

Multiphase Fractional-Order Sinusoidal Oscillator Design Using CFOA

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ABSTRACT

Fractional-order multiphase sinusoidal oscillator (FMSO) configurations realized by employing Current Feedback Operational Amplifiers (CFOA) as active elements are presented in this work. Compared with the corresponding already published literature, the proposed realizations offer the following advantages: a) independent tuning of the frequency and the condition of oscillations, b) reduced number of passive components, c) increased number of grounded components, and d) capability for high frequency operation. The operation of the proposed FMSO schemes has been evaluated through simulation and experimental results by utilizing the AD844 CFOA IC and discrete components.

Keywords: Fractional calculus, fractional-order circuits, multiphase sinusoidal oscillators, low-pass filters, Current Feedback Operational Amplifiers (CFOA)

I. INTRODUCTION

Non-integer calculus or fractional calculus research is gaining the pace as it is the natural generalization of the traditional integer order calculus. As a result, due to its feat in explanation of anomalous diffusion, it has been widely accepted as a strong tool for efficient modeling of most of the physical phenomena's and is employed in various areas of study like in bio-, electrical-, electronic- and mechanical-engineering, physics, chemistry, biology etc. [1]. As far as electrical and electronics engineers are concerned, the fractionalization of electronic circuits and systems are showing the promising results for the future trends in circuits and system design [2-11]. Among the various reported fractional-order electronic circuits and systems in the literature, the fractional order oscillators are frequently studied circuits that shows the most positive and promising results as far as their frequency of oscillation is concerned. Apart from the commonly studied and designed fractional-order oscillators, the class of oscillators known as multiphase sinusoidal oscillators (MSOs) are used as building blocks of various communication systems, power controllers, quadrature mixers, switched capacitor filters etc. [12-14].

As per the author's best knowledge, fractional-order MSO (FMSO) topologies have been reported in [13, 14]. In [13] integer and fractional-order all-pass filter (APF) stages have been employed to design the fractional/mixed MSO, where operational amplifiers (op-amps) have been used as active elements. The topologies in [13] suffer from the following drawbacks: a) the type of the realized filter function is determined by the matching of resistors in the negative feedback path, b) three of the four passive elements are floating, and c) the maximum frequency of operation is limited by the bandwidth of the employed op-amps. In [14] a multi feedback

approach, using Current Feedback Operational Amplifiers (CFOAs) as active devices, has been employed to design the fractional-order MSO. The topology in [14] suffer from the following drawbacks: a) The design has used fractional lossless integrators which like their integer counterparts are difficult to design in practice than the lossy/low-pass filter ones used in our case. b) The oscillation condition is complex as authors need to set the feedback coefficients to different polarities as can be seen in the captions of Figs. 4 (c)-(e) and is not the case in our paper where the gain of all filters are of same polarity and the entire gain could even be achieved from a single block. The complexity in the oscillation condition will increase as the order of the oscillator increases. c) The circuit complexity is more than the proposed one as the design has more floating components and active devices. The authors have cleverly shown the circuit of conventional oscillator only where all the feedback coefficients are of same polarity otherwise for fractional cases the use of active devices to achieve the inversions would have made it clear that the proposed design is much simpler, and d) The reported topology is valid for $\alpha > 0.5$ as has been mentioned by the authors whereas the proposed design is valid $\alpha < 0.5$ as well. Besides, the stability of the topology is not permanent as it depends on the feedback coefficient while the proposed design will be stable due to the fact that the low-pass filters and single feedback with gain coefficient equal to 1 for all cases have been used. Furthermore, the authors have mentioned the advantage that the asymmetric phase changes between the output nodes can be obtained by taking the different values of α for different stages which is true in our case as well.

In order to overcome the aforementioned drawbacks, FMSO topologies constructed from fractional-order low-pass filter stages are introduced in this paper. Current Feedback Operational Amplifiers (CFOAs) have been utilized as active elements due to their benefits in terms of maximum frequency of operation, slew-rate, and design versatility [15]. The paper is organized as follows: a brief description of the low-pass fractional-order filters is given in Section 2, while the proposed odd- and even-phase MSOs are presented in Section 3. In section 4 the non-idealities of CFOA for current design has been discussed while the performance of MSOs is evaluated through simulation and experimental results in Section 5. In addition, a comparison with the performance of the topologies introduced in [13] has been made.

