IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

TWO DIMENSIONAL DEFORMATION IN A HOMOGENEOUS MICROSTRETCH THERMOELASTIC MEDIUM

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ABSTRACT

The present investigation deals with the 2-dimensional deformation in a homogeneous microstretch thermoelastic medium with mass diffusion subjected to inclined loads. The normal mode analysis technique is used to obtain the components of displacement, micro rotation, micro stress, mass concentration and temperature change due to normal force and tangential force. The computed numerical results are shown graphically to depict the effect of microstretch, diffusion and relaxation times. Some particular cases of interest are also deduced from the present investigation.

Keywords: Inclined Loads, microstretch, thermoelastic, normal mode analysis technique.

I. INTRODUCTION

The micropolar theory of elasticity and theory of micropolar elastic solids with stretch was presented by Eringen [2]. He derived the equations of motion, constitutive equations and boundary conditions for a class of micropolar solids which can stretch and contract. This model introduced and explained the motion of certain class of rigid chopped fibers, liquid crystals, granular and composite materials. This theory considered the intrinsic rotations of the microstructures and supported body and surface couples. Eringen [3] developed a theory of thermo-microstretch elastic solids in which he included micro structural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translation and rotations. For example, composite materials reinforce with chopped elastic fibers, porous medium with pores filled with gases, asphalt or other inclusions are characterized as microstretch solids. A comprehensive review on this subject is given by Eringen [4]. Microstretch theory is generalization of the theory of micropolar elasticity and a special case of micromorphic theory. Thus a microstretch elastic solid posses seven degrees of freedom, three for translation, three for rotation (as in micropolar elasticity) and one for stretch, required by substructures. Such a model can catch more information about the micro deformation inside a material point, which is more suitable for modeling the overall properties of the foam matrix in the case of foam composites. Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low-concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law which does not take into consideration the mutual interaction between

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

the introduced substance and the medium into which it is introduced or the effect of temperature on this interaction. However, there is a certain degree of coupling with temperature and temperature gradients as temperature speeds up the diffusion process. The thermo diffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [5-8] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [9] and Olesiak and Pyryev [10], respectively, discussed the theory of thermo diffusion and coupled quasi stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross-effects arising from the coupling of the fields of temperature, mass diffusion, and strain due to which the thermal excitation results in additional mass concentration and that generates additional fields of temperature. Cicco [11] discussed the stress concentration effects in microstretch elastic bodies. Ezzat and Awad [12] adopted the normal mode analysis technique to obtain the temperature gradient, displacement, stresses and micro-rotation. Kumar et al [13] investigated the disturbance due to force in normal and tangential direction and porosity effect by using normal mode analysis in fluid saturated porous medium. Fundamental solution in the theory of thermomicrostretch elastic diffusive solids was developed by Kumar and Kansal [14].This present problem deals with the 2-dimensional deformation in a homogeneous microstretch thermoelastic medium with mass diffusion due to inclined loads. The normal mode analysis technique is used to obtain the expressions for the displacement components, couple stress, temperature, mass concentration and micro stress.

II. BASIC EQUATIONS

Following Eringen [3], the basic equations are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\ddot{\boldsymbol{u}} \quad (1.1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}} \quad (1.2)$$

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+\nu_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=\frac{\rho j_0}{2}\ddot{\phi}^* \quad (1.3)$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\beta_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla.\boldsymbol{u}+\nu_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C \quad (1.4)$$

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0 \quad (1.5)$$

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}$$

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

(1.6)

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} + b_0\epsilon_{mji}\phi^*_{,m}$$

Where symbols have their usual meanings

2.1 Formulation of the problem

We consider a micropolar generalized thermoelastic with mass diffusion medium with rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3-axis pointing vertically downward the medium. For two dimensional problems the displacement vector and micro rotation vector has been considered of the form:

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \qquad (2.1)$$

For further consideration it is convenient to introduce in equations (1.1)-(1.5) the dimensionless quantities defined as:

$$u_r' = \frac{\rho\omega^* c_1}{\beta_1 T_0} u_r \ , \quad x_r' = \frac{\omega^*}{c_1} x_r \ , \quad t' = \omega^* t \ , \quad \phi^{*'} = \frac{\rho c_1^2}{\beta_1 T_0}\phi^* \ , \quad T' = \frac{T}{T_0} \ , \quad \tau_1' = \omega^*\tau_1 \ ,$$
$$\tau_0' = \omega^*\tau_0 \ , \gamma_1' = \omega^*\gamma_1,$$

$$t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij} \ , \omega^* = \frac{\rho c^* c_1^2}{K^*}, \quad \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i \ , \quad \tau^{1'} = \omega^*\tau^1 \ , \quad c_1^2 = \frac{\lambda + 2\mu + k}{\rho} \ , \quad c_2^2 = \frac{\mu + k}{\rho} \ ,$$
$$c_3^2 = \frac{\gamma}{\rho j}, \quad c_4^2 = \frac{2\alpha_0}{\rho j_0},$$

$$\varepsilon = \frac{\gamma^2 T_0}{\rho^2 c^* c_1} \ , \ m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij} \ , \quad C' = \frac{\beta_2}{\rho c_1^2} C. \qquad \textbf{(2.2)}$$

With the aid of equation (2.2) and (2.3), the equations (1.1)-(1.5) reduce to:

$$a_1\frac{\partial e}{\partial x_1} + a_2\nabla^2 u_1 - a_3\frac{\partial\phi_2}{\partial x_3} + a_4\frac{\partial\phi^*}{\partial x_1} - \left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} - a_5\left(1 + \tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_1} = \ddot{u}_1, \qquad (2.3)$$

$$a_1\frac{\partial e}{\partial x_3} + a_2\nabla^2 u_3 + a_3\frac{\partial\phi_2}{\partial x_1} + a_4\frac{\partial\phi^*}{\partial x_3} - \left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} - a_5\left(1 + \tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_3} = \ddot{u}_3, \qquad (2.4)$$

$$\nabla^2\phi_2 - 2a_6\phi_2 + a_6\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = a_7\ddot{\phi}_2, \qquad (2.5)$$

$$\nabla^2\phi^* - a_8\phi^* - a_9 e + a_{10}\left(1 + \tau_1\frac{\partial}{\partial t}\right)T + a_{11}\left(1 + \tau^1\frac{\partial}{\partial t}\right)C = a_{12}\ddot{\phi}^*, \qquad (2.6)$$

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

$$\nabla^2 T = a_{13}\left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + a_{14}\left(\frac{\partial}{\partial t} + \varepsilon\tau_0 \frac{\partial^2}{\partial t^2}\right) e + a_{15}\left(\frac{\partial}{\partial t} + \varepsilon\tau_0 \frac{\partial^2}{\partial t^2}\right)\phi^* + a_{16}\left(\frac{\partial}{\partial t} + \gamma_1 \frac{\partial^2}{\partial t^2}\right) C \quad , \quad (2.7)$$

$$\nabla^2 e + a_{17}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla^2 T + a_{18}\left(\frac{\partial}{\partial t} + \varepsilon\tau^0 \frac{\partial^2}{\partial t^2}\right) C - a_{19}\left(1 + \tau^1 \frac{\partial}{\partial t}\right)\nabla^2 C = 0, \qquad (2.8)$$

The displacement components u_1 and u_3 are related to potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}\ , \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \qquad (2.9)$$

Using the relation (2.9), in the equations (2.3)-(2.8), we obtain:

$$(a_1 + a_2)\nabla^2\phi - \ddot{\phi} + a_4\phi^* - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T - a_2\left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = 0, \quad (2.10)$$

$$\left(\nabla^2 - a_8 - a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^* - a_9\nabla^2\phi + a_{10}\left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + a_{11}\left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = 0, \qquad (2.11)$$

$$a_{13}\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\dot{T} + a_{14}\left(\frac{\partial}{\partial t} + \varepsilon\tau_0 \frac{\partial^2}{\partial t^2}\right)\nabla^2\phi + a_{15}\left(1 + \varepsilon\tau_0 \frac{\partial}{\partial t}\right)\dot{\phi^*} + a_{16}\left(1 + \gamma_1 \frac{\partial}{\partial t}\right)\dot{C} - \nabla^2 T = 0 \quad , \quad (2.12)$$

$$\nabla^4\phi + a_{17}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla^2 T + a_{18}\left(\frac{\partial}{\partial t} + \varepsilon\tau^0 \frac{\partial^2}{\partial t^2}\right) C - a_{19}\left(1 + \tau^1 \frac{\partial}{\partial t}\right)\nabla^2 C = 0, \qquad (2.13)$$

$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0, \qquad (2.14)$$

$$\nabla^2\phi_2 - 2a_6\phi_2 - a_6\nabla^2\psi = a_7\ddot{\phi}_2, \qquad (2.15)$$

III. SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi, \psi, T, \phi_2, \phi^*, C\}(x_1, x_3, t) = \{\bar{\phi}, \bar{\psi}, \bar{T}, \overline{\phi_2}, \overline{\phi^*}, \bar{C}\}(x_3) e^{i(kx_1 - \omega t)} \qquad (3.1)$$

Here ω is the angular velocity and k is a complex constant.

