre open set;

[2.] Any intersection of IF-(ρ,σ)-semi pre closed set is an IF-(ρ,σ)-semi pre closed set.

Proof: (1) Let $\{A_i : i \in J\}$ be a collection of IF-(ρ,σ)-semi pre open sets in (X,τ). Then there exist IF-(ρ,σ)-pre open sets B_i such that

$$B_i \subseteq A_i \subseteq Cl_{\rho,\sigma}(B_i), \forall i \in J \qquad (3.2.1)$$

We know $\tau\left(\cup_{i\in J} B_i\right) \geq \wedge_{i\in J} \tau(B_i) \geq <\rho,\sigma>$. Thus $\cup_{i\in J} B_i$ is an IF-(ρ,σ)-open set. Further every IF-(ρ,σ)-open set is an IF-(ρ,σ)-pre open set. Therefore from (3.2.1), we have

$\cup_{i\in J} B_i \subseteq \cup_{i\in J} A_i \subseteq \cup_{i\in J} Cl_{\rho,\sigma}(B_i) \subseteq Cl_{\rho,\sigma}(\cup_{i\in J} B_i)$. It follows $\cup_{i\in J} B_i \subseteq \cup_{i\in J} A_i \subseteq Cl_{\rho,\sigma}(\cup_{i\in J} B_i)$. Thus $\cup_{i\in J} A_i$ is an IF-(ρ,σ)-semi pre open set in X.

(2): follows from (1) using Theorem 3.1.

Remark 3.3: The intersection of two IF-(ρ,σ)-semi pre open sets is not an IF- (ρ,σ)-semi pre open set in general as seen in the following example.

Example 3.3: Let $X = \{a, b\}$ and let $A, B, C \in \xi^X$ be IF-sets defined as

$$A = \{<a, 0.4, 0.6>, <b, 0.5, 0.3>\}$$
$$B = \{<a, 0.6, 0.3>, <b, 0.1, 0.4>\}$$
$$C = \{<a, 0.7, 0.2>, <b, 0.3, 0.4>\}$$

We define a IF-topology $\tau: \xi^X \to I \times I$ as follows:

$$\tau(F) = \begin{cases} <1, 0>, & if\ F = 0, 1 \\ <0.4, 0.6>, & if\ F = A \\ <0, 1>, & otherwise \end{cases}$$

Let $\rho = 0.2$ and $\sigma = 0.8$. We see that IF-sets C and A are IF-(ρ,σ)-semi pre open sets in X. But their intersection $A \cap C = \{<a, 0.4, 0.6>, <b, 0.3, 0.4>\}$ is not an IF-(ρ,σ)-semi pre open set.

Theoerem 3.3: Let (X,τ) be a So-IF-topological space and $B \in \xi^X$ be an IF-set in X. Then for a given $\rho \in I_0, \sigma \in I_1$ with $\rho + \sigma \leq 1$, IF-set B is an

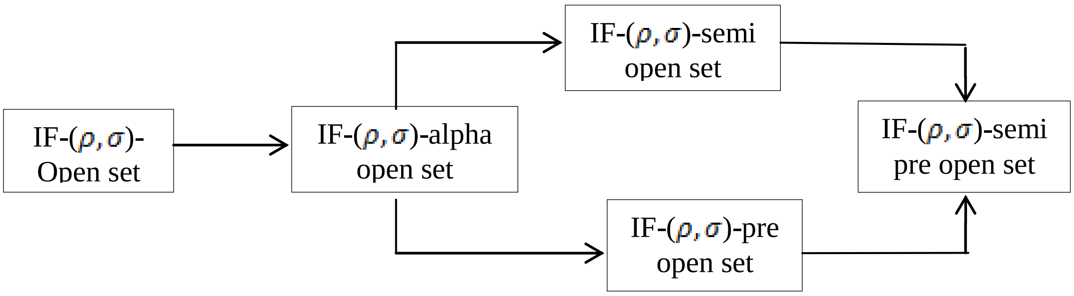
(i) IF-(ρ,σ)-semi pre open set in X if A is an IF-(ρ,σ)-semi pre open set such that $A \subseteq B \subseteq Cl_{\rho,\sigma}(A)$.