II. FRACTIONAL ORDER LOW-PASS FILTERS

The transfer function of an inverting fractional-order filter is that in (1)

$$H(s) = -\frac{K}{(\tau s)^\alpha + 1} \quad (1)$$

Where α is the order of the filter and τ is the corresponding time-constant. The gain and phase responses are given by the expressions in (2) and (3) respectively

$$|H(j\omega)| = \frac{K}{\sqrt{1 + (\omega\tau)^{2\alpha} + 2(\omega\tau)^\alpha \cos\left(\frac{\alpha\pi}{2}\right)}} \quad (2)$$

$$\arg[H(j\omega)] = -180 - \tan^{-1} \frac{(\omega\tau)^\alpha \sin\left(\frac{\alpha\pi}{2}\right)}{1 + (\omega\tau)^\alpha \cos\left(\frac{\alpha\pi}{2}\right)} \quad (3)$$

Note that when $\omega \rightarrow 0$, the phase response is equal to π while for $\omega \rightarrow \infty$ the phase varies asymptotically and will be equal to: $-\pi - \frac{\alpha\pi}{2}$. Therefore the maximum phase change offered by a fractional-order low-pass filter is

equal to $\frac{\alpha\pi}{2}$. The pole frequency of the filter (ω_h) is given by the formula: $\omega_0 = 1/\tau$, but it is not the half-power

frequency as in the case of integer-order filter. The half-power frequency will be calculated by setting $|H(j\omega)|_{\omega=\omega_h} = K/\sqrt{2}$; thus, from (2) it is derived that;

$$\omega_h = \frac{1}{\tau} \left[\sqrt{1 + \cos^2\left(\frac{\alpha\pi}{2}\right)} - \cos\left(\frac{\alpha\pi}{2}\right) \right]^{1/\alpha} \quad (4)$$

III. PROPOSED MULTIPHASE SINUSOIDAL OSCILLATOR

The Functional Block Diagram (FBD) of an odd-order MSO consisting of n fractional-order inverting low-pass filters is depicted in Fig.1a, where the open-loop gain is given by;

$$L(s) = -K^n \left[\frac{1}{(\tau s)^\alpha + 1} \right]^n \quad (5)$$

Taking into account that a phase change equal to $\frac{\alpha\pi}{2}$ it is concluded that $n \geq (2/\alpha)$.

Using (5) and (2) - (3), the condition and the frequency of oscillation, obtained from the criterion of Barkhausen, will be given by (6) and (7) respectively;

$$K = \sqrt{1 + (\omega_o\tau)^{2\alpha} + 2(\omega_o\tau)^\alpha \cos\left(\frac{\alpha\pi}{2}\right)} \quad (6)$$

$$\omega_o = \frac{1}{\tau} \left[\frac{\tan\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\alpha\pi}{2}\right) - \tan\left(\frac{\pi}{n}\right) \cdot \cos\left(\frac{\alpha\pi}{2}\right)} \right]^{1/\alpha} \quad (7)$$

Substituting (7) into (6), the condition of oscillation could be alternatively expressed as in (8);

$$K = \sqrt{1 + \left[\frac{\tan\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\alpha\pi}{2}\right) - \tan\left(\frac{\pi}{n}\right) \cdot \cos\left(\frac{\alpha\pi}{2}\right)} \right]^2 + 2 \frac{\tan\left(\frac{\pi}{n}\right) \cdot \cos\left(\frac{\alpha\pi}{2}\right)}{\sin\left(\frac{\alpha\pi}{2}\right) - \tan\left(\frac{\pi}{n}\right) \cdot \cos\left(\frac{\alpha\pi}{2}\right)}} \quad (8)$$

An important conclusion, from the design flexibility point of view, readily obtained from (8) is that the oscillation frequency could be adjusted through the realized time-constant without disturbing the condition for oscillation.

The FBD of the corresponding even-order MSO is depicted in Fig. 1b, where an extra inverting stage has been added in comparison to the case of the odd-order MSO. It should be mentioned at this point that the expressions in (5) - (8) are still valid.

