SEMI-REGULAR WEAKLY CLOSED AND SEMI-REGULAR WEAKLY OPEN MAPS IN TOPOLOGICAL SPACES

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ABSTRACT

In this article, we introduce the new weaker form of closed and open maps viz. Semi-regular weakly closed (briefly srw-closed) maps and Semi-regular weakly open (briefly srw-open) maps and some stronger form of srw-closed and srw-open maps viz. srw-closed and srw*-open maps in topological spaces. Also we study properties of newly formed maps as well as inter relationship with existed maps in topological spaces.*

Keywords: srw-closed maps, srw*-closed maps, srw-open maps and srw*-open maps.

I. INTRODUCTION

In general topology many researcher studied various closed and open maps. In 1982 Malghan [1] introduced and investigated some properties of generalized closed maps. Noiri [2], Biswas [3], Wali [4], Devi [5] and Dontchev [6] have introduced and studied semi-closed and semi-open maps, α-closed and α-open maps, αrw-closed and αrw-open maps, gs-closed and gs-open maps and gsp-open maps respectively. But in this article we introduce new class of weaker forms of closed and open maps i.e. srw-closed maps and srw-open maps and also stronger form of srw-closed and srw-open maps called srw*-closed and srw*-open maps. Here we discuss the properties of all newly formed maps and relationship with existed maps in topological spaces.

II. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) and (Z,γ) (or simply X, Y and Z) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. We denote the closure, semi-regular weakly closure, interior, semi-regular weakly interior of A by cl(A), srw-cl(A), int(A) and srw-int(A) respectively and neighbourhood of an element in any topological space is denoted as nbd of x.

Now we recall the following known definitions and results that are used in our work;

Definition 2.1 A subset A of a topological space X is called

(i) Regular open [7], if A=int(cl(A)) and regular closed if A=cl(int(A)).

(ii) Pre-open [8], if A⊆int(cl(A)) and pre-closed if cl(int(A))⊆A.

(iii) Semi open [9], if A⊆cl(int(A)) and semi-closed if int(cl(A))⊆A.

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(iv) α-open [10], ifA⊆int(cl(int(A))) andα-closed if cl(int(cl(A)))⊆A.

Definition 2.2 A subset A of a topological space X is called

(i) Generalized closed (briefly g-closed) [11], if cl(A)⊆U whenever A⊆U and U is open in X.

(ii) Generalized semi-closed (briefly gs-closed) [12], if scl(A)⊆U whenever A⊆U and U is open in X.

(iii) Generalized semi pre-closed (briefly gsp-closed) [6], if spcl(A)⊆U whenever A⊆U and U is open in X.

(iv) Weakly closed (briefly w-closed) [13], if cl(A)⊆U whenever A⊆U and U is semi-open in X.

(v) Regular weakly closed (briefly rw-closed) [14], if cl(A)⊆U whenever A⊆U and U is regular semi-open in X.

(vi) α-regular weakly closed (briefly αrw-closed) [15], ifαcl(A)⊆U whenever A⊆U and U is rw-open set in X.

The complements of above all closed sets are their respective open sets in the same topological space X.

Definition 2.3 A subset A of a space X is said to be semi regular weakly closed (briefly srw-closed) set [16], if scl(A)⊆U whenever A⊆U and U is rw-open set in X.

Definition 2.4 A subset A of X is called Semi Regular Weakly open (briefly srw-open) set [17], if X-A is srw-closed set in X.

Definition 2.5 For a subset A of a space X, srw-cl(A)= ∩{F:A⊆F and F is srw-closed set in X}is called srw-closure of A [17].

Definition 2.6 Let A is a subset of X. A point $x \in A$ is said to be srw-interior point of A, if A is a srw-nhd of x. The set of all srw-interior [17] of A and is denoted by srw-int(A).

Definition 2.7 For a subset A of X, srw-closure[17] of A is defined as srw-cl(A) to be the intersection of all srw-closed sets containing A.

Definition 2.8 Let X be any topological space and let x∈X. A subset N is said to be srw-nbd[18] of x, if and only if there exists a srw-open set G such that x∈G⊆**N**.

Definition 2.9 A subset N of X is a srw-nbd[17] of A⊆X in topological space (X, τ), if there exists a srw-open set G such that A⊆X⊆**N**.

