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Normal Shock Wave Diffraction for Carbon Dioxide (CO_2) Gas (Large Bends)

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ABSTRACT

Lighthill has considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4$, γ being the ratio of specific heats. Srivastava extended the work of Lighthill to monoatomic gases for which $\gamma = \frac{5}{3}$ both for small and large bends. In the present paper, pressure distribution over the diffracted for carbon dioxide gas ($\gamma = 1.29$) are presented for larger bends.

Keywords: Diffraction, Large bends, Pressure distribution, Carbon dioxide gas (CO_2), singular perturbation

I. INTRODUCTION

Lighthill (1949) considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4$, γ being the ratio of specific heats. Following Lighthill (1949), Sakurai etal (2002) have obtained pressure distribution over the diffracted shock wave. The work of Lighthill (1949) was extended to larger bends by Sakurai and Takayam (2005). Srivastava (2016) extended the work of Sakurai and Takayama (2005) for obtaining the pressure distribution over the diffracted shock wave. Srivastava (1963), (2016) extended the work of Lighthill (1949) for $\gamma = \frac{5}{3}$ and Srivastava (2017) carried forward the work of Sakurai and Takayama (2005) for $\gamma = \frac{5}{3}$.

In the present paper, the theory of Sakurai and Takayama (2005) has been used for carbon dioxide gas (CO_2) to obtain the pressure distribution over the diffracted shock wave. The Mach number of the shock wave is M=1.36 and $\gamma = 1.29$. The present results are applicable for larger bends.

We have also obtained results for carbon dioxide gas for lower bends (2017). Earlier Srivastava (2013) has given results for vorticity distribution over the diffracted shock wave both from Lighthill (1949) and Sakurai and Takayama (2005) theory. The book of Srivastava (1994) may be used for reference.

II. MATHEMATICAL FORMULATION

Let the velocity, pressure, density, sound speed behind the shock wave before it has crossed the bend be $q_1,\ \rho_1,\ p_1,\ a_1$ and ahead of the shock wave be $0,\ p_0,\ \rho_0,\ a_0$. Then applying the principle of conservation of mass, momentum and energy for general value of γ (γ being the ratio of specific heats)

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$$q_1 = \frac{2U}{(\gamma+1)}\left(1-\frac{a_0^2}{U^2}\right) \qquad \text{- (1)}$$

$$\rho_1 = \frac{\rho_0(\gamma+1)}{(\gamma-1)+2\frac{a_0^2}{U^2}} \qquad \text{- (2)}$$

$$p_1 = \frac{\rho_0}{(\gamma-1)}\left[2U^2 - \frac{a_0^2(\gamma-1)}{\gamma}\right] \qquad \text{- (3)}$$

U being the velocity of shock wave, Mach number of the shock $M = \frac{U}{a_0}$, $a_0 = \sqrt{\frac{\gamma\, p_0}{\rho_0}}$

For $\gamma = 1.29$ (carbon dioxide) and M = 1.36, the values of $q_1,\ \rho_1,\ p_1$ could be obtained from the above equations.

The wedge is made up of two walls having a small angle δ between them. After diffraction, the flow is two dimensional behind the shock wave. Let $\vec{q}_2,\ p_2,\ \rho_2$ and S_2 be the velocity vector, pressure, density and entropy at any point. We take the origin and Y axis lying on the leading edge of the wedge and X axis on the original wall produced.

Then the equations of conservation of mass, momentum and energy can be written as

$$\frac{D\rho_2}{Dt} + \rho_2 \operatorname{div} \vec{q}_2 = 0 \qquad \text{- (4)}$$

$$\frac{D\vec{q}_2}{Dt} + \frac{1}{\rho_2}\nabla \mathrm{p}_2 = 0 \qquad \text{- (5)}$$

$$\frac{DS_2}{Dt} = 0 \qquad \text{- (6)}$$

Now we introduce the following transformations

$$\frac{X - a_1 t}{a_1 t} = x \qquad \text{- (7)}$$

$$\frac{Y}{a_1 t} = y \qquad \text{- (8)}$$

$$\frac{\vec{q}_2}{q_1} = (1+u, v) \qquad \text{- (9)}$$

$$\frac{p_2 - p_1}{a_1\, q_1\, p_1} = p \qquad \text{- (10)}$$

Using small perturbation theory, the equations (4), (5), (6) and (7), (8), (9) and 10 yield an single second order partial equation in *p*. The equation is

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) \qquad \text{- (11)}$$

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The characteristics of the differential equation (11) are tangents to the unit circle $x^2 + y^2 = 1$. The disturbed region behind the diffracted shock is therefore enclosed by the arc of the unit circle, the diffracted shock and wedge surface.

