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A NEWLY INVENTED NUMERICAL INTEGRATION FORMULA: SIDDHALIC 4/10 RULE

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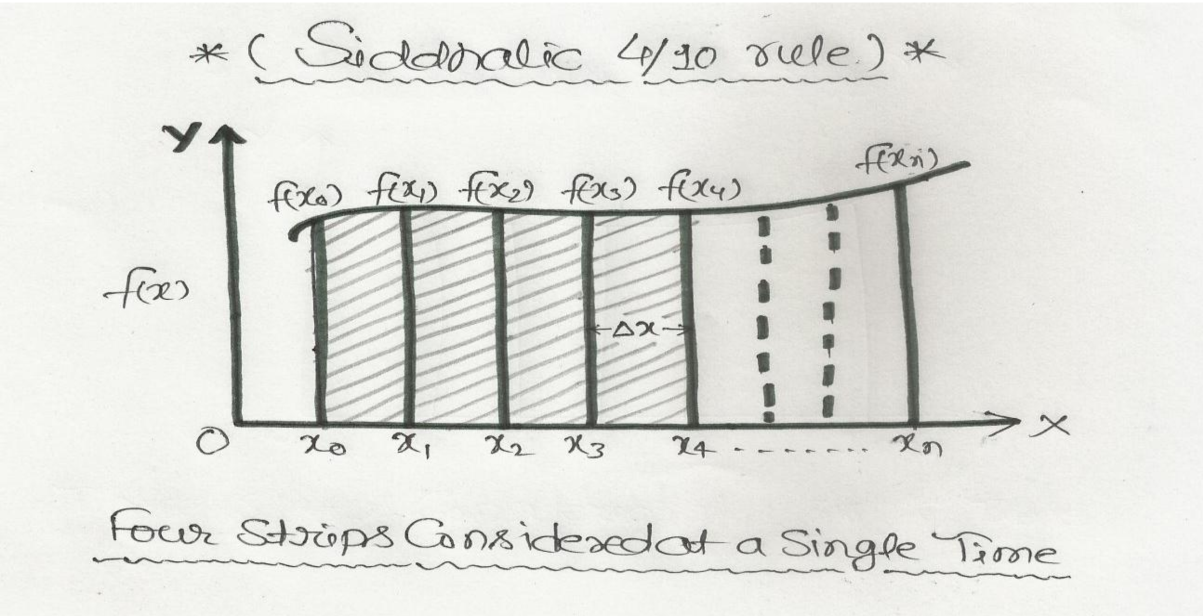
ABSTRACT

In numerical analysis there are methods of numerical integration namely simpson's $1/3^{rd}$ rule , simpson's 3/8 rule which are frequently used to find out the integration of some function numerically. These numerical integration formula are derived calculating the area under the curve between the integration limits with a conventional way. In this paper a new numerical integration formula namely siddhalic 4/10 rule is derived following a new approach which facilitates the field of numerical integration with a surplus tool to be used with the ease of method.

I INTRODUCTION

Below is the derivation of a newly discovered numerical integration formula namely, Siddhalic 4/10 rule, following the same approach, Simpson's 1/3 rd rule & Simpson's 3/8 rule are also derived ensuring a reasonable support to the method employed.

(1). SIDDHALIC 4/10 RULE :



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Let us consider a function f(x) drawn in the diagram shown above. The different values of this function namely $f(x_0)$, $f(x_1)$, $f(x_2)$, $f(x_3)$, $f(x_4)$……$f(x_n)$ are shown against the corresponding x-values namely x_0, x_1, x_2, x_3, x_4……x_n. In order to evaluate the value of $_{xo}\int^{xn} f(x)dx$,where x_0 & x_n represent the lower & upper limits of the integration. let us now divide the whole area under the curve into n-thin strips of equal width Δx, considering four such strips at a time as shown in the graph i.e. five x-values, x_0, x_1, x_2, x_3, x_4 , this way the combined area of first such combination is

= Total width of strip combination × average height of strip boundaries

$= (4\Delta x) \times \{f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)\}/5$

$= (4\Delta x/10) \times \{2f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)\}$