Definition 2.10 A function f: X→Y is said to be srw-continuous function [18], if f^{-1}(V) is srw-closed set of X for every closed set V of Y.

Definition 2.11 A function f: X→Y is called srw-irresolute [18], if $f^{-1}(V)$ is srw-closed set in X for every srw-closed subset V of Y.

Definition 2.12A function f: X→Yis said to be

(i) rw-irresolute (lemma 2.10)[18], if f^{-1}(V) is rw-open set in X for every rw-open set V of Y.

(ii) strongly srw-continuous [18], if f^{-1}(V) is open set in X for every srw-open set V of Y.

Definition 2.13 A function f: X→Y is called

(i) Regular closed [17], if f(F) is closed in Y for every regular closed set F of X.

(ii) Closed, if f(F) is closed in Y for every closed set F of X.

(iii) g-closed [1], if f(F) is g-closed in Y for closed set F of X.

(iv) w-closed [20], if f(F) is w-closed in Y for closed set F of X.

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(v) Semi-closed [2](resp. semi-open), if f(F) is semi-closed(resp. semi-open) in Y for every closed(resp. open) set F of X.

(vi) αrw-closed [4](resp. αrw-open), if f(F) is αrw-closed(resp. αrw -open)in Y for every closed(resp. open) set F of X.

(vii) gs-closed [5](resp. gs-open), if f(F) is gs -closed(resp. gs-open) in Y for every closed(resp. open) set F of X.

(viii) gsp-closed [6](resp. gsp-open), if f(F) is gsp-closed(resp. gsp-open) in Y for every closed(resp. open) set F of X.

(ix) Contra regular closed, if f(F) is r-closed in Y for every open set F of X.

Lemma 2.14 Let X be any topological space, in which

(i) Every closed (resp. semi-closed, αrw-closed) set is srw-closed set in X [16].

(ii) Every srw-closed set is gs-closed (resp. gsp-closed) set in X [16].

Definition2.15 A topological space (X, τ) is called

(i) T_{srw}-space [18] if every srw-closed set is closed.

(ii) $T_{½}$-space [1] if every g-closed set is closed.

(iii) $T_{\alpha r\omega}$-space [21] if every αrω-closed set is closed.

III. SEMI-REGULAR WEAKLY CLOSED MAPS IN TOPOLOGICAL SPACES

Definition 3.1 A map f: X→Y is said to be semi-regular weakly closed (srw-closed) map, if the image of every closed set in X is srw-closed in Y.

Theorem 3.2 Every closed map is srw-closed map but not conversely.

Proof: Let f: X→Yis closed map. Let F be any closed set in X. Then f(X) is closed but every closed set is srw-closed set [16]. Hence f is srw-closed map.

Example 3.3 Let X={x, y, z} with topology τ= {∅, {x}, {y}, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=a, f(y)=b and f(z)=c. Then f is srw-closed map but not closed, since the image of closed set {z} in X is {c}, which is not closed in Y.

Theorem 3.4 Every semi-closed map is srw-closed map but not conversely.

Proof: Let f: X→Yis semi-closed map. Let F be any closed set in X. Then f(X) is semi-closed but every semi-closed set is srw-closed set [16]. Hence f is srw-closed map.

Example 3.5 Let X={x, y, z} with topology τ= {∅, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=b, f(y)=c and f(z)=a. Then f is srw-closed map but not semi-closed, since the image of closed set {z} in X is {a}, which is not semi-closed in Y.

Theorem 3.6 Every αrw-closed map is srw-closed map but not conversely.

Proof: Let f: X→Yisαrw-closed map. Let F be any closed set in X. Then f(X) is αrw-closed but every αrw-closed set is srw-closed set [16]. Hence f is srw-closed map.

Example 3.7 Let X={x, y} with topology τ= {∅, {x}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d and f(y)=a. Then f is srw-closed map but not αrw-closed, since the image of closed set {y} in X is {a}, which is not αrw-closed in Y.

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Theorem 3.8 Every srw-closed map is gs-closed map but not conversely.

Proof: Let f: X→Yis srw-closed map. Let F be any closed set in X. Then f(X) is srw-closed but every gs-closed set is srw-closed set [16]. Hence f is gs-closed map.

Example 3.9 Let X={x, y, z} with topology τ= {∅, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=b, f(y)=c and f(z)=b. Then f is gs-closed map but not srw-closed, since the image of closed set {z} in X is {b}, which is not srw-closed in Y.

