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DYNAMIC BEHAVIOR OF FGM CIRCULAR PLATE UNDER THERMAL ENVIRONMENT

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ABSTRACT

This paper present dynamic behavior of functionally graded material circular plate under thermal environment. Material constants are assumed to be dependent on temperature and graded according to power law distribution in thickness direction. The FGM circular plate under thermal environment is carried out by imposing different temperatures on plate surfaces. A three-dimensional finite element analysis has been done by COMSOLMultiphysics software.

Keyword: Functionally Gradedmaterial, Thermal Effect, Power Law, COMSOL.

I. INTRODUCTION

Functionally graded materials (FGMs) is composites which prepared by combination of two or more constitutes phase. The FGMs have various material parameters which continuously vary through a function in certain direction [1]. Japan scientist was obtained functionally graded material since 1980. Functionally graded materials are recovering mechanical and thermal constants of structure components under high temperature condition.

Go J. and Afsar A.M. [2] studied of FGM circular disk under thermos-elastic field using finite element method. Dong C.Y. [3] investigated natural frequencies of FG annular plate with various boundary conditions using Chebyshev–Ritz method. Rastgo Abbas and EbrahimiFarzad [4] reported dynamic response of FG thin circular plate based on classical plate theory. Eisenberger M. and Efraim E. [5] presented exact analysis of thick FGM annular plate using first-order shear deformation theory.

Chin CD ans Reddy JN [6] examined dynamic response of FGMs cylindrical shell. Yang B. and LiewK.M. [7] investigated elastic solution of FGM annular plate based on three-dimensional elasticity.

The present paper proposed non-dimensional frequencies of FGMs circular plate under clamped-free boundary condition. Further, effect of temperature on FGMs circular plate under clamped-free boundary condition are studied. FGMs circular plate are prepared by silicon nitride and zirconia which dependent on temperature and graded according to power law distribution in thickness direction.

II. FUNCTIONALLY GRADED MATERIAL

Functionally graded materials are advanced material in which the material constants are assumed to vary continuously in accordance with mathematical function. FGMs are widely used in high temperature

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environment and material parameters are temperature dependent. So, material parameters $þ_i$ is expressed in terms of temperature environment T (K) is written as [8]

$$þ_i = þ_0\,(þ_{-1}T^{-1} + 1 + þ_1\,T + þ_2T^2 + þ_3T^3\,) \;\ldots\ldots\ldots.\; (1)$$

Table 1 shows the material constants of functionally graded materials (FGMs) which are used in vibration analysis of FGMs in following chapters.

Table 1 Material Constants of Silicon Nitride and Zirconia [6]

Material	Constants	$þ_0$	$þ_{-1}$	$þ_1$	$þ_2$	$þ_3$
Silicon nitride	E_s (Pa)	348.43e9	0	-3.070e-4	2.160e-7	-8.940e-11
	ρ_s (kg/m^3	3440	0	0	0	0
	ε_s	0.2400	0	0	0	0
Zirconia	E_z(Pa)	244.27e9	0	-1.371e-3	1.214e-6	-3.681e-10
	ρ_z (kg/m^3)	5700	0	0	0	0
	ε_z	0.2882	0	1.133e-4	0	0

Here, T = $T_0 + \Delta$ T, T_0 = 300K (room temperature) and $þ_0$, $þ_{-1}$,$þ_1$, $þ_2$and$þ_3$represent coefficient of temperature. The material parameters þ of FGMs and volume fraction of constitute phase are written as

$$þ_{fgm} = \sum_{i=1}^{k} þ_i\, Y_{fi} \qquad (2)$$

Volume fraction (Y_{fi}) defined volume of constitute phase at point z through the thickness 'h' according to power exponent value 'N' which regulates the shape function Y_{fi} (z).

$$Y_{fi}(z) = \left(\frac{z+\frac{h}{2}}{h}\right)^{N} \quad \text{For} - (h/2) \leq z \leq (h/2) \text{ and } 0 \leq N < \infty \quad (3)$$

For functionally graded material have two or more constitute phase the material constants are define as

$$E = (E_m - E_c)\, Y_f + E_c$$

$$\rho = (\rho_m - \rho_c)\, Y_f + \rho_c$$

$$\varepsilon = (\varepsilon_m - \varepsilon_c)\, Y_f + \varepsilon_c$$

$$\alpha = (\alpha_m - \alpha_c)\, Y_f + \alpha_c \qquad (4)$$

whereE_m and E_c are Young's modulus, ρ_m and ρ_c are density, ε_m and ε_c are Poisson's ratio and α_m and α_c are thermal expansion of metal and ceramic respectively.

