

Almost δ sp-Continuous Function

Jyoti

Department of Mathematics, R.C.C.V.Girls (P.G) College, Ghaziabad

ABSTRACT

A new class of functions is introduced in this paper. This class is called almost δ sp-continuity. This type of functions is seen to be strictly weaker than almost precontinuity. By using δ sp-open sets, many characterizations and properties of the said type of functions are investigated.

I. INTRODUCTION

The notion of δ sp-open set was introduced by E.Hatir and T. Noiri,[7] in 2006. We introduce here a new type of functions strictly weaker than almost precontinuity. We call this the almost δ sp-continuous functions. We investigate properties of such functions.

Throughout the present paper, spaces mean topological spaces and $f : (X, \tau) \rightarrow (Y, \sigma)$ (or simply $f : X \rightarrow Y$) denotes a function f of a space (X, τ) into a space (Y, σ) . Let A be a subset of a space X . The closure and the interior of A are denoted by $\text{cl}(A)$ and $\text{int}(A)$, respectively.

II. PRELIMINARIES

A subset A of a space X is said to be **regular open**[23] if $A = \text{int}(\text{cl}(A))$ and **δ -open**[24] if for each $x \in A$, there exists a regular open set W such that $x \in W \subset A$.

A subset A of a space X is said to be **α -open**[13] (resp. **semi-open**[8], **preopen** [10], **γ -open**[6] or **β -open**[1] or **semi-preopen**[2]) if $A \subset \text{int}(\text{cl}(\text{int}(A)))$ (resp. $A \subset \text{cl}(\text{int}(A))$, $A \subset \text{cl}(\text{int}(\text{cl}(A)))$).

The complement of a regular open set is said to be regular closed[23]. The complement of a semi-open set is said to be semi-closed[5]. The intersection of all

Semi-closed sets containing a subset A of X is called the semi-closure[5] of A and is denoted by $s\text{-cl}(A)$. The union of all semi-open sets contained in a subset A of X is called the semi - interior of A and is denoted by $s\text{-int}(A)$. A point $x \in X$ is called a **δ -cluster** (resp. **θ -cluster**) point of A [24] if $A \cap \text{int}(\text{cl}(U)) \neq \emptyset$ (resp. $A \cap \text{cl}(U) \neq \emptyset$) for each open set U containing x . The set of all δ -cluster (resp. θ -cluster) points of A is called the δ -closure (resp. θ -closure) of A and is denoted by $\delta\text{-cl}(A)$ (resp. $\theta\text{-cl}(A)$). If $\delta\text{-cl}(A) = A$ (resp. $\theta\text{-cl}(A) = A$), then A is said to be δ -closed (resp. θ -closed). The complement of a δ -closed (resp. θ -closed) set is said to be δ -open (resp. θ -open).

A subset S of a topological space X is said to be δ sp-open [7] iff $A \subseteq \text{cl}(\text{int}(\delta\text{-cl}(A)))$. The complement of a δ sp-open set is called a δ sp-closed set [7]. The union (resp. intersection) of all δ sp-open (resp. δ sp-closed) sets, each contained in (resp. containing) a set A in a topological space X is called the δ sp-interior (resp. δ sp-closure) of A and it is denoted by $\delta\text{sp-int}(A)$ (resp. $\delta\text{sp-cl}(A)$) [7].

The family of all δ sp-open (resp. regular open, preopen, β -open, α -open, semi-open, δ -open) sets of a space X will be denoted by δ SPO(X) (resp. RO(X), PO(X), β O(X), α O(X), SO(X), δ O(X)). The family of all δ sp-closed (resp. regular closed, δ -closed) sets in a space X is denoted by δ SPC(X) (resp. RC(X), δ C(X)). The family of all δ sp-open (resp. regular open, δ -open) sets containing a point $x \in X$ will be denoted by δ SPO(X, x) (resp. RO(X, x), δ O(X, x)).

Lemma 1. Let A be a subset of a space X . Then

- (1) δ sp-cl($X-A$) = $X - \delta$ sp-int(A),
- (2) $x \in \delta$ sp-cl(A) if and only if $A \cap U \neq \emptyset$ for each $U \in \delta$ SPO(X, x),
- (3) A is δ sp-closed in X if and only if $A = \delta$ sp-cl(A),
- (4) δ sp-cl(A) is δ -pclosed in X .