Corollary3.10 Every srw-closed map is gsp-closed map but not conversely.

Proof: Proof follows from Theorem 3.8 and also fact that, every gs-closed set is gsp-closed set [14]. Hence f is gsp-closed map.

Example 3.11 Let X={x, y, z} with topology τ= {∅, {x}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=b, f(y)=c and f(z)=c. Then f is gsp-closed map but not srw-closed, since the image of closed set {y, z} in X is {c}, which is not srw-closed in Y.

Theorem 3.12 A map f:X→Y is said to be srw-closed map if and only if for each subset A of Y and for each open set U containing $f^{-1}(A)$, there is a srw-open set V of Y such that A⊆V and f^{-1}(V)⊆U.

Proof: Suppose f is srw-closed map. Let A is a subset of Y and U is a open set of X such that f^{-1}(A)⊆U. Now X-U is a closed set in X. Since f is srw-closed map, f(X-U) is a srw-closed set in Y i.e. V=Y-f(X-U) V=Y-f(X-U) is srw-open set of Y. Note that f^{-1}(A)⊆U implies that A⊆V andf^{-1}(V)=X-f^{-1}(f(X-U))=X-(X-U)=U i.e.f^{-1}(V)⊆U.

Conversely, suppose that F is a closed set in X. Then f^{-1}(f(X-F))⊆X-F and X-F is open in X. By the hypothesis, there exists a srw-open set V in Y such that Y-f(F)⊆V andf^{-1}(V)⊆X-F. Therefore, F⊆X-f^{-1}(V). Hence Y-V⊆f(F)⊆f(X-f^{-1}(V))⊆Y-V which implies f(F)⊆V. Since Y-V is srw-closed, f(F) is srw-closed. Therefore f(F) is srw-closed in Y. Hence f is srw-closed map.

Theorem 3.13 If f: X→Y is contra-regular closed and srw-closed map, then f is semi-closed map.

Proof: Let V be a closed set in X, then f(V) is regular open and srw-closed. By Corollary 3.8(5) [16], f(V) is srw-closed. Thus f is semi-closed map.

Theorem 3.14 If f: X→Y is g-closed map and Y is a $T_{1/2}$-space, then f: X→Y is srw-closed map.

Proof: Let F be a closed set in X. Since f is g-closed map, f(F) is g-closed set in Y. As Y is a $T_{1/2}$-space, we have f(F) is closed in Y. As every closed set is srw-closed, f(F) is a srw-closed in Y. Thus f is a srw-closed map.

Theorem 3.15 If f: X→Y is αrw-closed map and Y is a $T_{\alpha rw}$-space, then f: X→Y is srw-closed map.

Proof: Let F be a closed set in X. Since f is αrw-closed map, f(F) is αrw-closed set in Y. As Y is a $T_{\alpha rw}$-space, we have f(F) is closed in Y. As every closed set is srw-closed, f(F) is a srw-closed in Y. Thus f is a srw-closed map.

Remark 3.16 w-closed maps and srw-closed maps are independent.

Example 3.17 Let X={x, y, z} with topology τ= {∅, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=c and f(z)=c. Then f is srw-closed map but not w-closed, since the image of closed set {z} in X is {b}, which is not w-closed in Y.

Example 3.18 Let X={w, x, y, z} with topology τ= {∅, {w}, {x}, {w, x}, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let f: X→Y is an identity map. Then f is w-closed map but not srw-closed, since the image of closed set {y, z} in X is {c, d}, which is not srw-closed in Y.

Remark 3.19 g-closed maps and srw-closed maps are independent.

Example 3.20 Let X={x, y, z} with topology τ= {∅, {z}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=a and f(z)=b. Then f is g-closed map but not srw-closed, since the image of closed set {x, y} in X is {a, d}, which is not srw-closed in Y.

Example 3.21 Let X={x, y, z} with topology τ= {∅, {x}, {y}, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=a, f(y)=b and f(z)=c. Then f is srw-closed map but not g-closed, since the image of closed set {z} in X is {c}, which is not g-closed in Y.

Theorem 3.22 If f: X→Y is srw-closed map, then srw-cl(f(A))⊆f(cl(A)) for every subset A of X.

