



Burr Type III Software Reliability Assessment Using SPC- An Order Statistics Approach

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ABSTRACT

Assessment of Software Reliability is a vital aspect to be considered during the software development process. Software reliability is the probability that given software functions work without failure in a specific environment during a specified time. It can be assessed using Statistical Process Control (SPC). SPC is a method of quality control that uses statistical methods to control and monitor a software process and thereby contributes significantly to the improvement of software reliability. Control charts are widely used SPC Tools to monitor software quality. The proposed model involves estimation of the parameters of the mean value function and hence these values are used to develop the control charts. The Maximum Likelihood Estimation (MLE) method is used to derive the estimators of the distribution. In this paper we propose a mechanism to monitor software quality based on order statistics of cumulative observations of time domain failure data using mean value function of Burr type III distribution based on Non-Homogeneous Poisson Process.

Keywords- Burr Distribution, Control Charts, Mean Value Function, Non-Homogeneous Poisson Process, Order Statistics, Probability Limits, Software Reliability, Statistical Process Control.

I. INTRODUCTION

Software reliability is one of the most important characteristics of software quality. Reliable software systems can be produced and maintained by employing quality measurement and management technologies during the software life cycle. Software Reliability is the probability of failure free operation of software in a specified environment during specified time [1].

The monitoring of Software reliability process is a far from simple activity. In recent years, several authors have recommended the use of SPC for software process monitoring. A few others have highlighted the potential pitfalls in its use [2].

The main thrust of the paper is to formalize and present an array of guidelines in a disciplined process with a view to helping the practitioner in putting SPC to correct use during software process monitoring.

Over the years, SPC has come to be widely used among others, in manufacturing industries for the purpose of controlling and improving processes. Our effort is to apply SPC techniques in the software development process so as to improve software reliability and quality [3]. It is reported that SPC can be successfully applied to

several processes for software development, including software reliability process. SPC is traditionally so well adopted in manufacturing industry. In general software development activities are more process centric than product centric which makes it difficult to apply SPC in a straight forward manner.

The utilization of SPC for software reliability has been the subject of study of several researchers. A few of these studies are based on reliability process improvement models. They turn the search light on SPC as a means of accomplishing high process maturities. Some of the studies furnish guidelines in the use of SPC by modifying general SPC principles to suit the special requirements of software development [3] (Burr and Owen[4]; Flora and Carleton[5]). It is especially noteworthy that Burr and Owen provide seminal guidelines by delineating the techniques currently in vogue for managing and controlling the reliability of software. Significantly, in doing so, their focus is on control charts as efficient and appropriate SPC tools.

It is accepted on all hands that Statistical process control acts as a powerful tool for bringing about improvement of quality as well as productivity of any manufacturing procedure and is particularly relevant to software development also. Viewed in this light, SPC is a method of process management through application of statistical analysis, which involves and includes the defining, measuring, controlling, and improving of the processes[6].

II. PROPOSED WORK

A. BURR Type III NHPP Model

NHPP software reliability growth models have been proposed to assess the reliability of software [13]. In these models, the number of software failures display the behavior of non-homogenous Poisson Process[20]. These models consider the debugging process as a counting process characterized by its mean value function. Software reliability can be estimated once the mean value function is determined. Model parameters are usually estimated using Maximum Likelihood method or Genetic Algorithms.

The various notations used in NHPP model are :

$\{N(t), t \geq 0\}$ represents the cumulative number of failures by time 't'.

$m(t)$ denotes the expected number of software failures by time 't'.

'a' represents the expected number of software failures eventually detected.

'b' denotes the failure detection rate.

$\lambda(t)$ corresponds to intensity function of software failures [9,13,23].

The Assumptions of NHPP Model are :

1. A Software system is subject to failures during execution caused by faults remaining in the system.
2. All faults are mutually independent from a failure detection point of view.
3. Failure rate of the software depends on the faults remaining in the system.
4. The number of faults detected at any time is proportional to the remaining number of faults in the software.