Lemma 2 (Noiri [17], [18]). For a subset of a space Y , the following hold:

- (1) α -cl(V) = cl(V) for every $V \in \beta$ O(Y).
- (2) p -cl(V) = cl(V) for every $V \in$ SO(Y).
- (3) s -cl(V) = int(cl(V)) for every preopen set V of a space X .

Definition 3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **almost δ sp-continuous** if for each $x \in X$ and each $V \in$ RO(Y) containing $f(x)$, there exists $U \in \delta$ SPO(X) containing x such that $f(U) \subset V$.

Definition 4. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

1. **R-map** [4] if $f^{-1}(V) \in$ RO(X) for every $V \in$ RO(Y).
2. **almost continuous** [21], $f^{-1}(V) \in \tau$, for every $V \in$ RO(Y).
3. **almost α -continuous** [16] $f^{-1}(V) \in \alpha$ O(X), for every $V \in$ RO(Y).
4. **almost precontinuous** [11], $f^{-1}(V) \in$ PO(X) for every $V \in$ RO(Y).
5. **δ -continuous** [14] if $f^{-1}(V) \in \delta$ O(X) for every $V \in$ RO(Y).

Remark 5. The following implications hold:

al. contin. $\Rightarrow$ al. α -contin. $\Rightarrow$ al. precontin. $\Rightarrow$ al. δ sp-contin.

The converses are not true in general.

Example 6. Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow X$ be a function defined by $f(a) = b, f(b) = a, f(c) = c$. Then, f is almost δ sp-continuous but not almost precontinuous.

Theorem 7. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:

- (1) f is almost δ sp-continuous;
- (2) for each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $U \in \delta$ SPO(X)

containing x such that $f(U) \subset \text{int}(\text{cl}(V))$;

(3) $f^{-1}(F) \in \delta\text{SPC}(X)$ for every $F \in \text{RC}(Y)$;

(4) $f^{-1}(V) \in \delta\text{SPO}(X)$ for every $V \in \text{RO}(Y)$.

(5) $f(\delta\text{sp-cl}(A)) \subset \delta\text{-cl}(f(A))$ for every subset A of X ;

(6) $\delta\text{sp-cl}(f^{-1}(B)) \subset f^{-1}(\delta\text{-cl}(B))$ for every subset B of Y ;

(7) $f^{-1}(F) \in \delta\text{SPC}(X)$ for every δ - closed set F of (Y, σ) ;

(8) $f^{-1}(V) \in \delta\text{SPO}(X)$ for every δ - open set V of (Y, σ) ;

(9) $\delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B)))) \subset f^{-1}(\text{cl}(B))$ for every subset B of Y ;

(10) $\delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(F)))) \subset f^{-1}(F)$ for every closed set F of Y ;

(11) $\delta\text{sp-cl}(f^{-1}(\text{cl}(V))) \subset f^{-1}(\text{cl}(V))$ for every open set V of Y ;

(12) $f^{-1}(V) \subset \delta\text{sp-int}(f^{-1}(s\text{-cl}(V)))$ for every open set V of Y ;

(13) $f^{-1}(V) \subset \text{int}(\delta\text{-cl}(f^{-1}(s\text{-cl}(V))))$ for every open set V of Y ;

(14) $f^{-1}(V) \subset \delta\text{sp-int}(f^{-1}(\text{int}(\text{cl}(V))))$ for every open set V of Y ;

(15) $f^{-1}(V) \subset \text{int}(\delta\text{-cl}(f^{-1}(\text{int}(\text{cl}(V))))$ for every open set V of Y ;

(16) $\delta\text{sp-cl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$ for each $V \in \beta\text{O}(Y)$;

(17) $\delta\text{sp-cl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$ for each $V \in \text{SO}(Y)$;

(18) $f^{-1}(V) \subset \delta\text{sp-pint}(f^{-1}(\text{int}(\text{cl}(V))))$ for each $V \in \text{PO}(Y)$;

(19) $\delta\text{sp-cl}(f^{-1}(V)) \subset f^{-1}(\alpha\text{-cl}(V))$ for each $V \in \beta\text{O}(Y)$;

(20) $\delta\text{sp-cl}(f^{-1}(V)) \subset f^{-1}(p\text{-cl}(V))$ for each $V \in \text{SO}(Y)$;

(21) $f^{-1}(V) \subset \delta\text{sp-int}(f^{-1}(s\text{-cl}(V)))$ for each $V \in \text{PO}(Y)$.

