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ON H-ADHERENCE AND MINIMAL $S_{2½.}$ SPACES

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ABSTRACT

In this paper we study the H-adherence based in terms of hyperclosure, of a filterbase obtain characterizations of some of the spaces in terms of H-adherence. Also variant of Minimal-P spaces and its relationships with other axioms is investigated.

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I INTRODUCTION

The adherence of a filterbase has been used to characterize some of the well known topological spaces such as compact spaces, H-closed spaces and various minimal topological spaces. Various forms of adherence of filterbase have been discussed in literature and these have found much importance place in topology. The minimal and maximal topologies with respect to a certain topological property in the lattice of topologies LT(X) defined on a set X partially ordered by inclusion have been of keen interest of topologists. In [1] Berri et.al, have given a survey on the considerable amount of work done in the field of minimal topological spaces. In [2] Herrington, introduced u-adherence of a filterbase and characterized Urysohn closed and Minimal Urysohn spaces in terms of u-adherence.

In this paper we study a variant of adherence based on hyperclosure of a set, namely H-adherence of a filterbase and characterize some spaces in terms of H-adherence. Also we investigate the class of minimal topological spaces with respect to the separation axiom $S_{2½}$ and the role of H-adherence in characterization of these spaces.

Notations: Throughout the paper, by a space X we mean a topological space, $N_{\Im}(x)$ is the filterbase of $\Im$-neighborhoods of some $x \in X$, $cl_{\Im}$ (A) denotes the closure of the subset $A \subset X$, X\A the complement of A in X and **Z** denotes the set of integers.

Definition 1.1: (1) A filter base B on a space X is said to be an open filter if each member of B is an open set.

(2) A point x is called a *adherent point* or *accumulation point* or *cluster point* of a filterbase B denoted as $x \in$ adh B, if for each open sets U containing x and each F in B, $F \cap U \neq \varnothing$.

(3) A filterbase B *θ-converges* to x if for each open set U containing x there exists an F in B such that $cl(F) \subset U$.

(4) A point x is called a *θ-adherent point* of a filterbase B denoted as $x \in adh_{\theta}$ B, if for each open sets U containing x and each F in B, $F \cap cl(U) \neq \varnothing$.

Definition 1.2 [3]: If X is a space, $A \subset X$, and $x \in X$, then

(1) $ker(A) = \cap\{ U : U \in \Im \text{ and } A \subset U\}$.

(2) $< x > = cl\{x\} \cap ker\{x\}$.

(3) $cl_H(A) = \{x \in X :$ For each open set U containing x and each open set V containing A, $cl(U) \cap cl(V) \neq \varnothing\}$

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(4) A is *hyperclosed* if A= $cl_H(A)$.

Definition 1.3 [4]:(1) An open filterbase B is a *Urysohn filterbase* if and only if for each x not in adherence of B, there is an open set U containing x and V in B such that, cl(U) ∩ cl(V) = ∅.

(2) A pair of open sets G and H containing a point x is called an ordered pair denoted by

(G, H) if, x ∈ G ⊂ cl(G) ⊂ H.

(3) For any subset A ⊂ X, $cl_u(A)$ = {x ∈ X: For each ordered pair of open sets (G, H) containing x, A ∩ cl(H) ≠ ∅}.

(4) A filterbase B *u-converges* to x if for each ordered pair of open sets (G, H) containing x there exists an F in B such that F ⊂ cl(H).

(5) A point x is called a *u-adherent point* or *u-accumulation point* of a filterbase B denoted as x ∈ $adh_u B$, if for each ordered pair of open sets (G, H) containing x and each F in B, F ∩ cl(H) ≠ ∅.

(6) An Urysohn space X is *Urysohn-closed* provided X is a closed set in every Urysohn space in which it can be embedded.

(7) A space (X, ℑ) is said to be *Minimal Urysohn* if ℑ is Urysohn and there exists no Urysohn topology on X strictly weaker than ℑ.

Definition 1.4 [5]: A space X is said to be,

(1) *Urysohn* ($T_{2½}$) *space* if for every pair of points x and y in X there exist neighborhoods U of x and V of y such that cl(U) ∩ cl(V) = ∅.