III. THEORETICAL FORMULATION

3.1 Displacement

Cylindrical coordinate system (r, θ, z) and three dimensional FGM circular plate made of two type functionally graded material with inner radius 'a', outer radius 'b', and thickness 'h' is shows in Figure 1. The deformation

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of plate with reference to cylindrical coordinate system (r, θ, z) at middle surface which are In-plane, radial and normal displacements respectively.

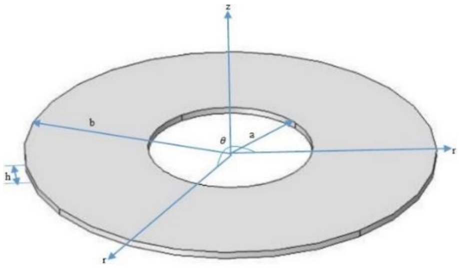

Fig. 1.FGM Circular Plate

For present study, clamped-free boundary condition are employed.

u = v = w = 0 at r=a

M = S.F.= 0 at r=b (5)

IV. RESULTS AND DISCUSSION

Dynamic behavior of FGMs circular plate is reported. Material constants are assumed to be dependent on temperature and graded according to power law distribution in thickness direction. In this study, COMSOLMultiphysics (version 4.2) software is used to evaluate the frequencies.

4.1 Convergence Study

Dynamic analysis of FGM circular plate (h/b=0.03, b/a=0.4, ΔT = 0) is considered for convergence study.Natural frequencies are calculated with varying meshing for clamped-free boundary condition. Results are obtained using COMSOLsoftware.The non-dimensional frequencies (ώ) is obtained using following expression

$$\acute{\omega} = \omega h \sqrt{\rho_s / E_s} * 10^3$$

Here, (m, n) represent the axial wavenumber and circumferential wavenumber respectively.

Table 2

Comparison of Non-dimensional Frequency (ώ) for an FGM Circular Plate under Clamped-Free Boundary Condition using COMSOL

(m,n)	Non-dimensional frequency (ώ)			
	Coarse	Normal	Fine	Finer
(0,0)	6.9683	6.7468	6.7302	6.7225
(1,0)	9.7639	9.7054	9.6578	9.6326
(2,0)	15.7366	15.6027	15.5077	15.4166
(3,0)	23.4222	23.1654	22.0035	22.9235
(4,0)	32.9323	32.4609	32.2019	32.0858
(0,1)	33.5275	33.1133	32.8503	32.7482

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Table 2 shows that the fine meshing gives better results as compare to results obtained with other meshing. Hence, fine meshing is used to further study.

4.2 Parametric Study

In this study, dynamic analysis of FGM circular plate (h/b=0.03, b/a=0.4) under thermal environment is considered. Material constants are dependent on temperature and vary in plate from silicon nitride at outer surface to zirconia at inner surface according to power-law distribution. For analysis, material constants are taken from Table I and calculate by equation (1) with different temperature. Natural frequencies are calculated for clamped-free boundary condition. This investigation shows that effect of temperature on natural frequency of FGM cylindrical shell.

The outer surface of the shell is kept at ambient temperature and temperature variation at inner surface of the shell. Results are presented in Table 3. The non-dimensional frequency (ώ) is given $\acute{\omega} = \omega h \sqrt{\rho_s / E_s} * 10^3$. Results are obtained for various mode (m, n) as (0, 0), (1, 0), (2, 0), (3,0), (4,0) and (0,1) with different power exponent value as N= 1, 3, 5. The mode shapes are shows in Figure 2.Figure 3 show that for all power exponent value, the natural frequencies are decrease with increase temperature for (1, 0) mode.

Table 3

Non-dimensional Frequency (ώ) for an FGM Circular Plate under Clamped-Free Boundary Condition under Thermal Environment (m, n, h/b=0.03, b/a=0.4, T_o = 300K)

T_{inner}(K)	Non-dimensional frequency ώ		
	N=1	N=3	N=5
320	6.5052 (0,0)	8.2585 (0,0)	8.7049 (0,0)
	9.5895 (1,0)	11.8743 (1,0)	12.5210 (1,0)
	15.4509 (2,0)	19.0191 (2,0)	20.0360 (2,0)
	22.9200 (3,0)	28.1413 (3,0)	29.6371 (3,0)
	32.0851 (4,0)	39.3603 (4,0)	41.4441 (4,0)
	32.7167 (0,1)	40.2317 (0,1)	42.3987(0,1)
340	6.4822 (0,0)	8.2528 (0,0)	8.7035 (0,0)
	9.4295 (1,0)	11.8660 (1,0)	12.5189 (1,0)
	15.4010 (2,0)	19.0060 (2,0)	20.0328 (2,0)
	22.8434 (3,0)	28.1222 (3,0)	29.6323 (3,0)
	31.9831 (4,0)	39.3406 (4,0)	41.4374 (4,0)
	32.6050 (0,1)	40.2039 (0,1)	42.3917 (0,1)
360	6.4660(0,0)	8.2474 (0,0)	8.7021 (0,0)
	9.3887 (1,0)	11.8606 (1,0)	12.5169 (1,0)
	15.3508 (2,0)	18.9970 (2,0)	20.0296(2,0)
	22.7699 (3,0)	28.1049 (3,0)	29.6285 (3,0)
	31.8800(4,0)	39.3155 (4,0)	41.4311(4,0)
	32.4980 (0,1)	40.1775 (0,1)	42.3850 (0,1)

