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NORMAL SHOCK WAVE DIFFRACTION FOR CARBON DIOXIDE (CO_2) GAS

R.S. Srivastava[1], Sanjay Srivastava[2]

[1]Formerly of Defence Science Centre, New Delhi (India)

[2]Bechtel (India)

ABSTRACT

Lighthill has considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4,$ γ being the ratio of specific heats. Srivastava extended the work of Lighthill to monoatomic gases for which $\gamma = \frac{5}{3}.$ In the present paper attempt has been made to present the results of normal shock wave diffraction for Carbon dioxide (CO_2) gases.

Keywords: Diffraction, Normal shock, Carbon dioxide (CO_2) gases, Pressure Distribution.

I. INTRODUCTION

Lighthill (1949) considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4,$ γ being the ratio of specific heats. In the present paper, the problem of Lighthill has been extended for Carbon dioxide (CO_2) gas for which $\gamma = 1.29.$ In the first instance the equations have been obtained for general value of γ and then the pressure distribution over the diffracted shock has been worked out for $\gamma = 1.29.$ The Mach number of the shock wave has been assumed to be 1.36. Earlier Srivastava (1963) and Srivastava (2016) have treated the analogous problem for $\gamma = \frac{5}{3}$ for the pressure distribution over the wall and the pressure distribution over the diffracted shock respectively. It may be mentioned here that Srivastava (2011) has obtained the vorticity distribution over the diffracted shock for monoatomic gases. Reference may be made to the book by Srivastava (1994) for detailed reading.

II.MATHEMATICAL FORMULATION

Let the velocity, pressure, density, sound speed behind the shock wave before it has crossed the bend be $q_1,\ \rho_1,\ p_1,\ a_1$ and ahead of the shock wave be $0,\ p_0,\ \rho_0,\ a_0$. Then applying the principle of conservation of mass, momentum and energy for general value of γ (γ being the ratio of specific heats)

$$q_1 = \frac{2U}{(\gamma+1)}\left(1 - \frac{a_0^2}{U^2}\right) \qquad - (1)$$

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$$\rho_1 = \frac{\rho_0(\gamma+1)}{(\gamma-1)+2\frac{a_0^2}{U^2}} \qquad - (2)$$

$$p_1 = \frac{\rho_0}{(\gamma-1)}\left[2U^2 - \frac{a_0^2(\gamma-1)}{\gamma}\right] \qquad - (3)$$

U being the velocity of shock wave, Mach number of the shock $M = \frac{U}{a_0}$, $a_0 = \sqrt{\frac{\gamma\, p_0}{\rho_0}}$

For $\gamma = 1.29$, we have

$$q_1 = \frac{2U}{2.29}\left(1 - \frac{1}{M^2}\right) \qquad - (4)$$

$$\rho_1 = \frac{\rho_0(2.29)}{0.29 + \frac{2}{M^2}} \qquad - (5)$$

$$p_1 = \frac{\rho_0 a_0^2}{0.29}\left[2M^2 - \frac{0.29}{1.29}\right] \qquad - (6)$$

$$a_1 = \sqrt{\frac{\gamma\, p_1}{\rho_1}} = \sqrt{\frac{(1.29)\, p_1}{\rho_1}}, \ \rho_1, p_1 \text{ are given by (5) and (6)} \qquad - (7)$$

The wedge is made up of two walls having a small angle δ between them. After the shock has suffered diffraction, the flow is two dimensional behind the shock wave. Let $\vec{q}_2, p_2, \rho_2$ and S_2 be the velocity vector, pressure, density and entropy at any point. We take the origin and Y axis lying on the leading edge of the wedge and X axis on the original wall produced.

If $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q}_2 \cdot \nabla$ signifies time rate change for a given fluid element, then the equation of conservation of mass, momentum and energy can be written as

$$\frac{D\rho_2}{Dt} + \rho_2 \operatorname{div} \vec{q}_2 = 0 \qquad - (8)$$

$$\frac{D\vec{q}_2}{Dt} + \frac{1}{\rho_2}\nabla \mathrm{p}_2 = 0 \qquad - (9)$$

$$\frac{DS_2}{Dt} = 0 \qquad - (10)$$

Now we introduce the following transformations

$$\frac{X - a_1 t}{a_1 t} = x \qquad - (11)$$

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$$\frac{Y}{a_1 t} = y \qquad - (12)$$

$$\frac{\vec{q}_2}{q_1} = (1+u, v) \qquad - (13)$$

$$\frac{p_2 - p_1}{a_1\, q_1\, p_1} = p \qquad - (14)$$

We assume that $\vec{q}_2, p_2, \rho_2$ differ by small quantities from the values $(q_1, 0),\ p_1,\ \rho_1$ which they had before diffraction, then using the equations (8), (9), (10) and (11), (12), (13), (14) we obtain a single second order partial differential equation in *p*. This equation is

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1 \right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} \right) \qquad - (15)$$

The characteristics of the differential equation (15) are tangents to the unit circle $x^2 + y^2 = 1,$ signifying that the disturbed region is enclosed by the arc of unit circle, the diffracted shock and the wedge surface.

