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FLOW SHOP SCHEDULING PROBLEM WITH LOADING AND UNLOADING TIME

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ABSTRACT

Flow shop scheduling is basically used in the industries where continuous manufacturing is going on. Research work in flow shop scheduling is very effective in production area. In flow shop scheduling problems total completion time of process that is make-span depends on the sequence of jobs selected in a schedule. In this paper we try to find the algorithm for flow shop scheduling problem of five machines arranged in series with n-jobs. Set up time, transporting time loading time and unloading time of machines are considered separately. Also effect of breakdown interval is observed on the schedule. Johnson's rule is used for finding the sequence of jobs. Numerical example is solved to show the application of the algorithm.

Keywords*: *Flow shop scheduling, Johnson's algorithm, loading time, setup time, transportation time, unloading time,.

I. INTRODUCTION

Scheduling is the way of completion of jobs involved in process of manufacturing in given assigned time interval. As there is development in modern technique of manufacturing, scheduling theory gets gradually developed with new inventions. Sequencing gives the proper order of arrangement of jobs. Sequencing is one of the important factors in a company where machines are less in number and heavy load of work. Resources include the machines and number of jobs operating on different machines. Effective sequencing and scheduling gives the final plan of wrapping up all steps and launch the ready product in the market. Also the machine environment is one of the important factors in scheduling. It gives the idea about the available machine and the job accessible for that particular machine as each machine required one job at a time or each job need one machine for its processing. There are different types of sequencing – first come first served, processing time, weightage of jobs, job size etc. Flow shop scheduling is the problem in which every job has to process through each machine and sequence of jobs is fixed. Time taken by each job may be different from each other. We have to choose the schedule which gives maximum output in minimum time. Also selection of schedule should involved proper employment of man power, equipments, services and minimum waiting time of jobs, waiting time of customer and inventory.

In recent trends of manufacturing computerised planning of scheduling plays vital role in designing of the problem, planning of execution, scheduling of this planning and controlling of the system. This helps to generate the desired amount of production in right time and to compete with the market demands. Mehrotra**[1]** in her

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thesis considered flow shop scheduling problem for two and three machines arranged in series with single transport agent in between and considered setup time, transportation time, returning time for all jobs on both machines with loading and unloading time. Ali Alahverdi **[2]** summarized the data of scheduling literature on models with setup times for more than 300 papers. According to Ajaykumar Agrawal et al.[**3]** used the concept of Flexible manufacturing system that is FMS scheduling system which is the most important part of Computer Integrated Manufacturing (CIM) system to minimize the make-span of production. Wassenhove and Jaikumar had done a survey of maximum number FMS all over the world out of which 93 in Japan, 35 in USA and 27 in Europe**[4].** There are different types of buffers used in industries like automated guided vehicle, cranes, robots, conveyors etc. Rotary table is one of them which is used as buffer for solving m-machines n-jobs flow shop scheduling problem by Huang et al.[**5]** for loading and unloading of the jobs finished at all machines. This table is in centre from which all the machines from which finished jobs which are to be loaded or unloaded are at same distance. All unloaded jobs are kept on the table and from there they will be loaded for other machines. Mohammad Sadaqa et al.[**6]** also used a concept of loading and unloading of jobs on machines. The flow shop problem is solved for two as well as three machines and objective of the solution is to reduce the make-span. The heuristic applied is meta heuristic for Randomised priority search. Loading concept of outbound containers was studied by Quingcheng Zeng et al.[**7]**. Seyedeh Sarah Zabihzadeh et al.[**8]** studied flexible flow shop scheduling problem with parallel machines which are not related. P.V.Senthiil et.al **[9]** have proposed new algorithm for N jobs M machines flow shop NP hard combinatorial problem. It is type of batch processing under category of two way transportation that is after completion of product in processing unit it will be transferred to dispatch section.

In this paper we try to prove the theorem which gives the algorithm for finding the optimum solution for the flow shop scheduling problem of five machines with consideration of loading and unloading time **[1, 10, 11].**

II. THEOREM FOR GETTING OPTIMAL SOLUTION

An optimal sequence is obtained by sequencing the item i-1, i, i+1 such that

Min(

$$P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i}, LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}} + V_{i+1}$$

)

$$< Min(P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}}, LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i} + V_i$$

)

Let X and X' denotes sequences of items.