(ii) IF-(ρ,σ)-semi pre closed set in X if A is an IF-(ρ,σ)-semi pre closed set such that $Int_{\rho,\sigma}(A) \subseteq B \subseteq A$.

Proof: (i) Let A be an IF-(ρ,σ)-semi pre open set and B be an IF-set in X such that $A \subseteq B \subseteq Cl_{\rho,\sigma}(A)$. Since A is an IF-(ρ,σ)-semi pre open set, then there exists an IF-(ρ,σ)-pre open set C such that $C \subseteq A \subseteq Cl_{\rho,\sigma}(C)$. Now we observe that $C \subseteq A \subseteq B \subseteq Cl_{\rho,\sigma}(A) \subseteq Cl_{\rho,\sigma}(C)$. It implies $C \subseteq B \subseteq Cl_{\rho,\sigma}(C)$. Hence B is an IF-(ρ,σ)-semi pre open set in X.

(ii): It can be proved in a similar manner.

Remark 3.4: The following diagram explains the relationship among the different types of IF-(ρ,σ)-sets



IV. INTUITIONISTIC FUZZY-(ρ,σ)-SEMI PRE CONTINUOUS MAPPING

In this section, we will introduce IF-(ρ,σ)-semi pre continuous mappings and investigate its characteristic properties. Firstly we define IF-(ρ,σ)-continous mapping as follows.

Let (X,τ) and (Y,δ) be two So-IF-topological spaces, where τ and δ are (ρ,σ)-level IF-topologies on X and Y respectively. Then a map $f: X \to Y$ is said to be IF-(ρ,σ)-continuous iff $\tau(f^{-1}(B)) \geq \delta(B)$ for each $B \in \xi^X$ such that $\delta(B) \geq \langle \rho,\sigma \rangle$.

Definition 4.1: Let (X,τ) and (Y,δ) be two So-IF-topological spaces and $f: X \to Y$ be a map. Then for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$, the map f is said to be an

[1.] IF-(ρ,σ)-semi continuous map if $f^{-1}(B)$ is an IF-(ρ,σ)-semi open set in X, for each IF-(ρ,σ)-open set B in Y.

[2.] IF-(ρ,σ)-alpha continuous map if $f^{-1}(B)$ is an IF-(ρ,σ)-alpha open set in X, for each IF-(ρ,σ)-open set B in Y.

[3.] IF-(ρ,σ)-pre continuous map if $f^{-1}(B)$ is an IF-(ρ,σ)-pre open set in X, for each IF-(ρ,σ)-open set B in Y.

Remark 4.1: It is clear for a given $\rho \in I_0, \sigma \in I_1$ with $\rho + \sigma \leq 1$ that

a) Every IF- (ρ,σ)-continuous mapping is an IF- (ρ,σ)-alpha continuous mapping.

b) Every IF- (ρ,σ)-alpha continuous mapping an is IF- (ρ,σ)-semi continuous mapping.

c) Every IF- (ρ,σ)-alpha continuous mapping is an IF- (ρ,σ)-pre continuous mapping.

But converse of (a), (b), (c) may not true in general (see [11]).

Now we will define IF- (ρ,σ)-semi pre continuous mapping in Sostak's IF-topological spaces as follows.

Definition 4.2: Let (X,τ) and (Y,δ) be two So-IF-topological spaces. A mapping $f: X \to Y$ is said to be an IF-(ρ,σ)-semi pre continuous mapping if $f^{-1}(B)$ is an IF-(ρ,σ)-semi pre open set in X for each $B \in \xi^Y$ such that $\delta(B) \geq <\rho,\sigma>$.