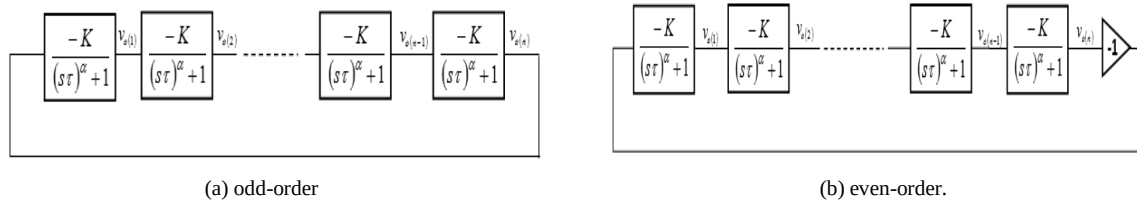


Fig. 1. FBD of an MSO constructed from low-pass fractional-order filters

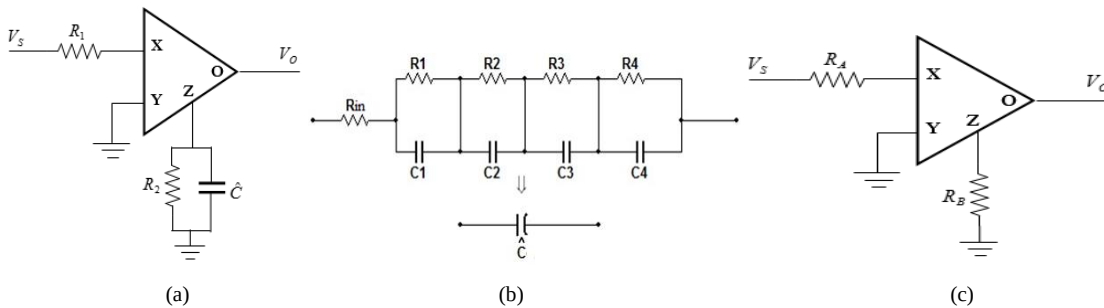


Fig. 2. Building blocks of MSO (a) CFOA based fractional-order low-pass filter, (b) RC network for approximating a fractional-order capacitor, and (c) CFOA based inverter stage.

To verify the operation of proposed MSO topologies, the fractional-order low-pass filter given in Fig. 2a will be employed. The low-frequency gain is: $K=R_2/R_1$, while the realized time constant is given by the formula $\tau=(R_2\hat{C})^{1/\alpha}$, where $\hat{C}$ is the pseudo-capacitance of the fractional-order capacitor. It should be mentioned at this point that the pseudo-capacitance, which is expressed in $F/\text{sec}^{1-\alpha}$, is related to the conventional capacitance (C) in Farads with the formula $C=\hat{C}/\omega^{1-\alpha}$. Due to the fact that fractional-order capacitors are not commercially available, a way for approximating this element is the utilization of the RC network depicted in Fig. 2b [3, 5-7, 13]. The CFOA realization of the inverter employed in even-order case is given in Fig. 2c, where the gain of this stage is equal to $K_I=R_B/R_A$.

IV. Parasitic effects of the non-ideal CFOA

The non-ideal model of showing various parasitic effects is shown in Fig. 3. Following the non-ideal model of CFOA of Fig. 3, the modified time constant and gain of the low-pass filter will be given by (9) and (10) respectively.

$$\tau'=(R_2'\hat{C}')^{1/\alpha} \quad (9)$$

$$K'=R_2'/R_1' \quad (10)$$

Where the non-ideal values of the components is given by $R_2'=R_2\parallel R_z$, $R_1'=R_1+R_x$, $\hat{C}'=\hat{C}+C_z$

Besides the gain of the inverter will be now given by (11);

$$K'_I = R'_B / R'_A \quad (11)$$

Where $R'_B = R_B \parallel R_z \parallel C_z$ and $R'_A = R_A + R_x$

Using (9)-(11), the open-loop gain of the MSO will be given by (12);