(2) $S_{2½}$ *space* if for every pair of points x and y in X, whenever cl{x} ≠ cl{y} then there exist neighborhoods U of x and V of y such that cl(U) ∩ cl(V) = ∅.

(3) S_2 *space* if for every pair of points x and y in X, whenever cl{x} ≠ cl{y} then there exist disjoint neighborhoods containing them.

(4) S_1 *space* if for every pair of points x and y, whenever x has a neighborhood not containing y, then y has a neighborhood not containing x.

Definition 1.5: A space X is said to be,

(1) *θ - point paracompact* [6] if for each open covering 𝒰 of X and each x ∈ X there exists an open refinement 𝒱 of 𝒰 and a θ-open set U containing x which intersects with only finite members of 𝒱.

(2) H(i) [2] if every open filter on X has non void adherence.

(3) U(i) [2] if every Urysohn filterbase on X has non void adherence.

The following results will be used in the next section.

Lemma 1.6: A filter F θ-adheres to a point x if and only if there exists a finer filter G which θ-converges to the point x.

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Lemma 1.7 [7]: A space X is S_1 if and only if one of the following conditions is satisfied:

(a) If U is open in X and $x \in U$ then $cl\{x\} \subset U$.

(b) If $x, y \in X$, then $cl\{x\} = cl\{y\}$ or $cl\{x\} \cap cl\{y\} = \varnothing$.

Lemma 1.8: For a space X and $x, y \in X$, following statements are equivalent:

(a) X is $S_{2½}$.

(b) Either, $<x> = <y>$ or $cl_H\{x\} \cap cl_H\{y\} = \varnothing$.

Lemma 1.9: For a space X and $x, y \in X$, $<x> = <y>$ if and only if $cl\{x\} = cl\{y\}$.

Lemma 1.10: Let X be Urysohn space. Then X is Urysohn closed if and only if every open filterbase has u-adherent point.

Lemma 1.11: X is U(i) if and only if every open filterbase has u-adherent point.

II H-ADHERNCE

First we give the relationships between H-adherence and some forms of adherence of filterbases known in literature. For this purpose, we prove that u-closure and hyper-closure of a set coincide.

Lemma 2.1: For a space X and a subset A of X, $cl_uA = cl_HA$.

Proof: Let $x \in X$, $x \in cl_HA \backslash cl_uA$. Then there exists an ordered pair of open sets (G, H) containing x such that, $A \cap cl(H) = \varnothing$. Then, $V = X \backslash cl(H)$ is an open set containing A, suvh that $cl(V) \cap H = \varnothing$. As (G, H) is an ordered pair containing x, we have $cl(G) \cap cl(V) = \varnothing$ and this implies that $x \notin cl_HA$ which is a contradiction.

Conversely, let $p \in cl_uA \backslash S$. Then there exists an open set U containing p and an open set V containing A such that, $cl(U) \cap cl(V) = \varnothing$. Thus $(U, X \backslash cl(V))$ is an ordered pair of open sets containing p such that $V \cap (X \backslash cl(V)) = \varnothing$. Therefore, $A \cap (cl(X) \backslash cl(V)) = \varnothing$ which implies $x \notin cl_uA$ which is a contradiction.

Definition 2.2: A point x is called a *H-adherent point* of a filterbase B denoted as $x \in adh_HB$, if for each open set U_x containing x and each open set V_F containing F, $cl(U_x) \cap cl(V_F) \neq \varnothing$ for all F in B.

Theorem 2.3: For a space X and filterbase B of X the following hold:

(a) $adh_HB = \cap \{cl_H(F) | F \in B\}$.

(b) $adh_uB = adh_HB$.

Proof: Part (a) is straightforward. Part (b) follows from part (a) and Lemma 2.1, since $adh_uB = \cap \{cl_u(F) | F \in B\}$.

Theorem 2.4: For a space X and a subset A of X, the following are equivalent:

(a) $x \in cl_H(A)$.