$= (4\Delta x/10) \times [\{f(x_0)+f(x_4)\}+f(x_0)+f(x_4)+2f(x_2) +\{f(x_1)+f(x_3)\}+f(x_1)+f(x_3)]$

now from figure by the definition of arithmetic mean, we have,

$$\{f(x_0)+f(x_4)\}/2 = f(x_2) = \{f(x_1)+f(x_3)\}/2$$

or $$f(x_0)+f(x_4) = 2f(x_2) = f(x_1)+f(x_3)$$

therefore the above expression becomes

$(4\Delta x/10) \times [2f(x_2)+f(x_0)+f(x_4)+2f(x_2)+2f(x_2)+f(x_1)+f(x_3)]$

$= (4\Delta x/10) \times [f(x_0)+f(x_1)+6f(x_2)+f(x_3)+f(x_4)]$

similarly the area of the next strip combination will be

$= (4\Delta x/10) \times [f(x_4)+f(x_5)+6f(x_6)+f(x_7)+f(x_8)]$

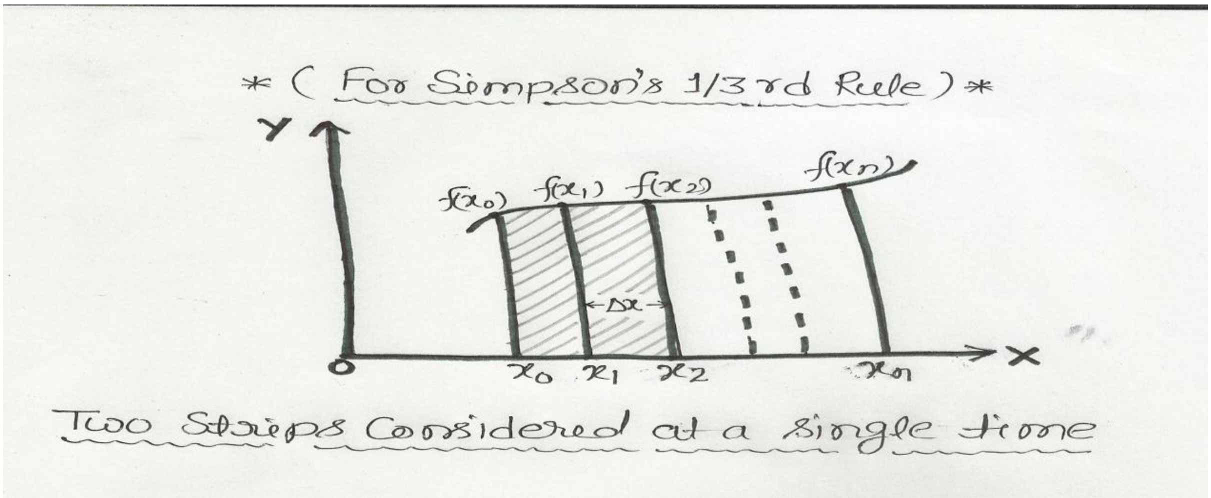
………………………………………..& so on

thus adding the combinations final area will be

$_{x0}\int^{xn} f(x)dx = (4\Delta x/10) \times [f(x_0)+f(x_1)+6f(x_2)+f(x_3)+2f(x_4)+\ldots\ldots\ldots + f(x_n)]$

Above formula represents the Siddhalic 4/10 rule for numerical integration technique where Δx is the width of one strip.

(2). SIMPSON'S 1/3RD RULE



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The diagram shows a graphical interpretation of a function f(x) in the manner shown with an objective to find out the value of $_{x0}\int^{xn} f(x)\,dx$ i.e. the area under the curve.

Considering two strips at a time as shown in graph i.e. three x-values namely x_0, x_1, x_2 . This way the combined area of first such combination is

= Total width of strip combination × average height of strip boundaries.

$= 2\Delta x \times [\{f(x_0)+f(x_1)+f(x_2)\}/3]$

$= (\Delta x/3) \times \{2f(x_0)+2f(x_1)+2f(x_2)\}$

$= (\Delta x/3) \times [f(x_0)+2f(x_1)+f(x_2)+\{f(x_0)+f(x_2)\}]$

now from the graph, if the arithmetic mean of $f(x_0)$ & $f(x_2)$ is $f(x_1)$, that is

