International Journal of Advance Research in Science and Engineering
Vol. No.6, Issue No. 03 , March 2017
www.ijarse.com
IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

DEA: A RECENT TREND IN MATHEMATICAL PROGRAMMING

Amit Bhatia

Department of Mathematics, Swami Premanand Mahavidyalaya, Mukerian, Punjab(India)

ABSTRACT

Transparent and more importantly measurable decision processes are the need of hour. Many social problems are confronted with evaluation procedures which involve their public and private decisions. Whether to improve quality of higher educations or understand concerning problems better or henceforth make good decisions, appropriate analysis tools are required. It is quite natural to assess by any similar body, whether the present investment of resources or adoption of policy is working fruitfully or not. Many methods and techniques for the design of transparent decision processes like Multi-Criteria Decision Analysis(MCDA) are available which clearly explain the rank-order positions of the alternatives, but they do not always indicate how a particular alternative could reasonably improve its position. This is typically a strong feature of Data Envelopment Analysis (DEA). DEA is the recent trend in mathematical programming which is developed for performance measurement and is being successfully employed for assessing the relative performance of a set of firms that use a variety of identical inputs to produce a variety of identical outputs.

Keywords: DEA, LPP, Mathematical Programming, Modeling.

I. INTRODUCTION

In mathematics, mathematical programming is the technique dealing with maximization of the desired objective from some set of available alternatives, keeping in mind the availability of limited resources. It is actually the most efficient use of resources to optimize(maximize or minimize) the objective function. In rigourous terms mathematical programming consists of maximizing or minimizing a real function under certain constraints i.e.

(1) : $Optimize\ f(X)$

$subject\ to\ g_i(X)\ (\leq or =\geq)\ 0,\ i = 1, 2, \ldots\ldots, m\ and\ X \geq 0$

$where\ X = [x_1\ x_2\ \ldots\ x_n]^T \epsilon R^n,$

$f(X)\ and\ g_i(X)$ **are real valued functions of** X

The problem is called a linear programming problem or LPP if the functions $f(X)$ and $g_i(X)$ are all linear. Nowadays a recent trend has emerged in the field of Mathematical Programmig which is known as Data envelopment analysis (DEA) in which productive efficiency of decision making units (or DMUs) is being measured empirically. It is a nonparametric, mathematics oriented method particularly a linear programming formulation that defines a correspondence between multiple inputs and multiple outputs. Since 1978, when the first paper on DEA was published, many articles, books and dissertations have appeared with wide applications in the field of education (schools, colleges and universities), banks, hospitals, clinics, agriculture, banking,

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

transportation, industry, and many others. In order to understand DEA better some glimpse of mathematical programming is necessary.

II. MATHEMATICAL PROGRAMMING

The 'Tableau economique' of Qnesney in the eighteenth centuary was the earliest attempt of studying programming problems as such. 'Tableau economique' or 'Economic Table' was an attempt to interrelate the rolls of three economic movers viz. landlords, peasants and artisans. So this table was also an attempt of econmists who felt the need of interpreting economic systems in mathematical language. After a gap of 150 years Walrasian equilibrium model of an economy came in 1874. Mathematical Programming emerged in its purest form when one of the crucial techniques of Mathematical Programming, Linear Programming (LP), was developed in 1939 by the Russian mathematician, Leonid Vitaliyevich Kantorovich (1912–1986). The advent of the 'New Deal' in the United States, the attempts to curve great economic depression and the need to provide necessary solutions to various strategic and tactical war problems during World War II, compelled everyone to find a systematic approach focusing on the specific decision making problems. As a result a new science named as Operations Research (O.R.) developed. In 1947, The American scientist, George Bernard Dantzig published the Simplex method. Afterwards the field of linear programming, together with its extensions (mathematical programming), grew by leaps and bounds. Linear programming problem also known as LPP may be defined as the problem of *maximizing or minimizing* a linear objective function subject to linear constraints. The constraints may be equalities or inequalities. Its feasible region come out to be a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Mathematically the LPP can be stated as:

(2) : $Maximize\ or\ minimize\ c^T X$

$subject\ to\ AX(\leq or =\geq)\ b,$

$and\ X \geq 0$

$where\ c = [c_1\ c_2\ \dots\ c_n]^T,\ b = [b_1\ b_2\ \dots\ b_n]^T,\ X = [x_1\ x_2\ \dots\ x_n]^T,$

$c,\ b,\ X \epsilon R^n,\ \ and\ A = \left[a_{ij}\right]_{m \times n}$

Afterwards further improvements to existing methods as well as new techniques related to LPP were discovered and developed. Dantzig, Lemke, Land and Doig, Gale, Kuhn and Tucker, Charnes, Cooper, and P. Wolfe contributed a lot in the development of various theoretical and computational aspects of linear programming.