Example 4.1: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $A, B \in \xi^X, C \in \xi^Y$ be IF-sets defined as follows:

$$A = \{<a, 0.2, 0.6>, <b, 0.3, 0.4>\}$$

$$B = \{<a, 0.4, 0.5>, <b, 0.6, 0.3>\}$$

$$C = \{<u, 0.7, 0.1>, <v, 0.6, 0.2>\}$$

We define IF-topologies $\tau: \xi^X \to I \times I$ and $\delta: \xi^Y \to I \times I$ as follows:

$$\tau(F) = \begin{cases} <1,0>, & if\ F = 0,1 \\ <0.2, 0.6>, & if\ F = A \\ <0,1>, & otherwise \end{cases}$$

$$\delta(F) = \begin{cases} <1,0>, & if\ F = 0,1 \\ <0.6, 0.2>, & if\ F = C \\ <0,1>, & otherwise \end{cases}$$

Consider the mapping $f: (X,\tau) \to (Y,\delta)$ defined as $f(a) = u, f(b) = v$. Suppose $\rho = 0.1, \sigma = 0.8$. We see that for IF-sets $f^{-1}(C) \equiv \{<a, 0.7, 0.3>, <b, 0.8, 0.2>\}$, $f^{-1}(0) \equiv 0\ and\ f^{-1}(1) \equiv 1$ are IF-(ρ,σ)-semi pre open sets. Thus f is an IF-(ρ,σ)-semi pre continuous map.

Now we will investigate some characteristic properties of IF-(ρ,σ)-semi pre continuous mappings.

Remark 4.2: We observe that for a given $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \leq 1$.

[1.] Every IF- (ρ,σ)-semi continuous mapping is an IF- (ρ,σ)-semi pre continuous map;

[2.] Every IF- (ρ,σ)-pre continuous map is an IF- (ρ,σ)-semi pre continuous map.

But converse of (1) and (2) may not true as shown in following examples.

Example 4.2: Considering Example 4.1, we see that f is an IF- (ρ,σ)-semi pre continuous mapping, but it is not an IF- (ρ,σ)-semi continuous map. We observe that IF-set $f^{-1}(C)$ is not an IF-(ρ,σ)-semi open set in X because $f^{-1}(C) \nsubseteq Cl_{\rho,\sigma}(Int_{\rho,\sigma}(f^{-1}(C)))$.

Example 4.2: Let $X = \{a,b\}$, $Y = \{u,v\}$ and let $A, B \in \xi^X, C \in \xi^Y$ be IF-sets defined as follows:

$$A = \{<a, 0.3, 0.4>, <b, 0.2, 0.5>\}$$

$$B = \{<a, 0.4, 0.2>, <b, 0.3, 0.2>$$

$$C = \{<u, 0.3, 0.3>, <v, 0.5, 0.3>\}$$

We define IF-topologies $\tau: \xi^X \to I \times I$ and $\delta: \xi^Y \to I \times I$ as follows:

$$\tau(F) = \begin{cases} <1,0>, & if\ F = 0,1 \\ <0.2, 0.5>, & if\ F = A \\ <0.3, 0.2>, & if\ F = B \\ <0,1>, & otherwise \end{cases}$$

$$\delta(F) = \begin{cases} <1,0>, & if\ F = 0,1 \\ <0.3,0.3>, & if\ F = C \\ <0,1>, & otherwise \end{cases}$$

Consider the mapping $f:(X,\tau) \to (Y,\delta)$ defined as $f(a) = u, f(b) = v$. Let $\rho = 0.2, \sigma = 0.7$. Then we see that f is an IF-(ρ,σ)-semi pre continuous map. But it is not an IF-(ρ,σ)-pre continuous map because $f^{-1}(C)$ is not an IF-(ρ,σ)-pre open set in X.

Theorem 4.1: Every IF-(ρ,σ)-continuous map is an IF-(ρ,σ)-semi pre continuous map.

Proof: Let (X,τ) and (Y,δ) be two So-IF-topological spaces and $f:X \to Y$ be an IF-(ρ,σ)-continuous map. Suppose $\rho \in I_0, \sigma \in I_1$ such that $\rho + \sigma \le 1$ and $B \in \xi^Y$ be an IF-set such that $\delta(B) \ge <\rho,\sigma>$, then $\tau(f^{-1}(B)) \ge \delta(B) \ge <\rho,\sigma>$. Therefore $f^{-1}(B)$ is an IF-(ρ,σ)-open set in X and every IF-(ρ,σ)-open set is an IF-(ρ,σ)-semi pre open set, hence $f^{-1}(B)$ is an IF-(ρ,σ)-semi pre open set in X for each $B \in \xi^Y$. Thus f is an IF-(ρ,σ)-semi pre continuous map.