The requirement of the problem is the determination of pressure distribution over the diffracted shock.

The position of the straight portion of the shock wave in *x, y* coordinates is given by $x = k,$ where $k = \dfrac{U - q_1}{a_1}$.The coordinates of the corner is $(-M_1, 0)$ where $\left(M_1 = \dfrac{q_1}{a_1} \right)$.Lighthill (1949) using Busemann transformation and complex variable techniques, obtained a function which satisfies all the boundary conditions. The function is given by

$$w(z_1) = \frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{C\delta\left[D(z_1 - x_0 - 1)\right]}{\left(z_1^2 - 1\right)^{1/2}\left[\alpha - i\left(z_1 - 1\right)^{1/2}\right]\left[\beta - i\left(z_1 - 1\right)^{1/2}\right](z_1 - x_0)} \qquad - (17)$$

$$z_1 = x_1 + iy_1$$

In the final z_1-plane, the imaginary part on the left hand side of (17) gives the pressure derivative which determines the pressure distribution over the diffracted shock. If one does that, then the expression for pressure derivative is given by

$$\frac{\partial p}{\partial x_1} = \frac{C\delta}{\left(x_1^2 - 1\right)^{1/2}}\left[D - \frac{1}{(x_1 - x_0)} \right]\frac{(\alpha + \beta)(x_1 - 1)^{1/2}}{\left[\alpha^2 + (x_1 - 1)\right]\left[\beta^2 + (x_1 - 1)\right]} \qquad - (18)$$

In (18), all the quantities are functions of the Mach number of the shock wave M except x_1 which runs from 1 to ∞ on the diffracted shock in the transformed plane and is connected to *y* in the physical plane through the relation

$$\frac{y}{k'} = \left(\frac{x_1 - 1}{x_1 + 1} \right)^{1/2},\ \ k' = \sqrt{1 - k^2} \qquad - (19)$$

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When $x_1 = 1, \frac{y}{k'} = 0$ (wall surface), when $x_1 \to \infty$, $\frac{y}{k'} = 1$ (point of intersection of shock and unit circle).

III. NUMERICAL SOLUTION:

The pressure distribution over the diffracted shock is obtained by integrating equation (18). The pressure *p*is zero at $x_1 = \infty$ i.e. at $\frac{y}{k'} = 1$ (the point ofintersection of shock wave and unit circle) and so pressure at other points could be known by integrating in intervals. The points chosen over the diffracted shock are

$$y/_{k'} = 0,\ y/_{k'} = 0.2,\ y/_{k'} = 0.4,\ y/_{k'} = 0.6,\ y/_{k'} = 0.8$$

The equations (18) and (19) have been used to get the results. The following table gives the results after integration. The table is for $y/_{k'}$ versus $-\frac{p}{k\delta}$ M is 1.36 and $\gamma = 1.29.$

Table-1

$y/_{k'}$	0	0.2	0.4	0.6	0.8	1
$-\frac{p}{k\delta}$	4.17	4.09	3.71	3.08	2.11	0

The table shows that $-\frac{p}{k\delta}$ is maximum at $y/_{k'} = 0$ i.e. at the point of intersection of the wall and shock. The value of $-\frac{p}{k\delta}$ falls from there and attain the value zero at $y/_{k'} = 1$ i.e. at the point of intersection of shock and unit circle. Physically this is consistent.

From the papers of Sakurai et al (2002) and Srivastava (2016) is could be seen that the value of $-\frac{p}{k\delta}$ is higher in the present case than those of earlier cases. In the interval of interest$\left(y/_{k'} = 0 \text{ to } y/_{k'} = 1 \right)$, $-\frac{p}{k\delta}$ for $\gamma = 1.4$ is lower than that for $\gamma = \frac{5}{3}$ and $-\frac{p}{k\delta}$ is lower for $\gamma = \frac{5}{3}$ than that for $\gamma = 1.29.$

IV. CONCLUSION

Diffraction of shock waves is very important aspect of aeronautical engineering. Particularly pressure variation over the diffracted shock is helpful in design work in aeronautics.

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IV. REFERENCES

1. Lighthill, M.J., The diffraction of blast-1, Proc. Roy. Soc. A, 198,454-470 (1949).
2. Srivastava, R.S.,Diffraction of plane straight shock wave. Ministry of Aviation British Aeronautical Research Council. Current Paper No. 603 (1963).
3. Srivastava, R.S., Normal shock wave diffraction for monoatomic gases. International Journal of Innovative Research in Science and Engineering Vol. No.-2, Issue 11, November 2016 (www.ijirse.com).
4. Srivastava, R.S., Diffraction of normal shock wave for monoatomic gases. Shock waves Vol. 21, No.1, pp 29-33 (2011).
5. Srivastava, R.S., Intersection of shock waves, Kluwer Academic Publishers Dordrecht (1994).
6. Sakurai A, Srivastava, R.S., Takahashi S., Takayama F., A note on Sandeman simple theory of weak Mach reflection, Shock waves 11, 409-411 (2002).