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PRIORI METHODS- A PRACTICAL APPROACH

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ABSTRACT

Multi Objective Optimization method is one of the most powerful tools of Operational Research. It is originally grew out of three areas: Economics Equilibrium and Welfare Theories, Game Theory, and Pure Mathematics.There are mainly three categories of methods:Methods with a Priori Articulation, Methods with Posteriori Articulation, Methods with No Articulation.Present paper deals with discussion of Priori methods. Advantages, Disadvantages and Applications of every method under Priori are discussed. Conclusions are drawn that reflects, often neglected ideas and applicability to some problems. It is found that no single approach is superior. The selection of specific method depends on the type of the information provided in the problems.

***Keywords**: Economic Equilibrium, Game Theory, Multi-objective Optimization, Priori Method, Welfare Theories .*

I. DEFINITION OF MULTI-OBJECTIVE OPTIMIZATION PROBLEM

When a Mathematical Optimization problem involve more than one objective functions which are to be optimized simultaneously. Such problem is called Multi-objective optimization problem.

The Multi-objective optimization problem is of following type:

$$\min_x F(x) = \left[F_1(x), F_2(x), \ldots\ldots. F_k(x)\right]^T$$

Subject to constraints $g_j(x) \leq 0,\ j = 1,2,\ldots\ldots\ldots m$,

$h_l(x) = 0, l = 1,2,\ldots\ldots\ldots\ldots e,$

Where 'k' is number of objective functions, 'm' is number of inequality constraints. $x \in E^n$ is a vector of decision variables, where 'n' is the number of independent variables x_i $F(x) \in E^K$ is a vector of objective functions $F_i(x): E^n \to E^1$, $F_i(x)$ are also called objective functions.

The feasible design space **X** is defined as the set

$$\{x : g_j(x) \le 0,\ j = 1,2,.........m \text{ and } \quad h_l(x) = 0, l = 1,2,..........e, \}.$$

The feasible criterion space **Z** is defined as the set $\{F(x) : x \in X\}$.This is also known as attainable set or cost space. Here **Z** is used to indicate points in the criterion space those are feasible means all constraints are satisfied and also attainable.

The main motive of Multi-objective optimization is to model a decision maker's preferences.

HERE ARE SOME BASIC CONCEPTS GIVEN BELOW:

1.1 Pareto Optimal Point

In Multi-objective optimization problem there does not exist single solution that optimizes all objective functions simultaneously, there exists a number of Pareto Optimal solutions. A point is called Pareto Optimal if it cannot improve an objective function without degrading at least one of the other objective functions.

i.e. A point $x^* \in X$ is Pareto optimal if and only if there does not exist another point $x \in X$, such that

$$F(x) \le F(x^*) \text{ and } \quad F_i(x) < F_i(x^*)$$ for at least one function.

1.2 Weakly Pareto Optimal Point

A point is weakly Pareto optimal point if there is no other point that improves all the objective functions simultaneously. In other words,

A point $x^* \in X$, is weakly Pareto optimal if and only if there does not exist another point, $x \in X$,such that $F(x) < F(x^*)$

1.3 Utility Function

Utility function represents an individual's degree of contentment. In terms of multi-objective optimization, an individual utility function is defined for each objective and represents relative importance of objective. It approximates the preference function.

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1.4 Utopia Point

A point $F^0 \in Z^K$,is Utopia point if for each

i=1,2,3,……,k, $\quad F_i^0 = \min_x \{F_i(x) : x \in X\}.$

F^0 is generally unattainable. The solution that is as close as possible to the utopia point is called compromise solution and is Pareto optimal. The close means minimizes the Euclidean distance N(x), which is defined as follows:

$$N(x) = \left|F(x) - F^0\right| = \left\{\sum_1^k \left[F_i(x) - F_i^0\right]^2\right\}^{\frac{1}{2}}$$

II. METHODS WITH PRIORI ARTICULATION OF PREFERENCES:

In priori articulation methods preferences of decision maker are asked and best solution according to given preferences are calculated.In these methods decision maker specify preferences i.e. relative importance of different objective functions. Some methods use parameters such as exponents, constraints, limits etc. If one considers more than one objective function additional degree of freedom will be introduced and we get a set of solution points instead of a single solution point if degrees of freedom are not constrained. For imposing such constraints, one has to develop utility functions.

2.1 Weighted Global Criterion Method

In weighted global criterion method all objective functions have to be combined to form a single function. A weighted global criterion is a type of utility function in which method parameters are used to model preferences. Here is given general utility function as weighted exponential sum:

$$U = \sum_{i=1}^k w_i [F_i(x)]^p, F_i(x) > 0 \forall i \qquad \ldots(1)$$

$$U = \sum_{i=1}^k \left[w_i F_i(x)\right]^p, F_i(x) > 0 \forall i. \qquad \ldots(2)$$

The extension of (1) and (2) are :

$$U = \left\{\sum_{i=1}^k w_i \left[F_i(x) - F_i^0\right]^p\right\}^{\frac{1}{p}}, \qquad \ldots(3)$$

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$$U=\left\{\sum_{i=1}^{k} w_i^{\,p}\left[F_i(x)-F_i^{\,0}\right]^p\right\}^{\frac{1}{p}} \qquad \ldots(4)$$

Here $\boldsymbol{w}$ is a vector of weights such that $\sum_{i=1}^{k} w_i = 1$ and **w> 0.**

Taking one or more of the weights equal to zero can result in weak Pareto Optimality where Pareto Optimality may be achievable.