Proof: Suppose that f is srw-closed and A⊆X. Then cl(A) is closed in X and so f(cl(A)) is srw-closed in Y. We have f(A)⊆f(cl(A)) and by Theorem [18], srw-cl(f(A))⊆srw-cl(f(cl(A)))…(i). Since f(cl(A)) is srw-closed set in Y, srw-cl(f(cl(A)))=f(cl(A)) …(ii), by Theorem [17]. From (i) and (ii), srw-cl(f(A))⊆f(cl(A)) for every subset A of X.

Corollary 3.23 If f: X→Y is a srw-closed then the image f(A) of closed set A in X is τ_{srw}-closed in Y.

Proof: Let A be a closed set in X. Since f is srw-closed, by Theorem 3.22, srw-cl(f(A))⊆f(cl(A)) … (i). Also cl(A)=A as A is a closed set and so f(cl(A))=f(A) … (ii). From (i) and (ii), srw-cl(f(A))⊆f(A). We know that f(A)⊆srw-cl(A) and so srw-cl(f(A))=f(A) . Therefore f(A) is τ_{srw}-closed in Y.

Theorem 3.24 Let X and Y are two topological spaces where 'srw-cl(A)=scl(A) for every subset A of Y' and f:X→Y be map, then the following are equivalent;

(i) f is srw-closed map.

(ii) srw-cl(f(A))⊆f(cl(A)) for every subset A of X.

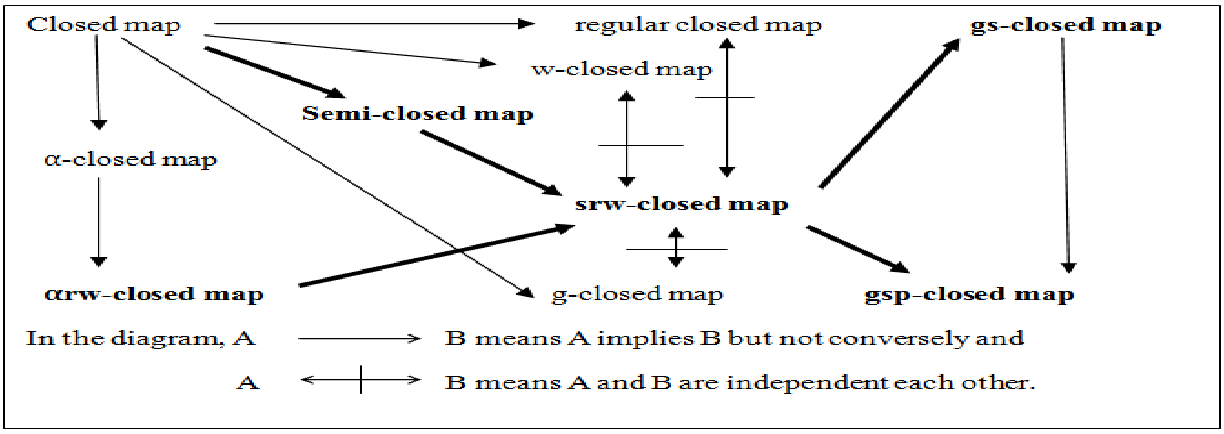
Proof: (i)⇒(ii) follows from the Theorem 3.22.

(ii)⇒(i), let A be any closed set of X then A=cl(A) and so srw-cl(f(A))⊆f(cl(A))=f(A), by hypothesis. We have f(A)⊆srw-cl(A) by [17]. Therefore f(A)=srw-cl(f(A)). Also f(A)=srw-cl(f(A))=scl(f(A)), by hypothesis. i.e. f(A)= scl(f(A)) and so f(A) is semi-closed set in Y. Thus f(A) is srw-closed set in Y. Hence f is srw-closed map.

Remark 3.25 The regular closed map and srw-closed maps are independent. This can be seen from following example.

Example 3.26 Let X={x, y, z} with topology τ= {∅, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=c and f(z)=b. Then f is regular closed map but not srw-closed, since the image of closed set {z} in X is {b}, which is not srw-closed in Y. For converse, let X= {w, x, y, z} with topology τ= {∅, {w}, {x}, {w, x}, {x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let a map f: f: X→Y be a defined by f(w)=a, f(x)=b, f(y)=a and f(z)=d. Then f is srw-closed map but not regular closed map, since the image of regular closed set {x, y, z} in X is {a, b, d}, which is not closed set in Y.

Remark 3.27 From the above discussion and known results we have the following implications.