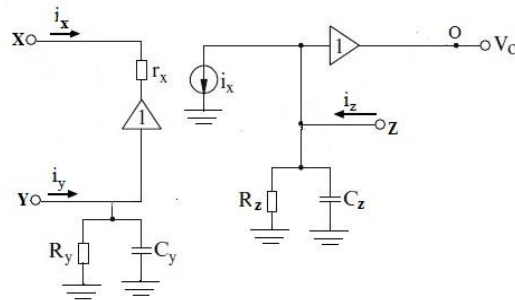


Fig. 3. Non-ideal model of CFOA.

$$L(s) = -(K')^n \left[\frac{1}{(\tau's)^\alpha + 1} \right]^n \quad (12)$$

Using (12), the condition and the frequency of oscillation, obtained from the criterion of Barkhausen, will be given by (13) and (14) respectively.

$$K' = \sqrt{1 + (\omega_o \tau')^{2\alpha} + 2(\omega_o \tau')^\alpha \cos\left(\frac{\alpha\pi}{2}\right)} \quad (13)$$

$$\omega_o = \frac{1}{\tau'} \left[\frac{\tan\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\alpha\pi}{2}\right) - \tan\left(\frac{\pi}{n}\right) \cdot \cos\left(\frac{\alpha\pi}{2}\right)} \right]^{1/\alpha} \quad (14)$$

V. EXPERIMENTAL AND SIMULATION RESULTS

As a first design example, a MSO with five fractional-order low-pass filter stages, $\alpha=0.5$ and $f_o=6\text{kHz}$ will be realized. Considering a fractional capacitor with $\hat{C} = 79.2\text{nF}/\text{sec}^{0.5}$ (i.e. $C=1\text{nF}$ @ 1kHz and $\alpha=0.5$), then using (7)-(8) and the formula $\tau = (R_2 \hat{C})^{1/\alpha}$, it is obtained that $R_2=244\text{k}\Omega$ and $K=4.52$. The values of passive elements of the RC network in Fig. 2b are summarized in Tab. 1. It is worth to mention here that the frequency of oscillation with the above values of R_2 and C in the case of an integer-order MSO would be only 8.23Hz which clearly shows that the frequency of oscillation of MSO gets increased by a large amount (depending upon the value of α) than its integer counterpart.

The operation of the MSO was verified through experimental and PSPICE simulation results by employing the AD844 discrete IC components as CFOAs with supply voltage of $\pm 10\text{V}$. The obtained simulated and sample experimental results are given in Fig. 4 and 5, respectively. The values of the simulated and experimentally obtained oscillation frequency were 5.71kHz and 6.57kHz , close to the theoretically predicted. Taking the output of the first low-pass filter as reference node, the theoretical, simulated and experimental values of phase angles of the outputs are given Table 2. In addition, the simulated values of the Total Harmonic Distortion

(THD) are provided in this Table 2. In AD-844 CFOA, $R_x = 50\Omega$, $R_y = 2M\Omega$, $C_y = 2pF$, $R_z = 3M\Omega$, $C_z = 4.5pF$. The values used in the design are such that they absorb the values of the parasitics. This is the reason that we see small deviations between the theoretical and simulation results.

As a second design example, a MSO with six fractional-order low-pass filter stages, $\alpha=0.5$ and $f_0=2$ kHz will be realized. Considering again that $\hat{C} = 79.2nF / \text{sec}^{0.5}$, the obtained values of R_2 and K will be 218 k Ω and 2.73, respectively. In addition, the values of resistors R_A and R_B have been set to 10k Ω . The obtained simulated and sample experimental results are given in Fig. 6 and 7, respectively. The oscillation frequency derived through simulation and experimentation was 2.12 kHz and 2.57 kHz. Taking the output of the inverter as reference node, the theoretical, simulation and experimental results about phase and simulated results of THD are summarized in Table 3. It is worth to mention here that the amplitudes of the generated signals are satisfactory and their stability if required can be controlled by adding amplitude control stages [16].

In contrast to [14], the design presented in [13] and the proposed design are single feedback designs. Therefore, it is fair to compare the proposed design with the one reported in [13]. An important benefit of the proposed MSO topology is that the mismatch of the resistor value only affects the low-frequency gain of the filter while in the case of the topology in [13] several frequency characteristics will be affected.