Let processing time and completion time of any item j on machine Y (ie. P, Q , R, S,V) for sequences X , X' be given by (Y_j, Y'_j) and (CY_j, CY'_j) respectvely.

Let (a_j, a'_j), (b_j, b'_j), (c_j, c'_j), (d_j, d'_j) are transportation time of item j from machine P to Q, Q to R , R to S and S to V resp.

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Let $p_j, p'_j, q_j, q'_j, r_j, r'_j, s_j, s'_j, v_j, v'_j$ set up time of item on machines P, Q ,R, S,V resp. for sequences X and X'.

The time required on machines Q, R, S and V for completion of j^th item is given by

CQ_j = max ($cP_j + LT_{PQ_j} + a_j + UL_{PQ_j}, cQ_{j-1}$) $+Q_j$

$= cP_j + LT_{PQ_j} + a_j + UL_{PQ_j} + Q_j$

CR_j = max ($cQ_j + LT_{QR_j} + b_j + UL_{QR_j}, cR_{j-1}$) $+R_j$

$= cQ_j + LT_{QR_j} + b_j + UL_{QR_j} + R_j$

CS_j = max ($cR_j + LT_{RS_j} + c_j + UL_{RS_j}, cS_{j-1}$) $+S_j$

$= cR_j + LT_{RS_j} + c_j + UL_{RS_j} + S_j$

CV_j = max ($cS_j + LT_{SV_j} + d_j + UL_{SV_j}, cV_{j-1}$) $+V_j$

= max

$(cP_j + LT_{PQ_j} + a_j + UL_{PQ_j} + Q_j + LT_{QR_j} + b_j + UL_{QR_j} + R_j + LT_{RS_j} + c_j + UL_{RS_j} +$

$S_j + LT_{SV_j} + d_j + UL_{SV_j}, cV_{j-1}) + V_j$ ------------- (1)

The sequence X will be selected in such a way that if $cV_m < c'V_m$ ------ (2)

Max(

$cP_m + LT_{PQ_m} + a_m + UL_{PQ_m} + Q_m + LT_{QR_m} + b_m + UL_{QR_m} + R_m + LT_{RS_m} + c_m + UL_{RS_m} + S_m +$

$LT_{SV_m} + d_m + UL_{SV_m}, cV_{m-1}$

$) + V_m$

$<$ max($c'P_m + LT'_{PQ_m} + a'_m + UL'_{PQ_m} + Q'_m + LT'_{QR_m} + b'_m + UL'_{QR_m} + R'_m + LT'_{RS_m} + c'_m +$

$UL'_{RS_m} + S'_m + LT'_{SV_m} + d'_m + UL'_{SV_m}, c'S_{m-1}$

$) + S'_m$

Now

$cP_m + LT_{PQ_m} + a_m + UL_{PQ_m} + Q_m + LT_{QR_m} + b_m + UL_{QR_m} + R_m + LT_{RS_m} + c_m + UL_{RS_m} + S_m +$

$LT_{SV_m} + d_m + UL_{SV_m}$

=

$c'P_m + LT'_{PQ_m} + a'_m + UL'_{PQ_m} + Q'_m + LT'_{QR_m} + b'_m + UL'_{QR_m} + R'_m + LT'_{RS_m} + c'_m + UL'_{RS_m} +$

$S'_m + LT'_{SV_m} + d'_m + UL'_{SV_m}$

We have $V_m = V'_m$ and Equation 2 will be satisfied only if $cV_{m-1} < c'V_{m-1}$ ----- (3)

Using all above results, we can say that equation 2 is true if $cV_j < c'V_j$ (j = i+1,i+2-----,m & i= 1,2,-----m-1)

---------------(4)

To find the values of cV_{i+1} & $c'V_{i+1}$

S_{i+1} = max { $cS_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}}, cV_i$ } $+V_{i+1}$