(b) There exists a filter F such that $A \in F$ and $x \in adh_H F$.

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(c) There exists a filter F which *θ-converges* to both x and A.

(d) There exists a filter F which *θ-adheres* to both x and A.

Proof: (a) ⇒ (b) Let $x \in cl_H(A)$ and B ={A} be a filterbase and F (B) = {F | A ⊂ F } be the filter generated by B. Now $x \in adh_H B$ which implies $x \in adh_H F (B)$.

(b) ⇒ (c) Let F be filter such that A ∈ F and $x \in adh_H F$. Let P be a filterbase where, P = $\{cl(U_F) \mid U_F$ are open sets containing F for all F in F}. Now as $x \in adh_H F$ the G(P) the filter generated from the filterbase P, θ-adheres to x. By Lemma 1.6 there exists a finer filter R which θ-converges to x. As A ∈ F, $\{cl(U_A) \mid U_A$ are open sets containing A} ⊂ P ⊂ G ⊂ R. Hence the filter R θ-converges to A also.

(c) ⇒ (d) is obvious

(d) ⇒ (c) follows from Lemma 1.6

(c) ⇒ (a) Let F be a filter which *θ-converges* to both x and A. Then for every open set U containing x and every open set V containing A there exists F_1 and F_2 in F such that $F_1 \subset cl(U)$ and $F_2 \subset cl(V)$. Since F is a filter $F_1 \cap F_2 \in$ F and $F_1 \cap F_2 \subset cl(V) \cap cl(U)$ and hence $cl(V) \cap cl(U) \neq \varnothing$. Thus $x \in cl_H(A)$.

Corollary 2.5: A set A of X is hyperclosed if and only if A contains all the H-adherent points of every filter containing A as a member.

As it is known that every H(i) space is U(i), we give a condition in terms of H-adherence of open filters for which the converse holds.

Lemma 2.6: If X is θ-point paracompact then every open filter with H-adherent point has an adherent point.

Proof: Let X be a θ-point paracompact space and F be an open filter with an H-adherent point x and no adherent point. Then, $\mathcal{U}$ = {X\cl(F): F ∈ F} is a directed open cover of X and so there exists an open refinement $\mathcal{V}$ of $\mathcal{U}$ and a θ-open set K containing x which intersects with only finite members of $\mathcal{V}$. Let P = $\cup\{U \in \mathcal{V}: U \cap K = \varnothing\}$. Hence, $P \cap K = \varnothing$. Now, $X \setminus P \subseteq \cup\{U \in \mathcal{V}: U \cap K \neq \varnothing\}$ and as $\mathcal{V}$ is an open refinement of $\mathcal{U}$ there exists finitely many O_i in $\mathcal{U}$ such that $\cup\{U \in \mathcal{V}: U \cap K \neq \varnothing\} \subseteq \bigcup_{i=1}^{n} \{O_i : O_i \in \mathcal{U}\}$. Since $\mathcal{U}$ is an open directed cover there exists some F∈ F such that $\bigcup_{i=1}^{n} \{O_i : O_i \in \mathcal{U}\} \subseteq X \setminus cl(F)$. Thus $X \setminus P \subseteq X \setminus cl(F)$ which implies $cl(F) \subseteq P$. Therefore we have, $cl(F) \cap K = \varnothing$ and as K is a θ-open set, by definition there exists an open set Q containing x such that cl (Q) ⊂ K. So, $cl(Q) \cap cl(F) = \varnothing$ and as F being a member of an open filter is an open set we have $x \notin cl_H(F)$ which implies that x is not an H-adherent point of X, a contradiction.

Theorem 2.7: A θ-point paracompact space X is H(i) if and only if it is U(i).

Proof: As H(i) space always implies U(i), the result follows from Lemma 2.6 and Lemma 1.11 above.

III MINIMAL $S_{2½}$ SPACES

Definition 3.1[1] : Given a topological property P in the lattice LT(X) on a set X a topology is said to be *minimal* if every weaker topology in LT(X) does not possess that property.