In order to verify this claim, the effect of mismatch of the components on the behavior of the oscillators will be evaluated through the Monte-Carlo analysis tool. The statistical plots with number of runs $N=100$ for oscillation frequency and phase angle for the design reported in [13] (case 1) and proposed design (case 1) are given in Figs 8(a) and 8 (b) respectively.

The percentage values of standard deviation of oscillation frequency and phase angle for the scheme in [13] are 5.57% and 5.78% respectively, while the corresponding values for the proposed MSO are 3.82% and 3.03% respectively. In addition, as can be seen from the filter topology in Fig. 2a two grounded conventional passive resistors and a grounded fractional-order capacitor are required, while in [13] three conventional passive resistors and a fractional-order capacitor are required. It should be mentioned at this point that at least two of the three resistors are floating in [13] while the fractional-order capacitor is floating in one of the two cases. Therefore, the proposed solution is advantageous in terms of passive component count as well as of the effect of parasitics compared to that offered in [13].

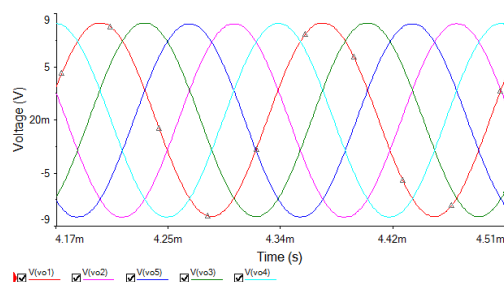


Fig. 4. Simulation results for the odd-order MSO.

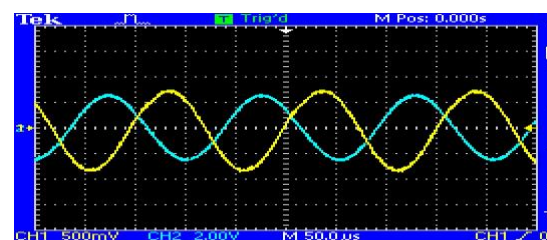


Fig. 5. Sample experimental results for the odd-order MSO ($V_{o(0)} \cdot V_{o(1)}$).

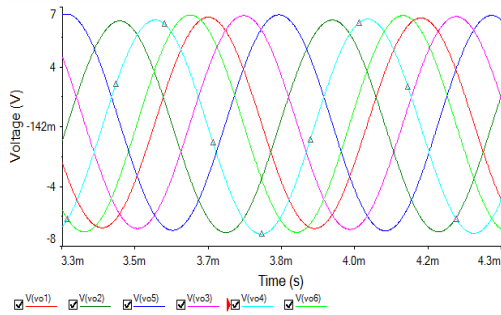


Fig. 6. Simulation results for the even-order MSO

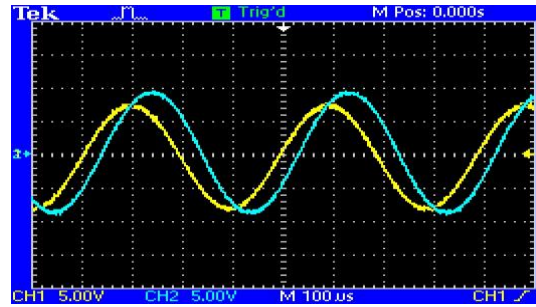


Fig. 7. Sample experimental results for the even-order MSO ($V_{o(1)}$, $V_{o(2)}$).

Table. 1 Values of the passive components in the RC network in Fig.2b.

Component	Simulations	Experimentation
R_{in}	17.68 k Ω	17.55 k Ω
R_1	39.9 k Ω	40.5 k Ω
R_2	59.9 k Ω	58.2 k Ω
R_3	143.05 k Ω	145.5 k Ω
R_4	1.17 M Ω	1.15 M Ω
C_1	0.53 nF	0.515 nF
C_2	1.88 nF	1.85 nF
C_3	3.33 nF	3.38 nF
C_4	4.36 nF	4.45 nF

Table. 2 Phase and THD characteristics of the MSO in Fig. 1(a).