But converse may not be true in general as seen in following example.

Example 4.3: Considering Example 4.1, we see that f is an IF-(ρ,σ)-semi pre continuous map, but f is not an IF-(ρ,σ)-continuous map because IF-set $f^{-1}(C)$ is not an IF-(ρ,σ)-open set in X.

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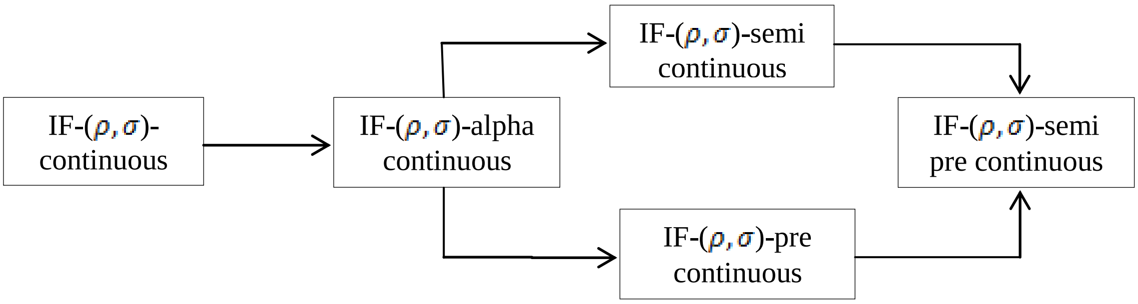
Theorem 4.2: Let (X,τ) and (Y,δ) be two So-IF-topological spaces and $f:X \to Y$ be a map. Then f is an IF-(ρ,σ)-semi pre continuous map iff $f^{-1}(B)$ is an IF-(ρ,σ)-semi pre closed set for each IF-(ρ,σ)-closed set B of Y.

Proof: Suppose (X,τ) and (Y,δ) are two So-IF-topological spaces and $f:X \to Y$ is an IF-(ρ,σ)-semi pre continuous map. Suppose $B \in I^Y$ is an IF-(ρ,σ)-closed set such that $\delta(B^c) \ge <\rho,\sigma>$, so that B^c is an IF-(ρ,σ)-open set in X. Therefore $f^{-1}(B^c)$ is an IF-(ρ,σ)-semi pre open set in X and also $\tau(f^{-1}(B^c)) \ge <\rho,\sigma>$. Since $(f^{-1}(B^c))^c = f^{-1}(B)$, thus $f^{-1}(B)$ is an IF-(ρ,σ)-semi pre closed set in X.

Conversely; Let $f:(X,\tau) \to (Y,\delta)$ be a map and let $f^{-1}(B)$ is an IF-(ρ,σ)-semi pre closed set in X for each IF-(ρ,σ)-closed set B in Y. Then $\delta(B^c) \ge <\rho,\sigma>$, therefore B^c is an IF-(ρ,σ)-open set in Y such that $\tau(f^{-1}(B))^c = \tau(f^{-1}(B^c)) \ge <\rho,\sigma>$. Since every IF-$(\rho,\sigma)$-open set is an IF-$(\rho,\sigma)$-semi pre open set. Hence $f^{-1}(B^c)$ is an IF-(ρ,σ)-semi pre open set in X. Thus f is an IF-(ρ,σ)-semi pre continuous map.

Remark 4.3: The following diagram explains the relationship among the different types of IF continuous maps.



V. CONCLUSION

In the present paper, we introduced the concept of IF-(ρ, σ)-semi pre open sets and IF-(ρ, σ)-semi pre continuous mapping in intuitionistic fuzzy topological space defined in sense of Sostak. We have also investigated the characteristic properties of them.

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