=max{

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$cP_{i+1} + LT_{PQ\,i+1} + a_{i+1} + UL_{PQ\,i+1} + Q_{i+1} + LT_{QR\,i+1} + b_{i+1} + UL_{QR\,i+1} + R_{i+1} + LT_{RS\,i+1} + c_{i+1} + UL_{RS\,i+1} + S_{i+1} + LT_{SV\,i+1} + d_{i+1} + UL_{SV\,i+1},$
$cV_i\} + V_{i+1}$

=max

$\{cP_{i+1} + LT_{PQ\,i+1} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR\,i+1} + b_{i+1} + UL_{QR\,i+1} + R_{i+1} + LT_{RS\,i+1} + c_{i+1} + UL_{RS\,i+1} + S_{i+1} + LT_{SV\,i+1} + d_{i+1} + UL_{SV\,i+1}$
,max($cS_i + LT_{SV_i} + d_i + UL_{SV_i}, cV_{i-1}$) $+V_i\}+V_{i+1}$

=max

$\{cP_{i+1} + LT_{PQ\,i+1} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR\,i+1} + b_{i+1} + UL_{QR\,i+1} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS\,i+1} + S_{i+1} + LT_{SV\,i+1} + d_{i+1} + UL_{SV\,i+1}$
, $cS_i + LT_{SV_i} + d_i + UL_{SV_i} + V_i, cV_{i-1} + V_i\} + V_{i+1}$

=max

$\{cP_{i+1} + LT_{PQ\,i+1} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR\,i+1} + b_{i+1} + UL_{QR\,i+1} + R_{i+1} + LT_{RS\,i+1} + c_{i+1} + UL_{RS\,i+1}$
$+S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}}$, max($cP_i + LT_{PQ_i} + a_i + UL_{PQ_i}\, cQ_{i-1}$) $+ Q_i +$
$LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS\,i} + c_i + UL_{RS_i} + S_i, LT_{SV_i} + d_i + UL_{SV_i} + V_i, cV_{i-1} + V_i\}$
$+ V_{i+1}$

cS_{i+1}= max

{

$cP_{i-1} + P_i + P_{i+1} + LT_{PQ\,i+1} + a_{i+1} + UL_{PQ\,i+1} + Q_{i+1} + LT_{QR\,i+1} + b_{i+1} + UL_{QR\,i+1} + R_{i+1} + LT_{RS\,i+1} + c_{i+1} + UL_{RS\,i+1} + S_{i+1} + LT_{SV\,i+1} + d_{i+1} + UL_{SV\,i+1} + V_{i+1}$

,

$cP_{i-1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i} + V_i + V_{i+1}, cV_{i-1} + V_i + V_{i+1}\}$
-------(5) Similarly

$c'S_{i+1}$ =max{

$c'P_{i-1} + P'_i + P'_{i+1} + LT'_{PQ_{i+1}} + a'_{i+1} + UL'_{PQ_{i+1}} + Q'_{i+1} + LT'_{QR_{i+1}} + b'_{i+1} + UL'_{QR\,i+1} + R'_{i+1} + LT'_{RS_{i+1}} + c'_{i+1} + UL'_{RS_{i+1}} + S'_{i+1} + LT'_{SV_{i+1}} + d'_{i+1} + UL'_{SV_{i+1}} + V'_{i+1}$

,

$c'P_{i-1} + P'_i + LT'_{PQ_i} + a'_i + UL'_{PQ_i} + Q'_i + LT'_{QR_i} + b'_i + UL'_{QR_i} + R'_i + LT'_{RS_i} + c'_i + UL'_{RS_i} + S'_i + LT'_{SV_i} + d'_i + UL'_{SV_i} + V'_i + V'_{i+1}, cV_{i-1} + V_i + V_{i+1}\}$
--------------(6)

Comparing sequences V & V', we get $c\,P_{i-1} = c'\,P_{i-1}$ & $c\,V_{i-1} = c'\,V_{i-1}$ & $Y_i = Y'_{i+1}$, $Y_{i+1} = Y'_i$

where Y = P, Q, R, S or V ------------ (7)