Summation argument in (3) and (4) can be explained as the component of distance between solution point and Utopia point in the criteria space. That is the reason that this method is also known as Utopia point method. Also decision maker has to compromise between final solution and Utopia point. So, Compromise programming method is another name of Global criteria method. For computational efficiency or in cases where a function's independent minimum may be unattainable, one may approximate utopia point by **z,** which is called an *aspiration point, reference point* or *target point.* In this case, we say that U is achievement function.

The solution to these approaches depends on the value of 'p', which is proportional to the amount of emphasis placed on minimizing the function with the largest difference between $F_i(x)$ and $F_i^{\,0}$.

2.1.1 Advantages of Global Criterion Method

- It clearly interpret the minimizing distance from utopia point
- It gives general formulation.
- It allows multiple parameters to be set to reflect preferences.
- It always gives a Pareto optimal solution when Utopia point is used.

2.1.2 Disadvantages of Global Criterion Method

- It is comparative more time consuming and computationally lengthy as use of utopia Point requires minimization of each objective function.
- Use of aspiration point requires that it be in order to yield a Pareto optimal solution.
- Setting of parameters is not intuitively clear when only one solution is desired.

2.2 Weighted sum method

The most common approach to multi-objective optimization is weighted sum method:

$$U=\sum_{i=1}^{k} w_i F_i(x) \qquad \ldots (5)$$

Which can be obtained from (1) or (2) with p = 1. If all of the weightsare positive, the minimum of (5) is Pareto optimal. Weights are mathematically related topreferences function of decision maker.With *ranking methods,* the different objectivefunctions are ordered by importance. The least important objective function receives a weight of one and increased weights are assigned to objectives which are more important.

The same approach is used with *categorization method*

2.2.1 Advantages of Weighted Sum Method

- This method is very simple.
- It is very easy to use.
- For convex problems it guarantees to find solution on entire Pareto-optimal set.

2.2.2 Disadvantages of Weighted Sum Method

- For mixed optimization problem we need to convert all objective functions into one type.
- A uniformly distributed weight does not guarantee a uniformly distributed set of Pareto optimal solutions.
- Two different set of weight vectors not necessarily lead to two Pareto optimal solutions.
- It is impossible to obtain points in non-convex portionsof the Pareto optimal set in the criterion space.
- Varying the weights consistently and continuouslymay not necessarily result in an even distribution ofPareto optimal points.

2.3 Lexicographic Method

With the lexicographic method, the objective functions are arranged in order of importance. Then, the following optimization problems are solved one at a time:

$$\min_{x \in X} imize \; F_i(x)$$

Subject to $F_j(x) \leq F_j(x_j^*)$, j=1, 2, ………….., i-1. i> 1. i=1, 2, 3, ….k.

Here *'i'* represents function's sequence according to the preference and $F_j(x_j^*)$ represents optimum of the j^{th} objective function, found in j^{th} iteration.

2.3.1 Advantages of Lexicographic Method

- It is unique approach to specifying preferences.
- It does not require that objective function should be normalized.
- It always provides a Pareto optimal solution.

2.3.2 Disadvantages of Lexicographic Method

- It requires the solutions of many single objective problems to obtain just one solution point.
- It requires additional constraints to be imposed.
- It can be expensive as it is most effective when used with global optimization engine.

2.4 Goal Programming Mrthod

In Goal programming method difference of jth objective function from specified goals b_j are considered. If d_j is the deviation from the goal d_j. Here $\sum_{j=1}^{k} |d_j|$.

d_j is split into positive and negative parts such that $d_j = d_j^+ - d_j^-$ where $d_j^+ \geq 0$, $d_j^- \geq 0$ and $d_j^+ d_j^- = 0$, $|d_j| = d_j^+ + d_j^-$, d_j^+ and d_j^- represents underachievement and overachievement respectively, where achievement means goal has been reached.

The optimization problem is $\min_{x \in X, d^+, d^-} imize \sum_{i=1}^{k} (d_i^+ + d_i^-)$

Subject to $F_j(x) + d_j^+ - d_j^- = b_j$, j=1,2,3,…..k,

$d_j^+ d_j^- = 0$ j=1, 2, 3 , ….. k

2.4.1 Advantages of Goal Programming

- It allows for ordinal ranking of goals where the low priority goals are considered after high priority goals are satisfied.
- This method is useful in situations where there is conflict in multiple goals and these multiple goals cannot be achieved fully.
- This method is suitable to find satisfactory solution where many objectives are to be considered.

2.4.2 Disadvantages of Goal Programming

- It takes time in construction of models hence it is comparative more time consuming.
- Require more decision makers.
- Giving weights to priority level to goal are subjective.
- Taking deviations is also a matter of concern.

2.4.3 Application of Goal Programming

Goal programming is extension of linear programming. It was first developed by A. Charnes and W. W. Cooper. Goal programming has wide applications in areas like academic planning, administration, accounting, financial planning, military strategies and planning, urban planning, predicting student performances, energy resource planning, economic-policy analysis.

III. CONCLUSION

In general, multi-objective optimization requires more computational effort than single-objective optimization. Unless preferences are irrelevant or completely understood, solution of several single objective problems may be necessary to obtain an acceptable final solution. Solutions obtained with no articulation of preferences are arbitrary relative to the Pareto optimal set. In this class of methods, the objective sum method is one of the most computationally efficient, easy-to-use, and common approaches. Consequently, it provides a benchmark approach to multi objective optimization. Methods with a priori articulation of preferences require the user to specify preferences only in terms of objective functions. Methods that provide both necessary and sufficient conditions for Pareto optimality are preferable. When one is interested in determining a single solution, the advantages of obtaining only Pareto optimal solutions are clear. Most General question is which method should be used? The answer to this question hinges, in part, on how accurately one is able to approximate the preference function. Physical programming is effective in this respect. Whereas a weight represents the simplest form of an individual utility function

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