Proof. (1) $\Rightarrow$ (2). Let $x \in X$ and $V \in \sigma$ containing $f(x)$. We have $\text{int}(\text{cl}(V)) \in \text{RO}(Y)$. Since f is almost δsp -continuous, then there exists $U \in \delta\text{SPO}(X, x)$ such

that $f(U) \subset \text{int}(\text{cl}(V))$.

(2) $\Rightarrow$ (1). Obvious.

(3) $\Leftrightarrow$ (4). Obvious.

(1) $\Rightarrow$ (4). Let $x \in X$ and $V \in \text{RO}(Y, f(x))$. Since f is almost δsp -continuous, then there exists $U_x \in \delta\text{SPO}(X, x)$ such that $f(U_x) \subset V$. We have $U_x \subset f^{-1}(V)$. Thus, $f^{-1}(V) = \bigcup U_x \in \delta\text{SPO}(X)$.

(4) $\Rightarrow$ (1). Obvious.

(1) $\Rightarrow$ (5). Let A be a subset of X . Since $\delta\text{-cl}(f(A))$ is δ -closed in Y , it is denoted by $\bigcap \{F_i : F_i \in \text{RC}(Y), i \in I\}$, where I is an index set. By (1) $\Leftrightarrow$ (3), we have $A \subset f^{-1}(\delta\text{-cl}(f(A))) = \bigcap \{f^{-1}(F_i) : i \in I\} \in \delta\text{SPC}(X)$ and hence $\delta\text{sp-cl}(A) \subset f^{-1}(\delta\text{-cl}(f(A)))$. Therefore, we obtain $f(\delta\text{sp-cl}(A)) \subset \delta\text{-cl}(f(A))$.

(5) $\Rightarrow$ (6). Let B be a subset of Y . We have $f(\delta\text{sp-cl}(f^{-1}(B))) \subset \delta\text{-cl}(f(f^{-1}(B))) \subset \delta\text{-cl}(B)$ and hence $\delta\text{sp-cl}(f^{-1}(B)) \subset f^{-1}(\delta\text{-cl}(B))$.

(6) $\Rightarrow$ (7). Let F be any δ - closed set of (Y, σ) . We have $\delta\text{sp-cl}(f^{-1}(F)) \subset$



$f^{-1}(\delta - \text{cl}(F)) = f^{-1}(F)$ and hence $f^{-1}(F)$ is δ sp-closed in (X, τ) .

(7) $\Rightarrow$ (8). Let V be any δ - open set of (Y, σ) . We have $f^{-1}(Y - V) = X - f^{-1}(V) \in \delta\text{SPC}(X)$ and hence $f^{-1}(V) \in \delta\text{SPO}(X)$.

(8) $\Rightarrow$ (1). Let V be any regular open set of (Y, σ) . Since V is δ - open in (Y, σ) , $f^{-1}(V) \in \delta\text{SPO}(X)$ and hence, by (1), (4), f is almost δ sp-continuous.

(1) $\Rightarrow$ (9). Let B be any subset of Y . Assume that $x \in X - f^{-1}(\text{cl}(B))$. Then

$f(x) \in Y - \text{cl}(B)$ and there exists an open set V containing $f(x)$ such that $V - B = \phi$ hence $\text{int}(\text{cl}(V)) \cap \text{cl}(\text{int}(\text{cl}(B))) = \phi$. Since f is almost δ ps-continuous,

there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset \text{int}(\text{cl}(V))$. Therefore, we have

$U \cap f^{-1}(\text{cl}(\text{int}(\text{cl}(B)))) = \phi$ and hence $x \in X - \delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B)))))$.

Thus we obtain $\delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B))))) \subset f^{-1}(\text{cl}(B))$.

(9) $\Rightarrow$ (10). Let F be any closed set of Y . Then we have

$\delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(F)))) = \delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(F))))) \subset f^{-1}(\text{cl}(F)) = f^{-1}(F)$.