Remark 3.28 The composition of two srw-closed maps need not be srw-closed map in general. This can be shown by the following example.

Example 3.29 Let X={x, y, z} with topology τ= {∅, {x}, {y}, {x, y}, X}, Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y} and Z ={p, q, r, s} with topology γ={∅, {p}, {q, r}, {p, q, r}, Z}. Let a map f: X→Y be defined as f(x)=a, f(y)=b and f(z)=c and g: Y → Z defined by g(a)=r, g(b)=p, g(c)=s and g(d)=q. Then f and g are two srw-closed maps but their composition g∘f: X → Z is not srw-closed map because F={z} is closed in X, but(g∘f)(F)=g(f({z}))=g({d})={q}, which is not srw-closed in Z.

Theorem 3.30 If f: X→Y is closed map and g: Y→Z is srw-closed map, then the g∘f: X→Z is srw-closed map.

Proof: Let F be any closed set in X. Since f is closed map, f(F) is closed set in Y. Since g is srw-closed map, g(f(F))=(g∘f)(F) is srw-closed set in Z. Hence g∘f is srw-closed map.

Remark 3.31 If f: X→Y is srw-closed map and g: Y→Z is closed map, then the composition need not be srw-closed map. This can be seen from following example.

Example 3.32 Let X={x, y, z} with topology τ= {∅, {x, y}, X}, Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y} and Z ={p, q, r, s} with topology γ={∅, {p}, {q, r}, {p, q, r}, Z}. Let a map f: (X, τ) →(Y, σ) be defined as f(x)=b, f(y)=d and f(z)=c and g:(Y, σ) →(Z, γ) defined by g(a)=p, g(b)=s, g(c)=r and g(d)=q. Then f is srw-closed map and g is a closed map but their composition g∘f: (X, τ) → (Z, γ) is not srw-closed map because F={c} is closed in X, but (g∘f)(F)=g(f({z}))=g({c})={r}, which is not srw-closed in Z.

Theorem 3.33 If f: X→Y and g: Y→Z are two srw-closed maps and Y be a T_{srw}-space then g∘f: X→Z is srw-closed map.

Proof: Let A be a closed set of X. Since f is srw-closed, f(A) is srw-closed in Y. Then by hypothesis, f(A) is closed. Since g is srw-closed, g(f(A)) is srw-closed in Z and g(f(A))=(g∘f)(A). Therefore g∘f is srw-closed map.

Theorem 3.34 If f: X→Y is g-closed map and g: Y→Z is srw-closed maps and Y be $T_{1/2}$-space then g∘f: X→Z is srw-closed map.

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Proof: Let A be a closed set of X. Since f is g-closed, f(A) is g-closed in Y. Since Y is $T_{1/2}$, f(A) is closed in Y. Since g is srw-closed, g(f(A)) is srw-closed in Z and g(f(A))=(g∘f)(A). Therefore g∘f is srw-closed map.

Theorem 3.35 Composition of closed maps is srw-closed map.

Proof: Proof is straight forward and fact that every closed set is srw-closed set.

Theorem 3.36 Let $f_i:(X_i,\tau_i) \to (X_{i+1},\tau_{i+1})$ be a map, then following are true;

(i) If $f_1,f_2,f_3,\ldots,f_n$ are closed maps then their compositions $f_n \circ f_{n-1} \circ f_{n-2} \circ\ldots\circ f_1$ is srw-closed map.

(ii) If $f_1,f_2,f_3,\ldots,f_{n-1}$ are closed maps and f_n is a srw-closed map then the compositions $f_n \circ f_{n-1} \circ f_{n-2} \circ\ldots\circ f_1$ is srw-closed map.

Proof: (i) The proof follows from the Theorem 3.35 and fact that every closed set is srw-closed set.

(ii) The proof follows from the Theorem 3.30.

Theorem 3.37 If f: X→Y and g: Y→Z are two mappings such that their composition g∘f: X→Z is srw-closed map, then the following statements are true;

(i) If f is continuous and surjective, then g is srw-closed map.

(ii) If g is srw-irresolute and injective, then f is srw-closed map.

(iii) If f is g-continuous, surjective and X is a, then g is srw-closed map.

(iv) If g is strongly srw-continuous and injective, then f is srw-closed map.