Also 1) $a_i = a'_{i+1}$, $a_{i+1} = a'_i$ 2) $b_i = b'_{i+1}$, $b_{i+1} = b'_i$

3) $c_i = c'_{i+1}$, $c_{i+1} = c'_i$ 4) $d_i = d'_{i+1}$, $d_{i+1} = d'_i$

1) $LT_{PQ_i} = LT'_{PQ_{i+1}}$, $LT_{PQ_{i+1}} = LT'_{PQ_i}$ 2) $LT_{QR_i} = LT'_{QR_{i+1}}$, $LT_{QR_{i+1}} = LT'_{QR_i}$

3) $LT_{RS_i} = LT'_{RS_{i+1}}$, $LT_{RS_{i+1}} = LT'_{RS_i}$ 4) $LT_{SV_i} = LT'_{SV_{i+1}}$, $LT_{SV_{i+1}} = LT'_{SV_i}$

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5) $UL_{PQ_i} = UL'_{PQ_{i+1}}$, $UL_{PQ_{i+1}} = UL'_{PQ_i}$ 6) $UL_{QR_i} = UL'_{QR_{i+1}}$, $UL_{QR_{i+1}} = UL'_{QR_i}$

7) $UL_{RS_i} = UL'_{RS_{i+1}}$, $UL_{RS_{i+1}} = UL'_{RS_i}$ 8) $UL_{SV_i} = UL'_{SV_{i+1}}$, $UL_{SV_{i+1}} = UL'_{SV_i}$

Writing eq. 4 for j = i+1 & using eq. 7, we get

max{

$cP_{i-1} + P_i + P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}} + V_{i+1}, cP_{i-1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i} + V_i + V_{i+1}, cV_{i-1} + V_i + V_{i+1}$ } } <

$max\{cP_{i-1} + P_{i+1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i} + V_i$

, $cP_{i-1} + P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}} + V_{i+1} + V_i, cV_{i-1} + V_{i+1} + V_i$ } -----------

(8)

Subtracting last term from both sides, and then subtracting

$cP_{i-1} + P_i + P_{i+1} + LT_{PQ_i} + a_i + UL_{PQ_i} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + LT_{QR_i} + b_i + UL_{QR_i} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}}$

$+LT_{RS_i} + c_i + UL_{RS_i} +$

$LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + LT_{SV_i} + d_i + UL_{SV_i} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}} + Q_i + Q_{i+1} + R_i + R_{i+1} + S_i + S_{i+1} + V_i + V_{i+1}$

from both the sides we get the required result.

Min(

$P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i},$
$LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}} + V_{i+1}$

)

$< Min(P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + LT_{SV_{i+1}} + d_{i+1} + UL_{SV_{i+1}}, LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + LT_{SV_i} + d_i + UL_{SV_i} + V_i$

)

III. PROBLEM DESIGNING

To prove the utility of above theorem numerical problem is solved.

In the given problem five machines namely P, Q, R, S,V are arranged in series. Loading time, unloading time and transport time of articles A_1, A_2, A_3, ------ A_n are considered. The transporting agent transport the items from machine P to machine Q, machine Q to machine R, machine R to machine S and machine S to machine V in such a way that after delivering the articles to machine V without delay come back to machine P for transferring the next item. All machines required time for loading items transported by transport agent so that machines will start processing, this time will be considered as loading time. After finishing the working of machine the articles will be transferred to the next machine, the time required for unloading of these articles on the particular machine is considered as unloading time.

LT_{PQi} :- loading time at machine P, $LT_{QRi,}$:- loading time at machine Q,

LT_{RSi} :- loading time at machine R, LT_{SVi} :- loading time at machine V,

UL_{PQi} :- unloading time at machine Q, UL_{QRi} :- unloading time at machine R

UL_{RSi} :- unloading time at machine S, UL_{SVi} :- unloading time at machine V

Step I: Suppose FOUR machines E, F , G and O are assumed with the service time $E_{i,}$ $F_{i\,,}$ G_i and O_i resp. where,

