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Mathematical Abstract Concepts Acting as a Language for Understanding Fundamental laws of Physical Sciences

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ABSTRACT

Mathematics is probably the only subject that can be classified both as art as well as science- former, because it is not constrained by the real world and latter because it is a logical system with precisely defined rules as well as primitives that lead to unambiguous nontrivial theorems. Indian (and of Indian origin) mathematicians have continued to do seminal work till present times. In such fabulous times, a non-mathematician ponders about the nature of mathematics, and revisits the question: why are fundamental laws of Nature inherently mathematical?

Keywords: Axiom, Entities, Non-Euclidean Geometry, Observables, Operators

I.INTRODUCTION

Inception of nascent topics of mathematics like arithmetic and geometry can be attributed to necessity, factoring in human evolution and survival. Archeological evidence suggests that Indus Valley people of 2500-1900 BC used vertical strokes to represent numbers [1]. Early humans not only had to keep track of their possessions through counting but also needed to estimate directions, distances, shapes and sizes, without which hunting, exploration, building cities like that of Harappa and Mohenjodaro, raising humongous Egyptian pyramids, etc, would not have been possible.

Homo sapiens who could assess numbers and sizes, and discern shapes and directions, had an evolutionary advantage for survival during the Darwinian struggle for existence. But as is the nature of human brain, brighter of the lot, dealing with this incipient subject of numbers and shapes, perceived interesting patterns in some of these entities, their interrelations, and posed interesting questions that entailed concepts such as the zero, decimal system, prime numbers, irrational numbers, the Pythagorean relation between sides and the hypotenuse of right triangles and so on [2].

Playing and tinkering are natural human instincts and, not surprisingly therefore, curious and innovative minds toyed around with patterns found among the arithmetical and geometrical entities to create rich logical systems from which one could establish (i.e. prove) non-trivial results (i.e. theorems) such as the number of prime numbers being infinite or the Pythagoras theorem, by deploying imaginative and clever tricks on a set of very few, almost self-evident assumptions (i.e. axioms) and by making use of logical rules of operations.

The sensitiveness of mathematical minds gave fillip to axiomatic systems, like for example Euclid's geometry, to grow wings as though of their own and to fly out to magical and intangible worlds. Numbers and geometrical

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concepts got transmuted and generalized to ever increasingly abstract but beautiful creatures, seemingly far removed from concrete reality. New kinds of numbers were imagined.

II. MATHEMATICS AS A TOOL FOR PHYSICS

Speaking of abstraction, the imaginary number $i = \sqrt{-1}$ (iota) arose as a mathematical device, representing an abstract solution of the quadratic equation $x^2 + 1 = 0$. It is interesting to note that we can give a geometrical meaning to iota (as the altitude) by interpreting $x^2 + 1 = 0$ to be a Pythagorean relation for an abstract right triangle having a unit base length but a hypotenuse of zero length!

Complex numbers, constructed out of a combination of real numbers and iota, gave birth to a rich structure consisting of elegant theorems involving analytic functions, conformal mappings, analytic continuation, Fourier transforms, Riemann zeta functions, etc. Before the arrival of iota, it was unimaginable that geometrical entities and functions related to logarithm could get linked up, as in the case of the amazing formula due to Euler,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Till the advent of quantum theory, complex numbers were thought of only as useful tools, having no correspondence with the actual world. After all, in Nature, measurable quantities are always real. But post 1920s, it was realized that the physical world is quantum mechanical in nature, in which the Schrodinger equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \qquad (1)$$

determines the time evolution of a physical system, given the Hamiltonian $\hat{H}$, while the commutator bracket of position and momentum operators,

$$[\,\hat{x}\,,\hat{p}\,] = i\hbar \qquad (2)$$

is responsible for Heisenberg's uncertainty principle $\Delta x\,\Delta p\ \geq \frac{\hbar}{2}$. Eqns.(1) and (2) emphasize that $i \equiv \sqrt{-1}$ indeed 'exists' in the real world, and it is probably as real as time [3]. Quantities measured in experiments are still real, as they invariably correspond to eigenvalues of hermitian operators representing the physical observables.

In another case, the measure of distance between any two points that relied on Pythagoras theorem in the standard Euclidean geometry got generalized to abstract ones involving metric tensors described by non-Euclidean geometry. Pioneers of these developments were titans like Gauss, Lobachevsky, Bolyai, Riemann, David Hilbert, Poincare and others [4].