Since the expected number of errors remaining in the system at any time is finite, $m(t)$ is bounded, non-decreasing function of 't' with the boundary conditions

$$m(t) = \begin{cases} 0, & t = 0 \\ a, & t \rightarrow \infty \end{cases}$$

For $t \geq 0$ $N(t)$ is known to have a Poisson Probability mass function with parameters $m(t)$ i.e.,

$$P\{N(t) = n\} = \frac{[m(t)]^n e^{-m(t)}}{n!} \quad n = 0, 1, 2, \dots, \infty$$

The behaviour of software failure phenomena can be illustrated through $N(t)$ process. Several time domain models exist in the literature which specify that the mean value function $m(t)$ will be varied for each NHPP process.

In this paper we consider the mean value function of Burr Type III software reliability growth model as

$$m(t) = a[1 + t^{-c}]^{-b} \quad (1)$$

Here, we consider the performance given by the Burr Type III software reliability growth model based on order statistics and whose mean value function is given by

$$m(t) = \left(a(1 + (t_i)^{-c})^{-b} \right)^r \quad (2)$$

Where $[m(t)/a]$ is the cumulative distribution function of Ordered Burr distribution model

$$P\{N(t) = n\} = \frac{m(t)^n e^{-m(t)}}{n!}$$

$$\lim_{n \rightarrow \infty} P\{N(t) = n\} = \frac{a^n e^{-a}}{n!}$$

This is considered as Poisson model with mean a . Let S_k be the time between $(k-1)^{th}$ and k^{th} failure of the software product. It is assumed that X_k be the time up to the k^{th} failure. We need to find out the probability of the time between $(k-1)^{th}$ and k^{th} failures. The Software Reliability function is given by

$$R = \frac{S_k}{X_{(k-1)}} (S/X) = e^{-[m(x+s) - m(s)]} \quad (3)$$

B. Parameter Estimation Based on Inter Failure Times

The mean value function of Order Burr Type III is given by

$$m(t) = \left(a(1 + (t_i)^{-c})^{-b} \right)^r \quad (4)$$

The constants a , b and c in the mean value function are called parameters of the proposed model. To assess the software reliability, it is necessary to compute the expressions for finding the values of a , b and c . For doing this, Maximum Likelihood estimation is used whose Log Likelihood function is given by

$$LLF = \sum_{i=1}^n \text{Log}[\lambda(t_i)^r - m(t_n)^r] \quad (5)$$

Differentiating $m(t)$ with respect to 't' we get $\lambda(t)$

$$\lambda(t) = \frac{rabc}{(t_i)^{(c+1)} * [1 + (ti)^{-c}]^{(br+1)}} \quad (6)$$

The log likelihood equation to estimate the unknown parameters a, b, c after substituting (4) in (5) is given by

$$\text{LogL} = -[a(1+(t_n)^{-c})^{-b}]^r + \sum_{i=1}^n [\log r + \log a + \log b + \log c] + \sum_{i=1}^n [-(br+1) \log(1+(t_i)^{-c}) - (c+1) \log(t_i)] \quad (7)$$

Differentiating LogL with respect to 'a' and equating to 0 (i.e. $\frac{\partial \log L}{\partial a} = 0$) we get

$$a^r = \frac{n(1+(t_n)^{-c})^{br}}{r} \quad (8)$$

Differentiating LogL with respect to 'b' and equating to 0 (i.e. $\frac{\partial \log L}{\partial b} = 0$) we get

$$g(b) = \frac{n}{b} + \sum_{i=1}^n r \log(1+(t_i)^{-1}) + \frac{n^2(1+(t_n)^{-1})^{br}}{r} \log(1+(t_n)^{-1}) \quad (9)$$

Again Differentiating g(b) with respect to 'b' and equating to 0 (i.e. $\frac{\partial^2 \log L}{\partial b^2} = 0$)

$$g'(b) = \frac{-n}{b^2} + n^2(1+(t_n)^{-1})^{br} \cdot \log^2(1+(t_n)^{-1}) \quad (10)$$

Differentiating LogL with respect to 'c' and equating to 0 (i.e. $\frac{\partial \log L}{\partial c} = 0$) we get

$$g(c) = \frac{n}{c} + \sum_{i=1}^n \left(\frac{(r+1)(t_i) - c}{1+(t_i) - c} - 1 \right) \log t_i - \frac{n(t_n) - c \log t_n}{(1+(t_n)^{-c})} \quad (11)$$