$$E_i = P_i + LT_{PQi} + a_i + UL_{PQi} + Q_i + LT_{QRi} + b_i + UL_{QRi}$$

$$F_i = LT_{PQi} + a_i + UL_{PQi} + Q_i + LT_{QRi} + b_i + UL_{QRi} + R_i$$

$$G_i = LT_{QRi} + b_i + UL_{QRi} + R_i + LT_{RSi} + c_i + UL_{RSi} + S_i$$

$$O_i = LT_{RSi} + c_i + UL_{RSi} + S_i + LT_{SVi} + d_i + UL_{SVi} + V_i$$

With conditions

1) Min ($Pi + LT_{PQi} + a_i + UL_{PQi}$) $\geq$ Max ($LT_{PQi} + a_i + UL_{PQi} + Q_i$)

2) Min ($LT_{QRi} + b_i + UL_{QRi} + R_i$) $\geq$ Max ($Q_i + LT_{QRi} + b_i + UL_{QRi}$)

3) Min ($LT_{RSi} + c_i + UL_{RSi} + S_i$) $\geq$ Max ($R_i + LT_{RSi} + c_i + UL_{RSi}$)

4) Min ($LT_{SVi} + d_i + UL_{SVi} + V_i$) $\geq$ Max ($S_i + LT_{SVi} + d_i + UL_{SVi}$)

Article	$E_i = P_i + LT_{PQi} + a_i + L_{PQi} + Q_i + LT_{QRi} + b_i + L_{QRi}$	$F_i = LT_{PQi} + a_i + UL_{PQi} + Q_i + LT_{QRi} + b_i + UL_{QRi} + R_i$	$G_i = LT_{QRi} + b_i + UL_{QRi} + R_i + LT_{RSi} + c_i + UL_{RSi} + S_i$	wt_i
A_1	E_1	F_1	G_1	wt_1
A_2	E_2	F_2	G_2	wt_2
A_3	E_3	F_3	G_3	wt_3
A_4	E_4	F_4	G_4	wt_4
A_5	E_5	F_5	G_5	wt_5

Step II: Now this problem will be reduced into two machine problem.

But first we have to convert it into three machines H, K, L and then by considering two fictitious machines M and N the problem will be converted into two machines problem.

Let $Hi = E_i + F_{i,}$, $Ki = F_i + G_i$, $Li = G_i + O_i$

Therefore M and N are represented as follows: $Mi = H_i + K_i$, $Ni = K_i + L_i$

Step III:

For getting the proper sequence the new problem is defined as follows;

$M'i = M'i/ wt_i$ and $N'i = N'i/ wt_i$ where

1) If min (M,N) = M_i then $M'_i = M_i - wt_i$ and $N'_i = N_i$

2) If min (M,N) = N_i then $M'_i = M_i$ and $N'_I = N_i + wt_i$

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Article	M'_i / wt_i	N'_i / wt_i	wt_i
A_1	M'_1 / wt_1	N'_1 / wt_1	wt_1
A_2	M'_2 / wt_2	N'_2 / wt_2	wt_2
A_3	M'_3 / wt_3	N'_3 / wt_3	wt_3
A_4	M'_4 / wt_4	N'_4 / wt_4	wt_4
A_5	M'_5 / wt_5	N'_5 / wt_5	Wt_5

Step IV: Considering the breakdown interval (u, v) and the effect of this time interval should be observed on all jobs. If the jobs are affected by this time interval then the difference of the interval will be added. With the effect of this breakdown interval, the problem will be redefined as follows:

1) If the job is affected by the breakdown interval then

$P'_i = P_i + (u-v)$, $Q'_i = Q_i + (u-v)$, $R'_i = R_i + (u-v)$ and $S'_i = S_i + (u-v)$

2) If the job is not affected by the breakdown interval then

$P'_i = P_i$, $Q'_i = Q_i$, $R'_i = R_i$ and $S'_i = S_i$

Step V: Applying steps I, II, III, IV the problem has been solved for getting the optimal sequence. The scheduling of all the course of action is in such a way that the minimum time should be required for getting the optimum solution or whole production For finding the algorithm the above information can be symbolized and represented in table for finding the sequence by using Johnson's rule of sequencing.