In this context of Minimal spaces we define Minimal $S_{2½}$ space and characterize them in terms of H-adherence,

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Definition 3.2: A topological space (X, $\Im$) is said to *Minimal $S_{2½}$ space* if $\Im$ is the minimal element of the lattice LT(X) which is $S_{2½}$.

Remark 3.3 [5]: The separation axioms like $S_{2½}$ which are non T_1 are vacuously satisfied by the indiscrete topology and in such cases of separation axioms the study of minimal topological in the lattice LT(X) becomes trivial. So it becomes necessary to restrict the interval of topologies in the lattice LT(X) to avoid such circumstances.

So to each $\rho \in LT(X)$ we associate the interval as in [2], $L_\rho = \{\Im \in LT(X): at(\rho) \leq \Im \leq \bar{\rho}\}$ where at(ρ) denotes the topology on X generated by the sets $\{X \backslash cl_\rho(P)$: P is a finite subset of X$\}$ and $\bar{\rho}$ denotes the closure of ρ in the power set 2^X.

Remark 3.4 [5]: (1) If $\rho \in LT(X)$ then $cl_{at(\rho)}\{x\} = cl_\rho\{x\} = cl_{\bar{\rho}}\{x\}$ for all $x \in X$.

(2) If $\Im, \rho \in Lt(X)$ then $\Im \in L_\rho$ if and only if $cl_\rho\{x\} = cl_\Im\{x\}$ for all $x \in X$.
(3) If ρ is S_1 and if X can be written as finite disjoint point closures then for each point $x \in X$, $cl\{x\} \in at(\rho)$ which leads to the consequence $at(\rho) = \rho = \bar{\rho}$ and thus $L_\rho = \{\rho\}$. To avoid this condition of triviality, in the further Section we assume that $\rho \in LT(X)$ is an S_1 topology such that X can be written as infinite union of disjoint point closures.

As every pair of non-empty at(ρ) - open sets intersect, at(ρ) cannot be $S_{2½}$. So the restriction L_ρ is well defined for characterizing minimal $S_{2½}$ spaces.

Lemma 3.5: A space (X, $\Im$) is $S_{2½}$ if and only if it is S_1 and $adh_H N_\Im(x) = cl\{x\}$ for all $x \in X$.
Proof: Let (X, $\Im$) be an $S_{2½}$ space. Hence it is S_1 also. Let $x \in X$ and by Lemma 1.7 in an S_1 space $cl\{x\} \subset V$ for all $V \in N_\Im(x)$. So, $cl\{x\} \subset cl_H V$ for all $V \in N_\Im(x)$ and thus $cl\{x\} \subset adh_H N_\Im(x)$. Now let us suppose some $y \in X$ be such that $y \in adh_H N_\Im(x) \backslash cl\{x\}$. Now $y \in adh_H N_\Im(x)$ implies that $y \in cl_H(V)$ for all $V \in N_\Im(x)$. Since V is an open set so it implies that for all $U \in N_\Im(y)$, $cl(U) \cap cl(V) \neq \emptyset$ for all $V \in N_\Im(x)$. As $cl\{x\} \neq cl\{y\}$ and the space is $S_{2½}$, it is not possible. Thus it follows that $adh_H N_\Im(x) = cl\{x\}$ for all $x \in X$ as x was chosen arbitrarily.
Conversely, suppose (X, $\Im$) is S_1 and $adh_H N_\Im(x) = cl\{x\}$ for all $x \in X$. Let x and y be such that $<x> \neq <y>$. Then by Lemma 1.9 we have $cl\{x\} \neq cl\{y\}$. Since space X is S_1 by Lemma 1.7 $cl\{x\} \cap cl\{y\} = \emptyset$. Thus $adh_H N_\Im(x) \cap adh_H N_\Im(y) = \emptyset$ which implies $cl_H\{x\} \cap cl_H\{y\} = \emptyset$. So by Lemma 1.8 the space (X, $\Im$) is $S_{2½}$.