III. DATA ENVELOPMENT ANALYSIS(DEA)

Transparent and more importantly measurable decision processes are the need of hour. Many social problems are confronted with evaluation procedures which involve their public and private decisions. Whether to improve quality of higher educations or understand concerning problems better or henceforth make good decisions, appropriate analysis tools are required. It is quite natural to assess by any similar body, whether the present investment of resources or adoption of policy is working fruitfully or not. Evidence can come in many forms but in the vast majority of cases will be reflected in some sort of quantitative measure. There is hardly any field in which the decision processes are not being measured. Many methods and techniques for the design of

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

transparent decision processes like Multi-Criteria Decision Analysis(MCDA) are available which clearly explain the rank-order positions of the alternatives, but they do not always indicate how a particular alternative could reasonably improve its position. This is typically a strong feature of Data Envelopment Analysis (DEA). DEA was originally developed for performance measurement and had been successfully employed for assessing the relative performance of a set of firms that use a variety of identical inputs to produce a variety of identical outputs. The principles of DEA date back to Farrel (1957). This technique aims to measure how efficiently a DMU uses the resources available to generate a set of outputs (Charnes et al. 1978). Besides firms or manufacturing units; universities, schools, banks, hospitals, power plants, police stations, defence bases, and even practising individual can be termed as Decision-making units. The concept of efficiency or productivity, which is the ratio of total outputs to total inputs, is fundamental in assessing the performance of DMUs. Further efficiencies estimated using DEA are *relative*, that is, relative to the best performing DMU or DMUs. The best-performing DMU is assigned an efficiency score of unity or 100 per cent, and the performance of other DMUs vary, between 0 and 100 percent relative to this best performance. DEA has certain advantages over other similar available technique. First of all it does not require any functional form of relationship between inputs and outputs. Further it does not require even any theoretical models (CAPM or APT) as measurement benchmarks. Instead DEA measures how well a DMU performs relative to the best set of DMUs within the declared objective category. Finally, one can use DEA to not only identify the inefficient firms, but also to estimate the magnitude of inefficiency. Such an analysis provide mathematical measures which calculate how to improve the efficiencies of inefficient firms. The DMUs which can not improve their performance are marked as inefficient or non-dominated. Otherwise the DMU can improve its performance by the convex combinations of more successful DMUs or in other words the improvement is pursued until the boundary of the convex hull of better DMUs is reached. These concepts can be undersood more clearly if represented in riguorous mathematical notations.

IV. MATHEMATICAL FORMULATION

Mathematically the general DEA CCR output miximization model is represented as

(3) : $Maximize_{I,O} Z = O_m^T Y_m$

$subject\ to$

$I_m^T X_m = 1, O_m^T Y - I_m^T X \leq 0,$

$O_m^T, I_m^T \geq \epsilon$

$where\ I\ and\ O\ denote\ input\ and\ output\ matrix\ respectively$

$and\ m\ denotes\ the\ mth\ DMU\ whose\ efficiency\ is\ to\ maximize$

This model is also known as Multiplier version of output miximization. Computationally more appropriate model is its dual input oriented envelopment version which is represented as

(4) : $Minimize_{\theta,\lambda} \theta_m$

$subject\ to$

$Y_\lambda \geq Y_m, \ X_\lambda \leq \ \theta_m X_m$

$\lambda \geq 0, \theta_m\ free$

These are the basic additive DEA models. Several variations of these basic models such as multiplicative DEA models have been proposed, as per the requirement.

V. CONCLUSION

THE GODS of the past are now faded. And they cannot be revived as they were. They have become less relevant to human consciousness; they were created by a less mature mind. Man has come of age. He needs a different vision of the gods; he needs a different kind of approach, because only then can the tomorrow become possible. Quest of optimization is the intrinsic quality of human consciousness and DEA is its recent manifestation.

REFERENCES:

[1] H. A. Eiselt, C. –L. Sandbloom, linear programming and its applications Springer, ISBN 978-3-540-73670-7.

[2] Kuhn HW and AW Tucker, "Nonlinear programming," in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, JNeyman (ed), University of California Press, Berkley, Calif, pp 481-492, 1950.

[3] Von Neumann J, "Uber ein okonomisches Gleichungssystem und ein Verallgemeinerung des Brouwerschen Fixpunktsatzes," Ergebnisse eines Mathematischen Kolloquiums, No 8, 1937. English translation, "A Model of General Economic Equilibrium," Rev Econ Studies, Vol. 13, pp 1-9, 1945-1946.

[4] G. Hadley, linear programming, Addison Wesley Publishing Company, 1962.