Again Differentiating g(c) with respect to 'c' and equating to 0

(i.e. $\frac{\partial^2 \log L}{\partial c^2} = 0$) we get

$$g'(c) = \frac{-n}{c^2} + \sum_{i=1}^n \frac{(r+1)(\log t_i)^2 (t_i)^{-c}}{(1+(t_i)^{-c})^2} + \frac{n \log(t_n)^2 (t_n)^{-c}}{(1+(t_n)^{-c})^2} \quad (12)$$

The parameters 'b' and 'c' are estimated by iterative Newton-Raphson Method using

$$b_{n+1} = b_n - \frac{g(b_n)}{g'(b_n)} \quad (13)$$

$$c_{n+1} = c_n - \frac{g(c_n)}{g'(c_n)} \quad (14)$$

which are substituted in (7) to determine 'a'.

C. Order Statistics

Order Statistics can be used in several applications like data compression, survival analysis, Study of Reliability and many others [11]. Let X denote a continuous random variable with probability density function $f(x)$ and cumulative distribution function $F(x)$, and let $(X_1, X_2, \dots, X_n)$ denote a random sample of size n drawn on X . The original sample observations may be unordered with respect to magnitude. A transformation is required to produce a corresponding ordered sample. Let $(X(1), X(2), \dots, X(n))$ denote the ordered random sample such that $X(1) < X(2) < \dots < X(n)$; then $(X(1), X(2), \dots, X(n))$ are collectively known as the order statistics derived from the parent X . The various distributional characteristics can be known from Balakrishnan and Cohen [11].

The inter-failure time data represent the time lapse between every two consecutive failures. On the other hand if a reasonable waiting time for failures is not a serious problem, we can group the inter-failure time data into non overlapping successive sub groups of size 4 or 5 and add the failure times with in each sub group.

For instance if a data of 100 inter-failure times are available we can group them into 20 disjoint subgroups of size 5. The sum total in each subgroup would denote the time lapse between every 5th order statistic in a sample of size 5. In general for inter-failure data of size 'n', if r (any natural number) less than 'n' and preferably a factor n , we can conveniently divide the data into 'k' disjoint subgroups ($k=n/r$) and the cumulative total in each subgroup indicate the time between every r^{th} failure. The probability distribution of such a time lapse would be that of the ordered statistic in a subgroup of size r , which would be equal to power of the distribution function of the original variable ($m(t)$).

The whole process involves the mathematical model of the mean value function and knowledge about its parameters. If the parameters are known they can be taken as they are for the further analysis, if the parameters are not known they have to be estimated using a sample data by any admissible, efficient method of estimation. This is essential because the control limits depend on mean value function, which in turn depends on the parameters. If software failures are quite frequent, keeping track of inter-failure is tedious. If failures are more frequent order statistics are preferable [11].

D. Monitoring the time between failures using control chart

Software process monitoring is an essential activity that has to be performed during software process improvement. Monitoring involves measuring a quantifiable characteristic of software process over time and detecting out anomalies. A process must be characterized before it is monitored, for example by using upper and lower threshold values for process performance limits. When the observed performance falls outside these limits one can understand that there is something wrong in the process. Statistical process control is a time series analysis technique that has been effective in manufacturing and recently used in software contexts. It uses control charts as a tool to establish operational limits for acceptable process variation [12].

Control charts are an essential tool used for continuous quality control. Control charts monitor processes to show how the process is performing and how the process and capabilities are affected by changes to the process. This information is then used to make quality improvements. Control charts are also used to determine the capability of the process. These charts have data points that are either averages of subgroup measurements or individual measurements plotted on the x/y axis and joined by a line. Time is always on the x-axis. These charts

have an indicator of the process performances as average line, Upper Control Limit (UCL) ,Lower Control Limit (LCL).

Control charts are mainly classified as attribute charts and variable charts. Attribute Control Charts are used to monitor an organization's progress at removing defects that are inherently present in a process. Attribute charts are based on data that can be grouped and counted as present or not. Attribute charts are also called count charts and attribute data is also known as discrete data.Examples of attribute charts are p-charts, np-chart, c-chart ,u chart.