$$f(x_1) = \{f(x_0)+f(x_2)\}/2$$

then we have , $f(x_0)+f(x_2) = 2f(x_1)$

so area of combination of strips is

$= (\Delta x/3) \times [f(x_0)+2f(x_1)+f(x_2)+2f(x_1)]$

$= (\Delta x/3) \times [f(x_0)+4f(x_1)+f(x_2)]$

similarly the area of the next strip combination will be

$= (\Delta x/3) \times [f(x_2)+4f(x_3)+f(x_4)]$

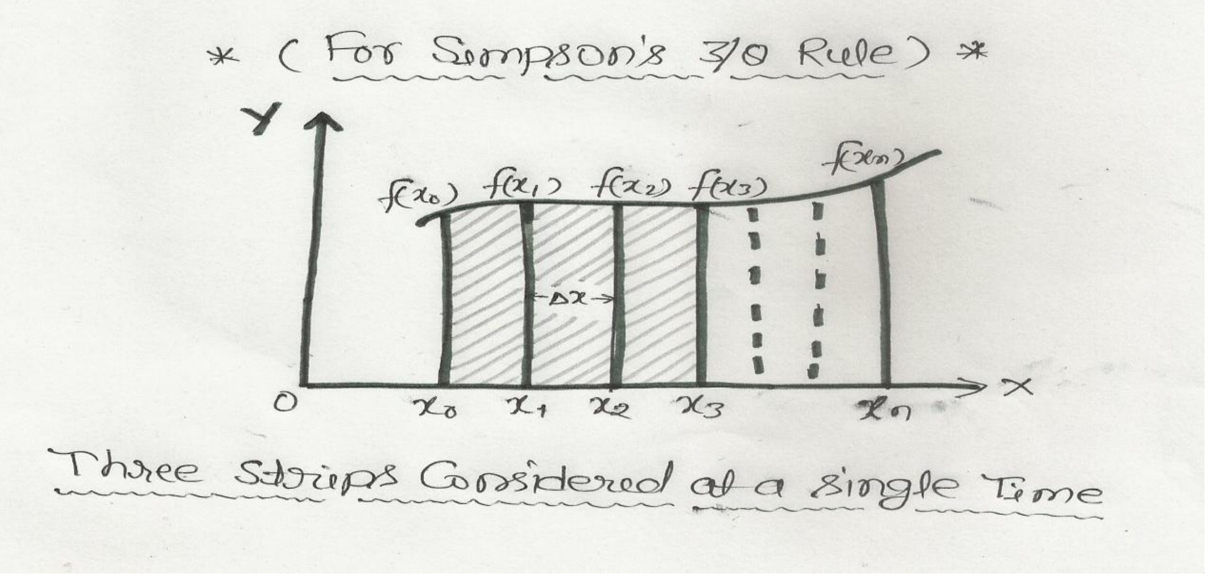
………………………………& so on.

thus adding the combinations, final area will be

$$_{x0}\int^{xn} f(x)\,dx = (\Delta x/3)\times\{f(x_0)+4f(x_1)+2f(x_2)+\ldots\ldots\ldots\ldots+f(x_n)\}$$

Above formula represents the famous Simpson's 1/3 rule for numerical integration technique where Δx is the width of one strip.

(3). SIMPSON'S 3/8 RULE :



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The diagram shows a graphical interpretation of a function f(x) in the manner shown with an objective to find out the value of $\int_{x0}^{xn} f(x)dx$ i.e. the area under the curve.

Considering three strips at a time as shown in the graph i.e. four x- values namely x_0, x_1 , x_2 , x_3.

This way the combined area of first such combination is

= Total width of strip combination × average height of strip boundaries.

$= (3\Delta x) \times \{f(x_0)+f(x_1)+f(x_2)+f(x_3)\}/4$

$= (3\Delta x/8) \times \{2f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)\}$

$= (3\Delta x/8)\times[f(x_0)+2f(x_1)+2f(x_2)+f(x_3)+\{f(x_0)+f(x_3)\}]$

from the definition of arithmetic mean, if we have

$$\{f(x_0)+f(x_3)\}/2 = \{f(x_1)+f(x_2)\}/2$$

therefore the above expression becomes

$(3\Delta x/8)\times[f(x_0)+2f(x_1)+2f(x_2)+f(x_3)+f(x_1)+f(x_2)]$

$= (3\Delta x/8) \times [f(x_0)+3f(x_1)+3f(x_2)+f(x_3)]$

similarly ,the area of next strip combination will be

$(3\Delta x/8) \times [f(x_3)+3f(x_4)+3f(x_5)+f(x_6)]$

…………………………….. & so on

Thus adding the combinations, final area will be

$\int_{x0}^{xn} f(x)dx = (3\Delta x/8)\times[f(x_0)+3f(x_1)+3f(x_2)+2f(x_3)+................+f(x_n)]$

Above formula represents the famous Simpson's 3/8 rule for numerical integration technique where Δx is the width of one strip.

II CONCLUSION

from the above derivations it is clear that siddhalic 4/10 rule can play a very important role in numerical integration technique as this is derived by a new approach employing which the Simpson's 1/3rd rule & 3/8 rule are also derived which ensures the strong support to this method of derivation.