For such a problem,Lighthill has derived a function which satisfies all the boundary conditions

$$w(z_1) = \frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{C\delta\left[D(z_1 - x_0) - 1\right]}{\left(z_1^2 - 1\right)^{1/2}\left[\alpha - i\left(z_1 - 1\right)^{1/2}\right]\left[\beta - i\left(z_1 - 1\right)^{1/2}\right]\left(z_1 - x_0\right)} \quad - (12)$$

In (12), $z_1 = x_1 + iy_1$

In the final z_1-plane, the imaginary part on the left hand side of (12) gives the pressure derivative which determines the pressure distribution over the diffracted shock. If one carries out this exercise, then we have

$$\frac{\partial p}{\partial x_1} = \frac{C\delta}{\left(x_1^2 - 1\right)^{1/2}}\left[D - \frac{1}{(x_1 - x_0)}\right]\frac{(\alpha + \beta)(x_1 - 1)^{1/2}}{\left[\alpha^2 + (x_1 - 1)\right]\left[\beta^2 + (x_1 - 1)\right]} \quad - (13)$$

In (13), all the quantities are functions of the Mach number of the shock wave M except x_1 which runs from 1 to ∞ on the diffracted shock in the transformed plane and is connected to *y* in the physical plane through the relation

$$\frac{y}{k'} = \left(\frac{x_1 - 1}{x_1 + 1}\right)^{1/2}, k' = \sqrt{1 - k^2} \quad - (14)$$

When $x_1 = 1, \frac{y}{k'} = 0$ (wall surface), when $x_1 \to \infty$, $\frac{y}{k'} = 1$ (point of intersection of shock and unit circle).

The theory of Lighthill (1949) was extended by Sakurai and Takayama (2005) to higher δ by considering second order terms through singular perturbation techniques. Sakurai and Takayama (2005) assumed y on the diffracted shock and computed $\bar{y}$ and $\bar{x}_1$. The relationship between $\bar{y}$ and $\bar{x}_1$ is the same as given by (14) in which y is replaced by $\bar{y}$ and x_1 by $\bar{x}_1$. The new $\bar{y}$ and $\bar{x}_1$ are used to calculate pressure distribution from equation (13). The modified results of Sakurai and Takayama (2005) required for calculations are given below.

$$\bar{y} = \sqrt{r^2 - k^2}\,,\ r = \xi + \delta r_1, \xi = \sqrt{\gamma^2 + k^2}$$

$$r_1 = K(\phi)\bar{R}\log\bar{R}\,,\ \bar{R} = \left[\rho^2 + \frac{2\bar{X}k\rho}{\xi} + X_0^2\right]^{1/2}$$

$$\rho = \frac{1 - \sqrt{1 - \xi^2}}{\xi}\,,\ \phi = \tan^{-1}\frac{y}{M_1 + k} \quad - (15)$$

$$K(\phi) = \frac{1}{\pi}.\frac{M_1^4}{1 - \sqrt{1 - M_1^2}}\frac{1}{\cos\phi\cos 2\phi}\left[1 + \frac{\gamma + 1}{2}\frac{M_1^2}{1 - M_1^2}\cos 2\phi\right]$$

where, ϕ in $K(\phi)$ is a variable

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$$X_0 = \frac{1-\sqrt{1-M_1^2}}{M_1} \qquad \text{-(16)}$$

y is the y coordinate on the diffracted shock and ξ is the strained variable and others are connected within themselves.

III. NUMERICAL SOLUTION

The pressure distribution over the diffracted shock is obtained by integrating equation (13). The pressure is zero at $x_1 = \infty$ i.e. at $\frac{y}{k'} = 1$ (the point of intersection of shock wave and unit circle) and so pressure at other points could be known by integration of equation (13) between the intervals.

The points chosen over the diffracted shock are

$$y/_{k'} = 0,\; y/_{k'} = 0.2,\; y/_{k'} = 0.4,\; y/_{k'} = 0.6,\; y/_{k'} = 0.8$$

The equations (18) and (19) have been used to get the results. The following table gives the results after integration. The table is for $y/_{k'}$ versus $-\frac{p}{k\delta}$ M is 1.36 and $\gamma = 1.29.$

Table-1

$y/_{k'}$	0	0.2	0.4	0.6	0.8	1
$-\frac{p}{k\delta}$	4.17	4.06	3.32	3.07	2.10	0

The table shows that $-\frac{p}{k\delta}$ is maximum at $y/_{k'} = 0$ i.e. at the point of intersection of the wall and shock. The value of $-\frac{p}{k\delta}$ falls from there and attain the value zero at $y/_{k'} = 1$ i.e. at the point of intersection of shock and unit circle.

IV. CONCLUSION

Shock diffraction problems are of immense importance in aeronautical engineering. Determination of pressure variation over diffracted shock will be useful for design purposes in aeronautics.

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