IV. NUMERICAL EXAMPLE

Ai	P_i	LT_{PQi}	a_i	UL_{PQi}	Q_i	LT_{QRi}	b_i	UL_{QRi}	R_i	LT_{RSi}	c_i	UL_{RSi}	S_i	LT_{SVi}	d_i	UL_{SVi}	V_i	wt_i
A_1	6	2	4	3	5	2	3	4	6	2	2	5	6	1	3	6	6	2
A_2	5	3	6	3	2	2	5	2	5	1	3	6	5	3	4	3	7	4
A_3	4	3	5	2	4	2	2	4	6	2	3	4	6	1	2	5	6	3
A_4	5	4	3	2	4	3	4	2	7	2	2	3	8	1	3	4	8	5
A_5	8	2	3	1	3	2	6	1	8	1	2	4	8	1	2	5	8	2

Step I: Suppose FOUR machines E, F , G and O are assumed with the service time E_i, F_i , G_i and O_i resp.

Ai	E_i	F_i	G_i	O_i	wt_i
A_1	29	29	30	31	2
A_2	28	28	29	32	4
A_3	26	28	29	29	3

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A_4	27	29	31	31	5
A_5	26	26	32	31	2

Step II:But first we have to convert it into three machines H, K, L and then by considering two fictitious machines M and N the problem will be converted into two machines problem.

Article	H	K	L	wt_i
A_1	58	59	61	2
A_2	56	57	61	4
A_3	54	57	58	3
A_4	56	60	62	5
A_5	52	58	63	2

Now, $Mi = H_i + K_i$, $Ni = K_i + L_i$

Ai	Mi	Ni	wt_i
A_1	117	120	2
A_2	113	118	4
A_3	111	115	3
A_4	116	122	5
A_5	110	121	2

Also,

Article	M'_i	N'_i	wt_i
A_1	115	120	2
A_2	109	118	4
A_3	108	115	3
A_4	111	122	5
A_5	108	121	2

Step III; Now the ratio of weights with Mi and Ni is taken so that we can decide the sequence of jobs according to Johnson's rule.

Article	M'_i / w_i	N'_i / w_i	wt_i
A_1	57.5	60	2
A_2	27.25	29.5	4
A_3	36	38.33	3
A_4	22.2	24.4	5
A_5	54	60.5	2

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By Johnson's rule the optimal sequence obtained for above reduced problem is 4, 2, 3, 5, 1. Then the time required for total processing of articles by using above scheduling sequence i.e minimum time for entire production can be calculated by considering the time required by the transporting agent when it returns back to machine $M_1$to load the next article and also the time when it reaches to machine M_2 for unloading of an article.

Ai	P_i		LT_{PQi}	a_i	UL_{PQi}	Q_i		LT_{QRi}	b_i	UL_{QRi}	R_i		LT_{RSi}	c_i	UL_{RSi}	S_i		LT_{SVi}	d_i	UL_{SVi}	V_i		wti
	I	O				I	O				I	O				I	O				I	O	
A_4	0	5	4	3	2	14	18	3	4	2	27	34	2	2	3	41	49	1	3	4	57	65	5
A_2	5	10	3	6	3	22	24	2	5	2	36	41	1	3	6	55	60	3	4	3	70	77	4
A_3	10	14	3	5	2	26	30	2	2	4	45	51	2	3	4	64	70	1	2	5	82	88	3
A_5	14	22	2	3	1	31	34	2	6	1	52	60	1	2	4	74	82	1	2	5	93	101	2
A_1	22	28	2	4	3	37	42	2	3	4	64	70	2	2	5	87	93	1	3	6	107	113	2

Let breakdown interval be [35, 41].The effect of breakdown interval is on jobs Q_i1 and R_i2, hence the original problem is converted into new problem.

If the job is affected by the breakdown interval then $P'_i = P_i + (u_2 - u_1)$, $Q'_i = Q_i + (u_2 - u_1)$, $R'_i = R_i + (u_2 - u_1)$ and $S'_i = S_i + (u_2 - u_1)$ Therefore repeating the same procedure of step 1,2,3 for finding the sequence and getting the optimal solution.