Non-Euclidean geometry was used by Einstein to formulate the relativistic theory of gravity (i.e. general relativity) during 1907-1915. General relativity took geometry of the four dimensional space and time to an exalted level, where it became as much dynamical as the matter itself, whose energy and momentum caused the geometry to be non-Euclidean[5,6].

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III. MATHEMATICAL FORMULATIONS IN PHYSICS

It is an established fact that all fundamental laws of Nature happen to be expressed in mathematical language. As early as 1959, Eugene Wigner, an eminent theoretical physicist, drove home the point about 'The unreasonable effectiveness of mathematics in the Natural sciences[7]. Abstract concepts and relations like tensors, Hilbert space, operators, matrices, commutators, Lie algebras and groups, Grassmann variables and many more, created by mathematicians for their own sake, turn out to describe fundamental truths concerning the actual universe.

General relativity and the bizarre quantum world of electrons, photons, atoms, etc. bear testimony to the fact that most of the aforementioned abstract concepts 'exist' in the real world. However, a crucial fact has been overlooked - only a miniscule portion of the vast body of mathematical creations gets to enjoy the status of being the language of fundamental laws of Nature. The supply of abstract concepts from mathematicians is incredibly larger than the demand from physics. Therefore, should one be surprised that a small subset of mathematical ideas based on logic, beauty, elegance and abstract generalization of concepts, which were rooted to reality once upon a time, gets realized in fundamental physics?

Any idea whose effect or consequences cannot be measured, in principle, is not part of science. It so happens that physical entities are measured quantitatively (i.e. in terms of numbers), and hence, it is natural that their interrelations, including temporal cause and effect connections, better be based on a language that is numeric, precise, logical and unambiguous. Mathematics is such a language and thus, is ideally suited for expressing laws and principles in physical sciences.

Furthermore, since mathematics deals with entities, relations and theorems that are abstract in character, it is naturally amenable to wider applications. For, one can substitute a physical concept in place of an abstract entity, use the established mathematical machinery and arrive at a non-trivial result that is concrete, provided the logical structure of the abstract system is 'isomorphic' to the basic framework of the concrete system. In other words, mathematical modeling of a physical system is often a case of moving from general to particular. By its very construction, mathematics encompasses such diverse, generalized as well as abstract elements and structures that it has a greater propensity to act as language for physical sciences.

The real world has continued to inspire mathematicians, be it the birth of calculus for finding trajectories of bodies moving in gravitational fields or of distribution theory that ensued from Dirac delta function. It is common wisdom that systematic analysis of gambling outcomes by Fermat, Pascal and Huygens had ushered in the mathematical theory of probability.

One could speculate whether the chance coincidence of angular sizes of the Sun and Moon being almost same at present (and also during the historical period) causing eclipses to occur, played a significant role in the development of mathematics in the past. After all, mathematicians of great repute exercised their minds to understand and predict eclipses, be it Hipparchus, Aryabhatta or Varahamihira. In other words, it is scientifically relevant to ask whether in the absence of Moon eclipsing the Sun, there would have been enough impetus and motivation to develop trigonometry and other useful computational techniques [8,9].

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The renowned astrophysicist S. Chandrasekhar arrived at many fundamental truths about gravitating systems in astronomy, employing beautiful and elegant mathematical analysis. His predictions concerning upper limit on white dwarf mass, dynamical friction, magnetohydrodynamics instabilities, etc. stood the tests of observational verification, vindicating the power of mathematical rigor [10,11].

V. CONCLUSION

Only a small portion of the vast body of mathematical creations gets to enjoy the status of being the language of fundamental laws of Nature. The supply of abstract concepts from mathematicians is incredibly larger than the demand from physics. Furthermore, since mathematics deals with entities, relations and theorems that are abstract in character, it is naturally amenable to wider applications. For, one can substitute a physical concept in place of an abstract entity, use the established mathematical machinery and arrive at a non-trivial result that is concrete, provided the logical structure of the abstract system is 'isomorphic' to the basic framework of the concrete system. In other words, mathematical modeling of a physical system is often a case of moving from general to particular. By its very construction, mathematics encompasses such diverse, generalized as well as abstract elements and structures that it has a greater propensity to act as language for physical sciences.

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