(10) $\Rightarrow$ (11). For any open set V of Y , $\text{cl}(V)$ is regular closed in Y and we have

$\delta\text{sp-cl}(f^{-1}(\text{cl}(V))) = \delta\text{sp-cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(V))))) \subset f^{-1}(\text{cl}(V))$.

(11) $\Rightarrow$ (12). Let V be any open set of Y . Then $Y - \text{cl}(V)$ is open in Y and by using

Lemma 2 we have $X - \delta\text{sp-int}(f^{-1}(s - \text{cl}(V))) =$

$\delta\text{sp-cl}(f^{-1}(Y - \text{int}(\text{cl}(V)))) \subset f^{-1}(\text{cl}(Y - \text{cl}(V))) \subset X - f^{-1}(V)$.

Therefore, we obtain $f^{-1}(V) \subset \delta\text{sp-int}(f^{-1}(s - \text{cl}(V)))$.

(12) $\Rightarrow$ (13). Let V be any open set of Y . We obtain

$f^{-1}(V) \subset \delta\text{sp-int}(f^{-1}(s - \text{cl}(V))) \subset \text{int}(\delta - \text{cl}(f^{-1}(s - \text{cl}(V))))$.

(13) $\Rightarrow$ (1). Let x be any point of X and V any open set of Y containing

$f(x)$. Then $x \in f^{-1}(\text{int}(\text{cl}(V))) \subset \text{int}(\delta - \text{cl}(f^{-1}(s - \text{cl}(\text{int}(\text{cl}(V))))) = \text{int}(\delta -$

$\text{cl}(f^{-1}(\text{int}(\text{cl}(V)))))$. Thus, $f^{-1}(\text{int}(\text{cl}(V))) \in \delta\text{SPO}(X)$. Take $U = f^{-1}(\text{int}(\text{cl}(V)))$.

We obtain $x \in U$ and $f(U) \subset \text{int}(\text{cl}(V))$. Therefore, f is almost δ sp-continuous.

(12) $\Rightarrow$ (14) and (13), (15). Obvious.

(1) $\Rightarrow$ (16). Let V be any β - open set of Y . It follows from [2, **Theorem 2.4**]

that $\text{cl}(V)$ is regular closed in Y . Since f is almost δ sp-continuous, by (1), (3),

$f^{-1}(\text{cl}(V))$ is δ sp-closed in X . Therefore, we obtain $\delta\text{sp-cl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$.

(16) $\Rightarrow$ (17). This is obvious since $\text{SO}(Y) \subset \beta\text{O}(Y)$.

(17) $\Rightarrow$ (1). Let F be any regular closed set of Y . Then F is semi - open in Y

and hence $\delta\text{sp-cl}(f^{-1}(F)) \subset f^{-1}(\text{cl}(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is

δ sp-closed. Therefore, by (1), (3), f is almost δ sp-continuous.

(1) $\Rightarrow$ (18). Let V be any semi-preopen set of Y . Then $V \subset \text{int}(\text{cl}(V))$ and $\text{int}(\text{cl}(V))$ is regular open in Y . Since f is almost δ sp-continuous, by (1), (4), $f^{-1}(\text{int}(\text{cl}(V)))$

is δ sp-open in X and hence we obtain that $f^{-1}(V) \subset f^{-1}(\text{int}(\text{cl}(V))) \subset \delta\text{sp-int}(f^{-1}(\text{int}(\text{cl}(V))))$.

(18) $\Rightarrow$ (1). Let V be any regular open set of Y . Then V is semi -preopen and $f^{-1}(V) \subset \delta\text{sp-int}(f^{-1}(\text{int}(\text{cl}(V)))) = \delta\text{sp-int}(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is δ sp-preopen in X and hence, by (1), (4), f is almost δ sp-continuous.

(16) $\Rightarrow$ (19) $\Rightarrow$ (17) $\Rightarrow$ (20) $\Rightarrow$ (18) $\vee$, (21). Obvious.

Lemma 8. A set S in X is δ sp-open if and only if $S \cap G \in \delta\text{SPO}(X)$ for every δ - open set G of X .

Definition 9. The δ sp-frontier of a subset A of X , denoted by $\delta\text{sp-fr}(A)$, is defined by $\delta\text{sp-fr}(A) = \delta\text{sp-cl}(A) \cap \delta\text{sp-cl}(X - A) = \delta\text{sp-cl}(A) - \delta\text{sp-int}(A)$.