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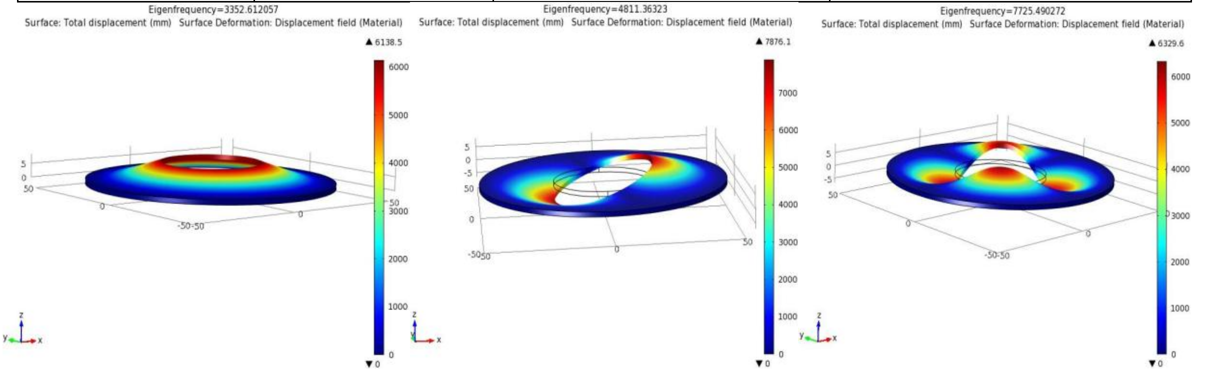
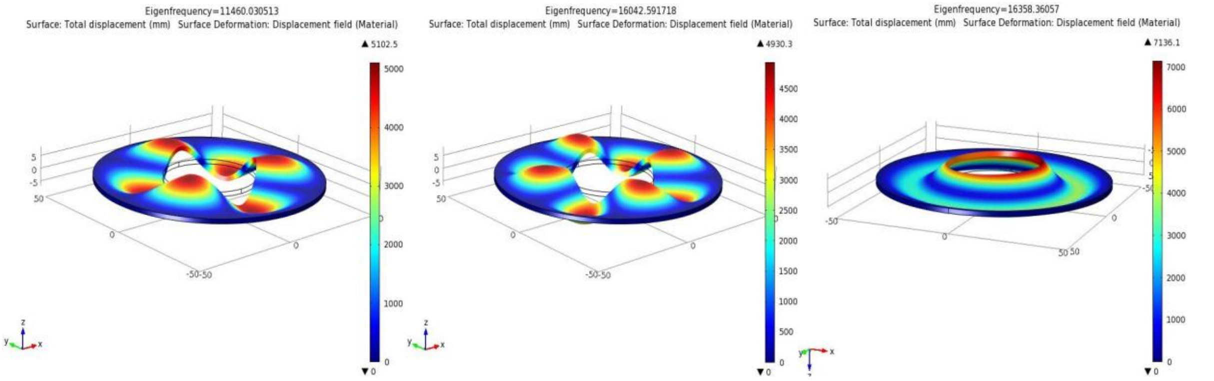
380	6.4393 (0,0)	8.2422 (0,0)	8.7008 (0,0)
	9.3284 (1,0)	11.8531 (1,0)	12.5150 (1,0)
	15.3023 (2,0)	18.9850 (2,0)	20.0266 (2,0)
	22.6984 (3,0)	28.0873 (3,0)	29.6241 (3,0)
	31.7807 (4,0)	39.2910 (4,0)	41.4250 (4,0)
	32.3957 (0,1)	40.1520 (0,1)	42.3786 (0,1)