Parameter	$V_{o(2)}$	$V_{o(3)}$	$V_{o(4)}$	$V_{o(5)}$
ϕ (theory)	216°	72°	288°	144°
ϕ (simulation)	215.5°	72.7°	287.5°	143.7°
ϕ (experiment)	212°	72°	276°	132°
Simulated THD	0.106%	0.175%	0.232%	0.132%

Table 3: Phase and THD characteristics of the MSO in Fig.1(b).

Parameter	$V_{o(1)}$	$V_{o(2)}$	$V_{o(3)}$	$V_{o(4)}$	$V_{o(5)}$	$V_{o(6)}$
ϕ (theory)	210°	60°	270°	120°	330°	180°
ϕ (simulation)	208.8°	59.6°	270°	119.5°	329.75°	180°
ϕ (experiment)	216°	54°	252°	117°	324°	180°
Simulated THD (%)	0.115	0.195	0.210	0.368	0.138	0.126

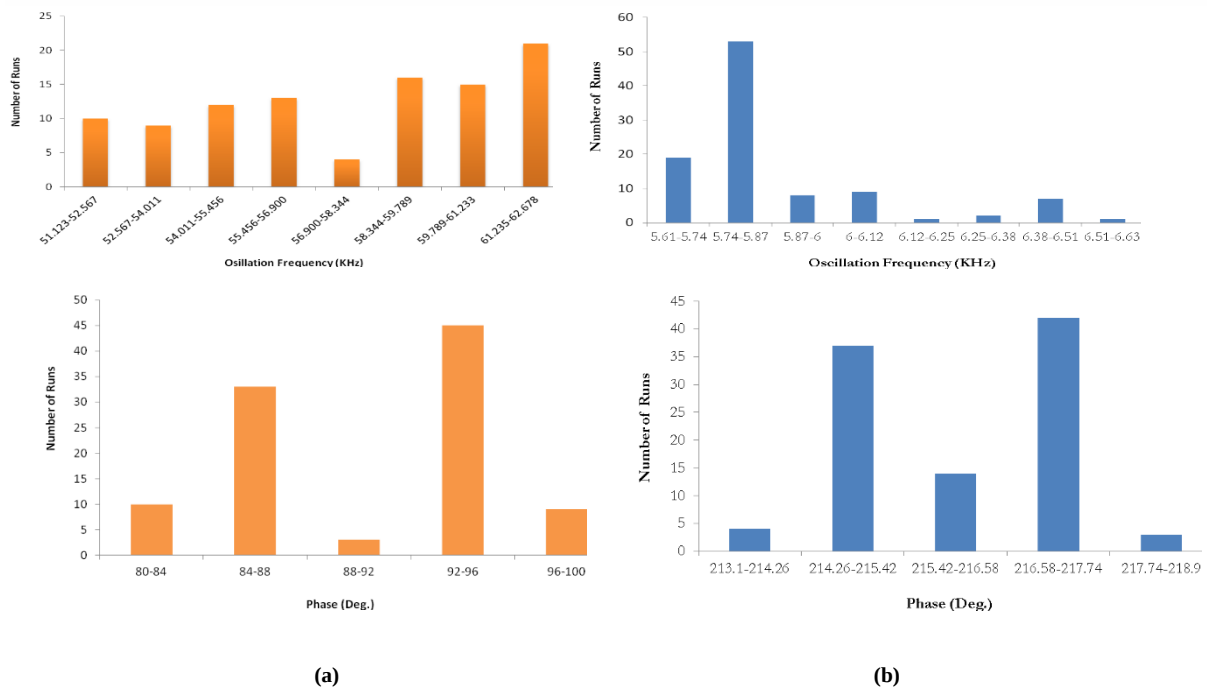


Fig. 8. The statistical results obtained from Monte-Carlo simulation with N=100 for (a) Oscillation Frequency and Phase angle for the design reported in [13], and (b) Oscillation Frequency and Phase angle for the proposed design.

VI. CONCLUSION

The proposed MSO topology is less sensitive to the effect of component values variation and offers independent adjustment of the oscillation frequency, reduced passive component count and effect of parasitics as compared to the one reported in the literature. Thus, the proposed oscillator could be an attractive candidate for realizing high-performance analog signal processing blocks.

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