Theorem 3.6: Let $\Im \in L_\rho$ be $S_{2½}$. Then the topology $\Im$ is Minimal $S_{2½}$ if and only if given any $\Im$-open filter F on X such that $adh_{H(\Im)} F = cl_\Im\{x\}$ for some $x \in X$, is convergent
(necessarily to every point of $cl_T\{x\}$).
Proof: Suppose $\Im$ is Minimal $S_{2½}$ and F is a $\Im$-open filter on X such that $adh_{H(\Im)}F = cl_\Im\{x\}$ for some $x \in X$ and which is not convergent to some point $z \in cl_\Im\{x\}$. Since $\Im$ is $S_{2½}$ and hence S_1 so $cl_\Im\{x\} = cl_\Im\{z\}$. So we can assume that F is not convergent to the point x.
Let us consider the topology δ generated by the neighborhood base,

$$U'(y) = \begin{cases} U, \text{where } U \in N_\tau(y), y \neq x \\ U \cup F, \text{where } U \in N_\tau(y) \text{ and } F \in F, y = x \end{cases}.$$

Then δ is a topology which is strictly coarser than $\Im$. We shall prove that δ is an $S_{2½}$ in L_ρ.

(1) $\underline{\delta \in L_\rho}$: Since $adh_{H(\Im)}F = cl\{x\}$ and $y \notin cl_T\{x\}$ then $y \notin adh_{H(\Im)}F$. So there exists a $V \in N_\Im(y)$ and $F \in F$ such that $cl(V) \cap cl(F) = \emptyset$. Since $cl_T\{y\} \subseteq cl(V)$ thus $F \subseteq X \backslash cl_T\{y\}$. Therefore $X \backslash cl_T\{y\} \in F$ and hence $N_{at(\rho)}(x) \subseteq F$. Also, if $N_{at(\rho)}(x) \subseteq F$ then $N_{at(\rho)}(x) \subseteq N_\delta(x)$ for all $x \in X$ which implies $\delta \in L_\rho$.

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(2) δ is $S_{2½}$: Now let y, z $\in$ X be distinct points from given x such that $y \notin cl_\delta\{z\}$. Thus $y \notin cl_\Im\{z\}$ and since $\Im$ is $S_{2½}$ there exists $\Im$ -open sets $U_1 \in N_\Im(y)$ and $V_1 \in N_\Im(z)$ such that,
$cl_\Im(U_1) \cap cl_\Im(V_1) = \varnothing$.

Case I: If $x \notin cl_T\{y\}$ and $x \notin cl_\Im\{z\}$ then $y, z \notin cl_\Im\{x\}$. Then $y, z \notin adh_{H(\Im)}F$ and so there exists $F_1, F_2 \in$ F and $U_2 \in N_\Im(y)$ and $V_2 \in N_\Im(z)$ such that, $cl_\Im(U_2) \cap cl_\Im(F_1) = \varnothing$ and $cl_\Im(V_2) \cap cl_\Im(F_2) = \varnothing$. Now since $cl_\Im\{x\} \neq cl_\Im\{y\}$ and $cl_\Im\{x\} \neq cl_\Im\{z\}$ and $\Im$ is $S_{2½}$ there exists $U_3 \in N_\Im(y), V_3 \in N_\Im(z)$ and $W_1, W_2 \in N_\Im(x)$ such that, $cl_\Im(U_3) \cap cl_\Im(W_1) = \varnothing$ and $cl_\Im(V_3) \cap cl_\Im(W_2) = \varnothing$. Then $O_y = U_1 \cap U_2 \cap U_3$ and $O_z = V_1 \cap V_2 \cap V_3$ are δ-open sets such that $cl_\Im(O_y) \cap cl_\Im(O_Z) = \varnothing$. Since $cl_\delta(O_y) = cl_\Im(O_y)$ and $cl_\delta(O_Z) = cl_\Im(O_Z)$ we have $cl_\delta(O_y) \cap cl_\delta(O_Z) = \varnothing$.