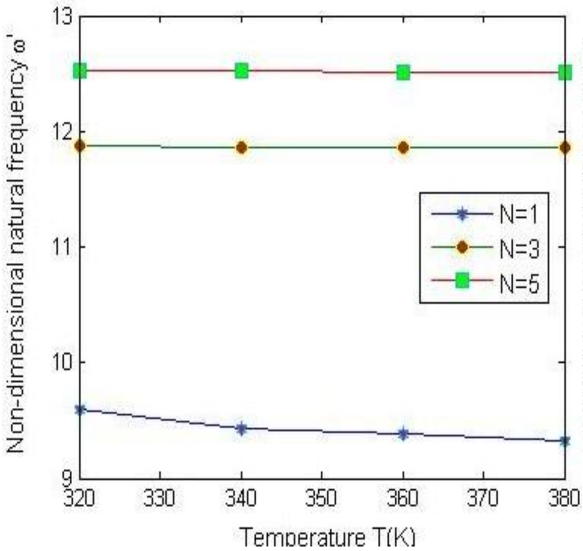
Eigen frequency= 3352.61 Hz (0,0) **Eigen frequency = 4811.36 Hz (1,0)** **Eigen frequency = 7725.49 Hz (2,0)**



Eigen frequency = 11460.03 Hz (3,0) **Eigen frequency = 16042.59 Hz (4,0)** **Eigen frequency = 16358.36 Hz (0,1)**

Fig. 2. Mode shapes of FGM

circular plate under clamped-free boundary condition under thermal environment (N=1)



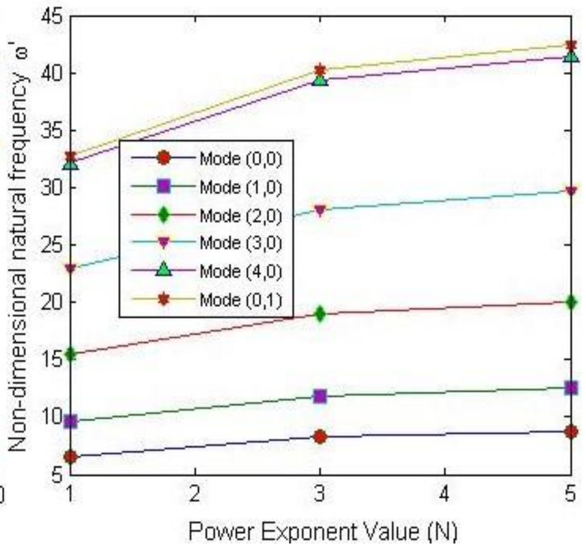


Fig. 3. Variation of natural frequency of FGM cylindrical Fig. 4. Variation of natural frequency with power

shell under clamped-free boundary conditions under exponent value (N) of FGM cylindrical shell

thermal effect with different power exponent value under clamped-free boundary condition under thermal effectfor mode (1,0).effect

Figure 4 shows that the natural frequencies are increase with increase in power exponent value for (0, 0), (1, 0), (2, 0), (3,0), (4,0) and (0,1) modes.

V. CONCLUSION

Dynamic behavior of functionally graded circular plate under thermal environment is reported. Material constants are assumed to be dependent on temperature and graded according to the power law distribution in thickness direction. It is observed that the variation of natural frequency under thermal environment, for all power exponent value, the natural frequencies are decrease with increase temperature for (0, 0), (1, 0), (2, 0), (3,0), (4,0) and (0,1) modes. The natural frequencies are increase with increase in power exponent value for (0, 0), (1, 0), (2, 0), (3,0), (4,0) and (0,1) modes. Dynamic behavior of FGMcircular plate under acoustic effect is to be studied for future work.

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REFERENCES

[1] Victor Birman and Larry W. Byrd, Modeling and Analysis of functionally graded materials and structures,
Applied Mechanics Reviews, 60, 2007, 195-216.

[2] Go J. and Afsar A.M., Finite element analysis of thermoelastic field in a rotating FGM circular disk, Applied Mathematical Modelling, 34, 2010, 3309–3320.

[3] Dong C.Y., Three-dimensional free vibration analysis of functionally graded annular plates using the Chebyshev–Ritz method, Materials and Design, 29, 2008, 1518–1525.

[4] Rastgo Abbas and EbrahimiFarzad, An analytical study on the free vibration of smart circular thin FGM plate based on classical plate theory, Thin-Walled Structures, 46, 2008, 1402– 1408.

[5] Eisenberger M. and Efraim E., Exact vibration analysis of variable thickness thick annular isotropic and FGM plates, Journal of Sound and Vibration, 299, 2007, 720–738.

[6] Chin CD ans Reddy JN, Thermo mechanical analysis of functionally graded cylinders and plates, Journal of Thermal Stresses,21, 1998, 593-626.

[7] Yang B. and LiewK.M., Elasticity solutions for free vibrations of annular plates from three-dimensional analysis, International Journal of Solids and Structures, 37, 2000, 7689-7702

[8] Farid M, Zahedinejad P and Malekzadeh P, “Three dimensional temperature dependent free vibration analysis of functionally graded material curved panels resting on two parameter elastic foundation using a hybrid semi-analytic, differential quadrature method”, Mater Des (2010), vol. 31, pp. 2-13.