Ai	P_i	LT_{PQi}	a_i	UL_{PQi}	Q_i	LT_{QRi}	b_i	UL_{QRi}	R_i	LT_{RSi}	c_i	UL_{RSi}	S_i	LT_{SVi}	d_i	UL_{SVi}	V_i	wt_i
A_1	6	2	4	3	11	2	3	4	6	2	2	5	6	1	3	6	6	2
A_2	5	3	6	3	2	2	5	2	11	1	3	6	5	3	4	3	7	4
A_3	4	3	5	2	4	2	2	4	6	2	3	4	6	1	2	5	6	3
A_4	5	4	3	2	4	3	4	2	7	2	2	3	8	1	3	4	8	5
A_5	8	2	3	1	3	2	6	1	8	1	2	4	8	1	2	5	8	2

As given in step I

Ai	E_i	F_i	G_i	O_i	wt_i
A_1	35	35	30	31	2
A_2	28	34	35	32	4
A_3	26	28	29	29	3
A_4	27	29	31	31	5

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A_5	26	26	32	31	2

According to step II the problem will be converted into three machines problem and then into two machines problem.

Ai	$Hi = E_i + F_i$	$Ki = F_i + G_i$	$Li = G_i + O_i$	wt_i
A_1	70	65	61	2
A_2	62	69	67	4
A_3	54	57	58	3
A_4	56	60	62	5
A_5	52	58	63	2

Now,

Article	$Mi = H_i + K_i$	$Ni = K_i + L_i$	wt_i
A_1	135	126	2
A_2	131	136	4
A_3	111	115	3
A_4	116	122	5
A_5	110	121	2

Also,

1) If min (M,N) = M_i then $M'_i = M_i - wt_i$ and $N'_i = N_i$
2) If min (M,N) = N_i then $M'_i = M_i$ and $N'_I = N_i + wt_i$

Ai	M'_i	N'_i	wt_i
A_1	135	128	2
A_2	127	136	4
A_3	108	115	3
A_4	111	122	5
A_5	108	121	2

Step III; The ratio of weights with Mi and Ni is taken so that we can decide the sequence of jobs according to Johnson's rule.

Article	M'_i / wt_i	N'_i / wt_i	wt_i
A_1	67.5	64	2
A_2	31.75	34	4
A_3	36	38.33	3
A_4	22.2	24.4	5

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A_5	54	60.5	2

By Johnson's rule the optimal sequence obtained for above reduced problem is 4, 2, 3, 5, 1.

Ai	P_i		LT_{POi}	a_i	UL_{POi}	Q_i		LT_{QRi}	b_i	UL_{QRi}	R_i		LT_{RSi}	c_i	UL_{RSi}	S_i		LT_{SVi}	d_i	UL_{SVi}	V_i		wt_i
	I	O				I	O				I	O				I	O				I	O	
A_4	0	5	4	3	2	14	18	3	4	2	27	34	2	2	3	41	49	1	3	4	57	65	5
A_2	5	10	3	6	3	22	24	2	5	2	36	47	1	3	6	57	62	3	4	3	72	79	4
A_3	10	14	3	5	2	26	30	2	2	4	51	57	2	3	4	66	72	1	2	5	84	90	3
A_5	14	22	2	3	1	31	34	2	6	1	58	66	1	2	4	76	84	1	2	5	95	103	2
A_1	**22**	**28**	2	4	3	37	48	2	3	4	70	76	2	2	5	89	95	1	3	6	109	115	2

Minimum weighted flow time (MWFT)

= (65*5) + (79- 5) *4 + (90-4)*3 + (103-8)*2 + (115- 6)*2

= 325 + 296+ 258 + 190 + 218 / 5+4+3+2+2

= 1287 / 16

= 80 hours

V. CONCLUSION

From above table it is shown that the time gets reduced for total production by using the sequence obtained with the help of Johnson's rule. The total elapsed time for the complete process is 115 hrs. Also the minimum weighted flow time is 80 hrs. By using the above method of reduction of number of machines the algorithm is useful in finding the total make-span of the process. We can find the solution of the problem by considering the more number of jobs with same machines.

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