Theorem 10. The set of all points x of X at which a function $f : X \rightarrow Y$ is not almost δ sp-continuous is identical with the union of the δ sp-frontiers of the inverse images of regular open sets containing $f(x)$.

Proof. Let x be a point of X at which f is not almost δ sp-continuous. Then, there exists a regular open set V of Y containing $f(x)$ such that $U \cap (X - f^{-1}(V)) \neq \emptyset$ for every $U \in \delta\text{SPO}(X, x)$. Therefore, we have $x \in \delta\text{sp-cl}(X - f^{-1}(V)) = X - \delta\text{sp-int}(f^{-1}(V))$ and $x \in f^{-1}(V)$. Thus, we obtain $x \in \delta\text{sp-fr}(f^{-1}(V))$.

Conversely, suppose that f is almost δ sp-continuous at $x \in X$ and let V be a regular open set containing $f(x)$. Then there exists $U \in \delta\text{SPO}(X, x)$ such that $U \subset f^{-1}(V)$; hence $x \in \delta\text{sp-int}(f^{-1}(V))$. Therefore, it follows that $x \in X - \delta\text{sp-fr}(f^{-1}(V))$. This completes the proof.

Definition 11. A function $f : X \rightarrow Y$ is said to be weakly δ sp-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset \text{cl}(V)$.

Definition 12. Let (X, τ) be a topological space. The collection of all regular open sets forms a base for a topology τ_s . It is called the **semi regularization**. In case when $\tau = \tau_s$, the space (X, τ) is called semi-regular [23].

Theorem 13. Let (X, τ) be a semi-regular space. Then a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost sp-continuous if and only if it is almost δ sp-continuous.

Definition 14. A function $f : X \rightarrow Y$ is said to be **δ - almost continuous** [20] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \delta\text{PO}(X, x)$ such that $f(U) \subset V$.

Definition 15. A function $f : X \rightarrow Y$ is said to be **δ sp-irresolute** if for each $x \in X$ and each δ sp-open set V of Y containing $f(x)$, there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset V$.

Definition 16. A function $f : X \rightarrow Y$ is said to be **almost δ sp-open** if $f(U) \subset \text{int}(\text{cl}(f(U)))$ for every δ sp-open set U of X .

Theorem 17. If $f : X \rightarrow Y$ is an almost δ sp-open and weakly δ sp-continuous function, then f is almost δ sp-continuous.

Proof. Let $x \in X$ and let V be an open set of Y containing $f(x)$. Since f is weakly δ sp-continuous, there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset \text{cl}(V)$. Since f is almost δ sp-open, $f(U) \subset \text{int}(\text{cl}(f(U))) \subset \text{int}(\text{cl}(V))$ and hence f is almost δ sp-continuous.

Definition 18. A space X is said to be (1) **almost regular** [22] if for any regular closed set F of X and any point $x \in X - F$ there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$, (2) **semi - regular** if for any open set U of X and each point $x \in U$ there exists a regular open set V of X such that $x \in V \subset U$.

Theorem 19. If $f : X \rightarrow Y$ is a weakly δ sp-continuous function and Y is almost regular, then f is almost δ sp-continuous.

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the almost regularity of Y , there exists a regular open set G of Y such that $f(x) \in G \subset \text{cl}(G) \subset \text{int}(\text{cl}(V))$ [22, Theorem 2.2]. Since f is weakly δ sp-continuous, there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset \text{cl}(G) \subset \text{int}(\text{cl}(V))$. Therefore, f is almost δ sp-continuous.

Theorem 20. If $f : X \rightarrow Y$ is an almost δ sp-continuous function and Y is semi regular, then f is δ - almost continuous.

Proof. Let $x \in X$ and let V be an open set of Y containing $f(x)$. By the semi regularity of Y , there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost δ sp-continuous, there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset \text{int}(\text{cl}(G)) = G \subset V$ and hence f is δ - almost continuous.

Theorem 21. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions. Then the following hold:

- (1) If f is almost δ sp-continuous and g is an R - map, then the composition $go f : X \rightarrow Z$ is almost δ sp-continuous,
- (2) If f is δ sp-irresolute and g is almost δ sp-continuous, the composition $go f : X \rightarrow Z$ is almost δ sp-continuous.