Variable charts are based on variable data that can be measured on a continuous scale .Variables Control Charts monitor process parameters or product features. A variable's measurement can indicate a significant change in process performance without producing a non-conformance. Variables Control Charts are more sensitive to change and are more efficient than Attribute Control Charts. Two primary statistics are measured and plotted on a Variables Control Chart: central tendency and process dispersion.Examples of variable charts are X-bar , R charts and multivariate charts. We have named the control chart as **Failures Control Chart** in this paper. The said control chart helps to assess the software failure phenomena on the basis of the given inter-failure time data[15].

E. Distribution of Time Between Failures

For a software system during normal operation, failures are random events caused by, for example, problem in design or analysis and in some cases insufficient testing of software. In this paper we applied Burr Type III to time between failures data. This distribution uses cumulative time between failure data for reliability monitoring.

The equation for mean value function of Burr Type III from equation [1] is

$$m(t) = a[1 + t^{-c}]^{-b}$$

Equate the pdf of above $m(t)$ to 0.99865, 0.00135, 0.5 and the respective control limits are given by.

$$T_u = [1 + t^{-c}]^{-b} = 0.99865$$

$$T_c = [1 + t^{-c}]^{-b} = 0.5$$

$$T_l = [1 + t^{-c}]^{-b} = 0.00135$$

These limits are converted to $m(t_u)$, $m(t_c)$ and $m(t_l)$ form. They are used to find whether the software process is in control or not by placing the points in control charts.

III. DATA ANALYSIS AND RESULTS

The procedure of a failures control chart for failure software process will be illustrated with an example here.

Table 1 shows the time between failures of a software product.

Table:1 Software failure data documented in Lyu(1996)



Failure No	Time Between Failures(hrs)	Failure No	Time Between Failures(hrs)	Failure No	Time Between Failures(hrs)
1	33	36	5	71	55
2	9	37	66	72	409
3	4	38	289	73	36
4	66	39	3	74	15
5	0.5	40	9	75	573
6	18	41	12	76	583
7	149	42	18	77	60
8	14	43	9	78	19
9	15	44	75	79	20
10	50	45	15	80	79
11	81	46	291	81	24
12	34	47	212	82	540
13	85	48	4	83	52
14	54	49	5	84	1596
15	3	50	308	85	314
16	15	51	269	86	1
17	6	52	276	87	763
18	8	53	1	88	10
19	130	54	400	89	20
20	19	55	294	90	144
21	19	56	227	91	28
22	112	57	118	92	56
23	15	58	13	93	476
24	16	59	47	94	65
25	154	60	89	95	98
26	50	61	242	96	884
27	10	62	99	97	212
28	2	63	607	98	287
29	22	64	83	99	53
30	53	65	2	100	3
31	19	66	26	101	831
32	58	67	586	102	43
33	20	68	708	103	55
34	3	69	6	104	109
35	92	70	4		

Table: 2 Successive Differences of 4th order mean value function (m(t))

Failure No.	4-Order Cumulative	m(t)	Successive Difference of m(t)	Failure No.	4-Order Cumulative	m(t)	Successive Difference of m(t)
1	112	7.0270383	0.0258214	14	4226.5	7.1165707	0.0013331
2	293.5	7.0528598	0.0122561	15	4493.5	7.1179038	0.0044536
3	473.5	7.0651159	0.0071614	16	5524.5	7.1223575	0.0045579
4	630.5	7.0722773	0.0056553	17	6846.5	7.1269154	0.0014080
5	793.5	7.0779326	0.0045077	18	7320.5	7.1283235	0.0031854
6	955.5	7.0824403	0.0048821	19	8527.5	7.1315090	0.0004285

7	1171.5	7.0873225	0.0028910	20	8705.5	7.1319375	0.0046554
8	1323.5	7.090213	0.0020425	21	10917.5	7.1365930	0.0019309
9	1443.5	7.0922560	0.0052756	22	12005.5	7.1385239	0.0004138
10	1810.5	7.0975317	0.0014083	23	12253.5	7.1389378	0.0023596
11	1924.5	7.098940	0.0054792	24	13776.5	7.1412975	0.0007910
12	2446.5	7.1044193	0.0067383	25	14331.5	7.1420885	0.0013949
13	3304.5	7.1111576	0.0054130	26	15369.5	7.1434835	

Table: 3 Successive Differences of 5th order mean value function(m(t))