Proof: (i) Let A be a closed set of X. Since f is continuous, $f^{-1}(A)$ is closed in X and since g∘f is srw-closed, (g∘f)($f^{-1}(A)$) is srw-closed in Z. i.e. g(A) is srw-closed in Z, since f is surjective. Therefore g is srw-closed map.

(ii) Let B be a closed set of X. Since g∘f is srw-closed, (g∘f)(B) is srw-closed in Z. Since g is srw-irresolute, g^{-1}((g∘f)(B)) is srw-closed set in Y implies that f(B) is srw-closed in Y, since f is injective. Therefore f is srw-closed map.

(iii) Let C be a closed set of Y. Since f is g-continuous is g-closed set in X. Since X is a $T_{1/2}$-space, f^{-1}(C) is srw-closed set in X. Since g∘f is srw-closed, (g∘f)(f^{-1}(C)) is srw-closed in Z implies g(C) is srw-closed in Z, since f is surjective. Therefore g is srw-closed map.

(iv) Let D be a closed set of X. Since (g∘f)(D) is srw-closed in Z. Since g is strongly srw-continuous, g^{-1}((g∘f)(D)) is closed set in Y implies f(D) is closed set in Y, since g is injective. Therefore f is closed map.

Theorem 3.38 If f: X→Y is rw-irresolute, srw-closed and A is a srw-closed subset of X, then f(A) is a srw-closed set in Y.

Proof: Let f(A)⊆G, where G is a regular weakly open set in Y. Since f is rw-irresolute, f^{-1}(G) is rw-open in X by definition 2.12 and A⊆ f^{-1}(G). Since A is a srw-closed set in X, scl(A)⊆ f^{-1}(G)[18]. Since f is srw-closed, f(scl(A)) is srw-closed set contained in rw-open set G implies that scl(f(scl(A)))⊆f(scl(A))⊆G and so scl(f(A))⊆G. Hence f(A) is srw-closed set in Y.

Corollary 3.39 If f: X→Y be a srw-closed map and g: Y→Z be srw-closed and irresolute map, then their composition $g \circ f: X \to Z$ is srw-closed map.

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Proof: Let A be a closed set of X. Since f is a srw-closed map, f(A) is srw-closed in Y. Since g is both srw-closed and rw-irresolute, g(f(A)) is srw-closed in Z by Theorem 3.38. Also g(f(A))=(g∘f)(A). Therefore g∘f is srw-closed map.

Theorem 3.40 If f: X→Y is an open, continuous, srw-closed, surjection and scl(F)=F for every srw-closed set in Y, where X is regular, then Y is regular.

Proof: Let U be an open set in Y and p∈U. Since f is surjection, there exists a point x∈X such that f(x)=p. Since X is regular and f is continuous, there is an open set V in X such that x∈V⊆scl(V) ⊆ f^{-1}(U). Here p∈f(V)⊆f(scl(V))⊆U … (i). Since f is srw-closed, f(scl(V)) is a srw-closed set contained in the open set U. By hypothesis scl(f(scl(A)))=f(scl(V)) and scl(f(V))⊆scl(f(scl(V))) … (ii). From (i) and (ii), p∈f(V)⊆f(scl(V))⊆U and f(V) is open, since f is open. Hence Y is regular.

Theorem 3.41 If f: X→Y is srw-closed and A is closed set of X, then its restriction f_A: (A, τ_A) →Y is srw-closed map.

Proof: Let F be a closed set of A. Then F=A∩E for some closed set E of X and so F is closed set of X. Since f is srw-closed, f(F) is srw-closed set in Y. But f(F)=f_A(F). Therefore f_A:(A, τ_A) →Y is srw-closed map.

Now we define the new class of stronger form of srw-closed maps is called srw*-closed maps in topological spaces.

Definition 3.42 A map f: X→Y is said to be **srw*-closed maps**, if the image of every srw-closed set of X is srw-closed set in Y.

Theorem 3.43 If f: X→Y is srw*-closed map, then which is srw-closed map, but not conversely.

Proof: Let F be a closed set in X and the fact that every closed set is srw-closed se. Hence F is srw-closed set in X. Since f: X→Y be a srw*-closed map, f(F) is srw-closed set in Y. Therefore f is srw-closed map.

Example 3.44 Let X= {w, x, y, z} with topology τ= {∅, {w}, {x, y}, {w, x, y}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X →Y be defined as f(w)=c, f(x)=d, f(y)=a and f(z)=b. Then f is srw-closed map but not srw*-closed, since the image of srw-closed set {x, y} in X is {a, d}, which is not srw-closed in Y.