Case II: If $x \in cl_\Im\{y\}$ then $cl_\Im\{x\} = cl_\Im\{y\} = adh_{H(\Im)}F$. Since $z \notin cl_\Im\{y\}$ there exist $V_4 \in N_\Im(z)$ and $F_3 \in$ F such that $cl_\Im(V_4) \cap cl_\Im(F_3) = \varnothing$. Since $cl_\Im\{x\} \neq cl_\Im\{z\}$ $W_3 \in N_\Im(x)$ and $V_5 \in N_\Im(z)$ such that, $cl_\Im(W_3) \cap cl_\Im(V_5) = \varnothing$. Then $P_y = W_3 \cup F_3$ and $P_z = V_4 \cap V_5$ are δ-open sets such that $cl_\Im(P_y) \cap cl_\Im(P_Z) = \varnothing$. Since $cl_\delta(P_y) = cl_\Im(P_y)$ and $cl_\delta(P_Z) = cl_\Im(P_Z)$ we have $cl_\delta(P_y) \cap cl_\delta(P_Z) = \varnothing$.

Thus δ is an $S_{2½}$ topology coarser than $\Im$ in L_ρ which is a contradiction and the result follows.

Conversely, let every $\Im$-open filter F on X such that $adh_{H(T)}F = cl_\Im\{x\}$ for some $x \in X$, is convergent. Let $\Im^* \in L_\rho$ be $S_{2½}$ such that $\Im^* \leq \Im$. Let $x \in U$ for some set $U \in \Im$. Then $adh_{H(\Im^*)} N_{\Im^*}(x) = cl_\Im\{x\}$and so $\Im$-open filter $N_{\Im^*}(x)$ is $\Im$-convergent to x. So $N_\Im(x) \subseteq N_{\Im^*}(x)$ and hence $U \in \Im^*$. As U was chosen arbitrarily we have $\Im^* = \Im$ and the result follows.

As a consequence of Theorem 3.6 we obtain that every Minimal $S_{2½}$ space is U(i).

Corollary 3.7: If $\Im \in L_\rho$ is Minimal $S_{2½}$ then it is U(i).

Proof: Suppose there exists an open filter F such that $adh_{H(\Im)}F = \varnothing$. Then for each $x \in X$ there exists a $V \in N_\Im(x)$ and $F \in$ F such that $cl_\Im(V) \cap cl_\Im(F) = \varnothing$. On the other hand since $cl_\Im\{x\} \subset V$ then $X \backslash cl_\Im\{x\} \in$ F. Thus $N_{at(\rho)}(x) \subseteq$ F and $N_\Im(x) \nsubseteq$ F for each $x \in X$. Now fix $x \in X$ and δ be defined as in Theorem 3.6 above. Then $\delta \in L_\rho$ and $\delta \leq \Im$. For proving δ is $S_{2½}$ we shoe that F contains an open filter F* such that $adh_{H(\Im)}F^* = cl_\Im\{x\}$.

Let F* ={F $\in$ F: $cl_\Im(U) \cap cl_\Im(F) \neq \varnothing$ for all $U \in N_\Im(x)$}. Then F* is a proper sub-filter of F and $cl_\Im\{x\} \subset adh_{H(\Im)}F^*$. Now let $y \notin cl_\Im\{x\}$ and as $\Im$ is $S_{2½}$ there exists $\Im$ -open sets $U_1 \in N_\Im(x)$ and $V_1 \in N_\Im(y)$ such that, $cl_\Im(U_1) \cap cl_\Im(V_1) = \varnothing$. Since $y \notin adh_{H(\Im)}F$ there exist $W_1 \in N_\Im(y)$ $F_1 \in$ F such that, $cl_\Im(W_1) \cap cl_\Im(F_1) = \varnothing$. Then $O = W_1 \cap V_1$ and $G = U_1 \cup F_1$ are such that $cl_\Im(O) \cap cl_\Im(G) = \varnothing$ where $O \in N_T(y)$ and $G \in$ F*. So $y \notin adh_{H(\Im)}F^*$ and hence $adh_{H(\Im)}F^* = cl_\Im\{x\}$. Then by similar steps as in Theorem 3.6 δ is $S_{2½}$ which implies that $\Im$ is not Minimal $S_{2½}$ which is a contradiction and the result follows.

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