Definition 22. A function $f : X \rightarrow Y$ is said to be **faintly δ sp-continuous** if for

each $x \in X$ and each θ - open set V of Y containing $f(x)$, there exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset V$.

Theorem 23. Let $f : X \rightarrow Y$ be a function. Suppose that Y is regular. Then, the following properties are equivalent:

- (1) f is δ - almost continuous,
- (2) $f^{-1}(\delta\text{-cl}(B))$ is δsp -closed in X for every subset B of Y ,
- (3) f is almost δsp -continuous,
- (4) f is weakly δsp -continuous,
- (5) f is faintly δsp -continuous.

Proof. (1) $\Rightarrow$ (2). Since $\delta\text{-cl}(B)$ is closed in Y for every subset B of Y , $f^{-1}(\delta\text{-cl}(B))$ is δsp -closed in X .

(2) $\Rightarrow$ (3). For any subset B of Y , $f^{-1}(\delta\text{-cl}(B))$ is δsp -closed in X and hence we have $\delta\text{sp-cl}(f^{-1}(B)) \subset \delta\text{sp-cl}(f^{-1}(\delta\text{-cl}(B))) = f^{-1}(\delta\text{-cl}(B))$. It follows that

f is almost δsp -continuous

(3) $\Rightarrow$ (4). This is obvious.

(4) $\Rightarrow$ (5). Let A be any subset of X . Let $x \in \delta\text{sp-cl}(A)$ and V be any open set of Y containing $f(x)$. There exists $U \in \delta\text{SPO}(X, x)$ such that $f(U) \subset \text{cl}(V)$. Since $x \in \delta\text{sp-cl}(A)$, we have $U \cap A \neq \emptyset$ and hence $\emptyset \neq f(U) \cap f(A) \subset \text{cl}(V) \cap f(A)$.

Therefore, we have $f(x) \in \theta\text{-cl}(f(A))$ and hence $f(\delta\text{sp-cl}(A)) \subset \theta\text{-cl}(f(A))$.

Let B be any subset of Y . We have $f(\delta\text{sp-cl}(f^{-1}(B))) \subset \theta\text{-cl}(B)$ and

$\delta\text{sp-cl}(f^{-1}(B)) \subset f^{-1}(\theta\text{-cl}(B))$. Let F be any θ - closed set of Y . It follows that $\delta\text{sp-cl}(f^{-1}(F)) \subset f^{-1}(\theta\text{-cl}(F)) = f^{-1}(F)$. Therefore $f^{-1}(F)$ is δsp -closed in X and hence f is faintly δsp -continuous.

(5) $\Rightarrow$ (1). Let V be any open set of Y . Since Y is regular, V is θ - open in Y . By the faint δsp -continuity of f , $f^{-1}(V)$ is δsp -open in X . Therefore, f is δ - almost continuous.

Recall that a space (X, τ) is said to be (1) submaximal [3] if every dense subset of X is open in X , (2) extremally disconnected [3,15] if $\text{cl}(U) \in \tau$ for every $U \in \tau$.

Definition 24. A function $f : X \rightarrow Y$ is said to be

1. **faintly continuous** [9], if $f^{-1}(V)$ is open in X for each θ - open set V of Y .
2. **faintly semi – continuous** [19], if $f^{-1}(V)$ is semi-open in X for each θ - open set V of Y
3. **faintly precontinuous** [19], if $f^{-1}(V)$ is pre-open in X for each θ - open set V of Y
4. **faintly β - continuous** [12], [19], if $f^{-1}(V)$ is β -open in X for each θ - open set V of Y
5. **faintly α - continuous** [12] if $f^{-1}(V)$ is α -open in X for each θ - open set V of Y .

Theorem 25. If (X, τ) is submaximal extremally disconnected semi-regular and (Y, σ) is regular, then the following are equivalent for a function $f : (X, \tau) \rightarrow (Y, \sigma)$:

- (1) f is faintly continuous,



- (2) f is faintly α - continuous,
- (3) f is faintly semi - continuous,
- (4) f is faintly precontinuous,
- (5) f is faintly δ sp-continuous,
- (6) f is faintly γ - continuous,
- (7) f is faintly β - continuous,
- (8) f is δ - almost continuous,
- (9) f is almost δ sp-continuous,
- (10) f is weakly δ sp-continuous.

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