Theorem 3.45 If f: X→Y and g: Y→Z are two srw*-closed maps, then their composition g∘f: X→Z is srw*-closed map.

Proof: Let F be a srw-closed set in X. Since f is srw*-closed map, f(F) is srw-closed set in Y. Since g is srw*-closed map, g(f(F)) is srw-closed set in Z. Hence g∘f is srw*-closed map.

Theorem 3.46 If f: X→Y is irresolute and srw-closed map then f is srw*-closed map.

Theorem 3.47 If f: X→Y be a closed map and g: Y→Z be srw*-closed, then their composition g∘f: X→Z is srw-closed map.

Proof: Let F be a closed set in X. Then f(F) is closed in Y. The fact that every closed set is srw-closed set implies that f(F) is srw-closed set in Y. Since g is srw*-closed map, g(f(F))=(g∘f)(F) is srw-closed set in Z. Hence g∘f is srw-closed map.

Theorem 3.48 If f: X→Y be a srw-closed map and g: Y→Z be srw*-closed, then their composition g∘f: X→Z is srw-closed map.

Proof: Let F be a closed set in X. Since f is srw-closed map, f(F) is srw-closed set in Y. Since g is srw*-closed map, g(f(F))= (g∘f)(F) is srw-closed set in Z. Hence g∘f is srw-closed map.

IV. SEMI-REGULAR WEAKLY OPEN MAPS IN TOPOLOGICAL SPACES

Definition 4.1 A map f: X→Y is said to be semi-regular weakly open (srw-open) map, if the image of every open set in X is srw-open in Y.

From the definition 4.1 we have following results;

Theorem 4.2 (i) Every open map is srw-open map, but not conversely.

(ii) Every semi-open map is srw-open map, but not conversely.

(iii) Every αrw-open map is srw-open map, but not conversely.

(iv) Every srw-open map is gs-open map, but not conversely.

(v) Every srw-open map is gsp-open map, but not conversely.

Proof: Proofs follow from Definition 4.1 and fact that lemma 2.14.

Example 4.3 Let X={x, y, z} with topology τ= {∅, {x}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=a and f(z)=b. Then f is srw-open map but not open, since the image of open set {x} in X is {d}, which is not open in Y.

Example 4.4 Let X={x, y, z} with topology τ= {∅, {z}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=b and f(z)=b. Then f is srw-open map but not semi-open, since the image of open set {z} in X is {b}, which is not semi-open in Y.

Example 4.5 Let X={x, y, z} with topology τ= {∅, {x, y}, X} and Y= {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=d and f(z)=b. Then f is srw-open map but not αrw-open, since the image of open set {x, y} in X is {d}, which is not αrw-open in Y.

Example 4.6 Let X={x, y, z} with topology τ= {∅, {x, z}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=c, f(y)=d and f(z)=b. Then f is gs-open map but not srw-open, since the image of open set {x, z} in X is {b, c}, which is not srw-open in Y.

Example 4.7 Let X={x, y, z} with topology τ= {∅, {x}, {x, z}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b, c}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=c and f(z)=b. Then f is gsp-open map but not srw-open, since the image of open set {x, z} in X is {b, d}, which is not srw -open in Y.

Theorem 4.8 If f: X→Y is srw-open, then f(int(A))⊆srw-int(f(A)) for every subset A of X.

Proof: Let f: X→Yis an open map and A is any subset of X. Then int(A) is open in X and so f(int(A)) is srw-open set in Y. We have f(int(A))⊆f(A). Therefore by the Remark 5.11(2) [17], f(int(A))⊆srw-int(f(A)).

Theorem 4.9 A map f: X→Y be srw-open if and only if for any subset S of Y and any closed set of X containing f^{-1}(S), there exists a srw-closed set T of Y containing S such that f^{-1}(T)⊆F.

Proof: Suppose f: X→Yis srw-open map. Let S⊆Y and F be a closed set of X such that f^{-1}(S)⊆F. Now X-F is an open set in X. Since f is srw-open map, f(X-F) is srw-open set in Y. Then T=Y-f(X-F) is a srw-closed set in Y. Note that f^{-1}(S)⊆F implies S⊆T and f^{-1}(T)=X-f^{-1}(X-F)⊆X-(X-F)=F. i.e. f^{-1}(T)⊆F.