Failure No.	5-Order Cumulative	m(t)	Successive Difference of m(t)	Failure No.	5-Order Cumulative	m(t)	Successive Difference of m(t)
1	112.5	4.442267	0.020125	11	3999.5	4.499509	0.001635
2	358.5	4.462393	0.008869	12	4493.5	4.501144	0.00287
3	615.5	4.471262	0.004056	13	5526.5	4.504014	0.002944
4	793.5	4.475318	0.005245	14	6856.5	4.506958	0.001984
5	1109.5	4.480563	0.001793	15	7944.5	4.508942	0.001221
6	1246.5	4.482356	0.002186	16	8705.5	4.510163	0.003356
7	1438.5	4.484542	0.003463	17	11231.5	4.513519	0.001043
8	1810.5	4.488005	0.001025	18	12169.5	4.514563	0.000747
9	1939.5	4.489031	0.005174	19	12892.5	4.515309	0.00136
10	2759.5	4.494205	0.005304	20	14331.5	4.51667	

Table : 4 4th and 5th order Parameter Estimates and control limits

Order	a	b	c	m(t _a)	m(t _c)	m(t _l)
4	7.371836	0.099992	0.50032	7.361884	0.009952	3.685918
5	4.655946	0.099988	0.108240	4.64966	0.006286	2.327973

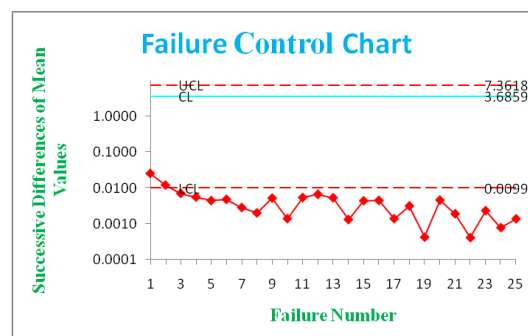


Figure 1 : Failure Control Chart of Table 2

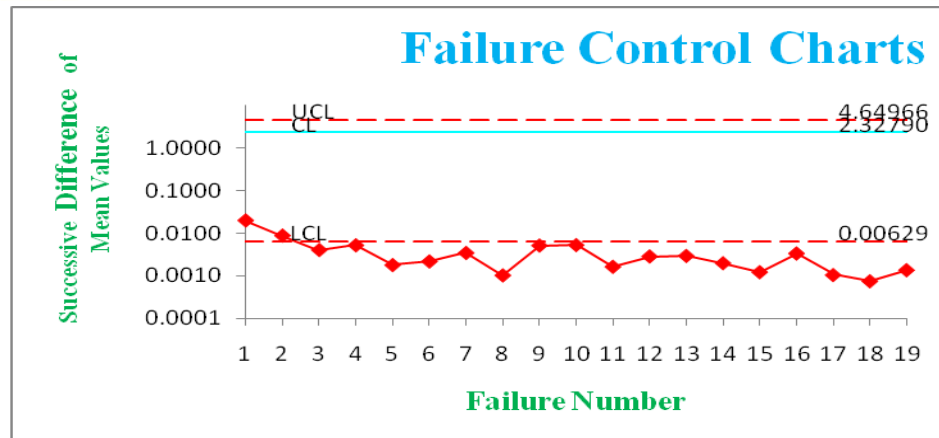


Figure 2 :Failure Control Chart of Table 3

IV. CONCLUSION

The 25 of 4th-order, 19 of 5th-order samples successive differences were plotted through the estimated mean value function against the failure number. The parameter estimation is carried out by Newton Raphson Iterative method for Burr model. The graphs have shown out of control signals i.e. below the LCL. Hence we conclude that our method of estimation and the control chart are giving a +ve recommendation for their use in finding out preferable control process or desirable out of control signal. By observing the Mean value Control chart we identified that the failure situation is detected at 3rd to 25th points of table-2 for the corresponding $m(t)$ in 4th-order statistics and at 3rd to 19th point of table-3 for the corresponding $m(t)$ in 5th-order statistics, which is below $m(t_L)$. It indicates that the failure process is detected at an early stage. The early detection of software failure will improve the software Reliability. When the time between failures is less than LCL, it is likely that there are assignable causes leading to significant process deterioration and it should be investigated. On the other hand, when the time between failures has exceeded the UCL, there are probably reasons that have lead to significant improvement.

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