Conversely, suppose U be an open set of X. Then (Y-f(U))⊆X-U is a closed set in X. By hypothesis, there exists a srw-closed set T of Y such that Y-f(U)⊆T and f^{-1}(T)⊆X-U and so U⊆X-f^{-1}(T) . Hence Y-T⊆f(U) ⊆Y-f(f^{-1}(T)⊆Y-T which implies f(U)=Y-T. Since Y-T is a srw-open, f(U) is srw-open in Y and therefore f is srw-open map.

Theorem 4.10 If f: X→Y is srw-open, then f^{-1}(srw-cl(A)) ⊆cl(f^{-1}(A)) for each subset A of Y.

Proof: Let f: X→Yis a srw-open map and A be any subset of Y. Then f^{-1}(A)⊆cl(f^{-1}(A)) and cl(f^{-1}(A)) is closed set in X. Then by Theorem 4, there exists a srw-closed set B of Y such that A⊆B and f^{-1}(B)⊆cl(f^{-1}(A)). Now srw-cl(A)⊆srw-cl(B)=B, by Remark 6.3(ii) [17], as B is srw-closed set of Y. Therefore f^{-1}(srw-cl(A))⊆ f^{-1}(B) and so f^{-1}(srw-cl(A))⊆ f^{-1}(B)⊆ cl(f^{-1}(A)). Thus f^{-1}(srw-cl(A))⊆cl(f^{-1}(A)) for each subset of A of Y.

Remark 4.11 The converse of above Theorem generally not true, which can be shown from the following example.

Theorem 4.12 If f: X→Y is srw-open, then for each neighbourhood U of x in X, there exists a srw-neighbourhood **N** of f(x) in Y such that **N**⊆f(U).

Proof: Let f: X→Yis a srw-open map. Let x∈X and U be an arbitrary neighbourhood of x in X. Then there exists, an open set G in X such that x∈G⊆U. Now f(x)∈f(G)⊆f(U) and f(G) is srw-open set in Y, as f is srw-open map. By Theorem 4.6 [17] f(G) is srw-neighbourhood of each of its points. By taking f(G)=**N**, **N**is a srw-nbd of f(x) in Y such that **N**⊆f(U).

Theorem 4.13 For any bijection map f: X→Y, the following statements are equivalent;

(i) f^{-1}: Y→X is srw-continuous.

(ii) F is srw-open map.

(iii) F is srw-closed map.

Proof: (i)⟹(ii), let U be an open set of X. By assumption, $(f^{-1})^{-1}$(U)=f(U) is srw-open in Y and so f is srw-open.

(ii)⟹(iii), let F be a closed set of X, X-F is open set in X. By assumption, f(X-F) is srw-open in Y i.e. f(X-F) is srw-open set in Y since every open set is srw-open Corollary 3.8[17] and therefore f(F) is srw-closed in Y. Hence f is srw-closed map.

(iii)⟹(i), let F be a closed set of X. By the assumption, f(F) is srw-closed in Y. But f(F)=$(f^{-1})^{-1}$(F) and therefore f^{-1} is continuous.

Remark 4.14 The composition of two srw-open maps need not be a srw-open map.

Now we define the new class of stronger form of srw-open maps is called srw*-open maps in topological spaces.

Definition 4.15 A map f: X→Y is said to be **srw*-open map**, if the image f(A) is srw-open set in Y for every srw-open set A in X.

Remark 4.16 Since every open set is a srw-open set, we have every srw*-open map is srw-open map. The converse is not true generally as seen from the following example.

Example 4.17 Let X={x, y, z} with topology τ= {∅, {x}, X} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: X→Y be defined as f(x)=d, f(y)=a and f(z)=b. Then f is srw-open map but not srw*-open, since the image of srw-open set {x} in X is {d}, which is not srw-open in Y. Hence f is not srw*-open map.

Theorem 4.18 If f: X→Y and g: Y→Z be two srw*-open maps, then their composition g∘f: X→Z is srw*-open map.

Proof: Proof is similar to the Theorem 3.45.

Theorem 4.19 For any bijective map f: X→Y, the following statements are equivalent;

(i) f^{-1}: Y→X is srw-irresolute map.

(ii) f is srw*-open map.

(iii) f is srw*-closed map.

Proof: Proof is similar to the Theorem 4.13.

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