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

Making use of (3.1), the relations (2.10)-(2.15) yields:

$$\left[b_1\left(\frac{d^2}{dx_3{}^2}-k^2\right)+\omega^2\right]\bar{\phi}+b_2\overline{\phi^*}+b_3\bar{T}+b_4\bar{C}=0, \qquad (3.2)$$

$$b_5\left(\frac{d^2}{dx_3{}^2}-k^2\right)\bar{\phi}+\left(\frac{d^2}{dx_3{}^2}+b_6\right)\overline{\phi^*}+b_7\bar{T}+b_8\bar{C}=0, \qquad (3.3)$$

$$b_9\left(\frac{d^2}{dx_3{}^2}-k^2\right)\bar{\phi}+b_{10}\overline{\phi^*}+\left(b_{11}-\frac{d^2}{dx_3{}^2}\right)\bar{T}+b_{12}\bar{C}=0, \qquad (3.4)$$

$$\left(\frac{d^2}{dz^2}-k^2\right)^2\bar{\phi}+b_{13}\left(\frac{d^2}{dx_3{}^2}-k^2\right)\bar{T}+\left(b_{14}+b_{15}\left(\frac{d^2}{dx_3{}^2}-k^2\right)\right)\bar{C}=0 \qquad (3.5)$$

$$\left[b_{16}\left(\frac{d^2}{dx_3{}^2}-k^2\right)+\omega^2\right]\bar{\psi}+b_{17}\overline{\phi_2}=0, \qquad (3.6)$$

$$b_{20}\left(\frac{d^2}{dx_3{}^2}-k^2\right)\bar{\psi}+\left[b_{18}\left(\frac{d^2}{dx_3{}^2}-k^2\right)+b_{19}\right]\overline{\phi_2}=0, \qquad (3.7)$$

$$b_{18}=\frac{\gamma}{\gamma\rho j c_1^2},\ b_{19}=-\frac{2K}{\rho j\omega^{*2}}+\omega^2,\ b_{20}=-\frac{Kc_1^2}{j\omega^{*2}\beta_1 T_0}. \qquad (3.8)$$

On solving equations (3.2)-(3.5), we obtain:

$$\left[A_0\frac{d^8}{d\,x_3{}^8}+A_1\frac{d^6}{d\,x_3{}^6}+A_2\frac{d^4}{d\,x_3{}^4}+A_3\frac{d^2}{d\,x_3{}^2}+A_5\right](\bar{\phi},\overline{\phi^*},\bar{T},\bar{C})=0\,, \qquad (3.9)$$

And on solving equations (3.6)-(3.7), we obtain:

$$\left[\frac{d^4}{d\,x_3{}^4}+A_6\frac{d^2}{d\,x_3{}^2}+A_7\right](\overline{\phi_2},\bar{\psi})=0, \qquad (3.10)$$

The solution of the above system satisfying the radiation conditions that

$(\bar{\phi},\bar{\psi},\bar{T},\overline{\phi_2},\overline{\phi^*},\bar{C})\rightarrow 0$ as $x_3\rightarrow\infty$ are given as following:

$$(\bar{\phi},\overline{\phi^*},\bar{T},\bar{C})=\sum_{i=1}^{4}(1,\alpha_{1i},\alpha_{2i},\alpha_{3i})M_i e^{-m_i x_3}, \qquad (3.11)$$

$$(\bar{\psi}, \bar{\phi_2}) = \sum_{i=5}^{6}(1, \beta_{1i})N_i e^{-m_i x_3}, \qquad (3.12)$$

Here M_i and N_i are functions depending on k and ω, $m_i^2(i = 1,2,3,4)$ are the roots of the equation (3.9) and $m_i^2(i = 5,6)$ are the roots of equation (3.10).

Here $\alpha_{1i} = \frac{D_{1i}}{D_{0i}}\ \alpha_{2i} = \frac{D_{2i}}{D_{0i}}\ \alpha_{3i} = \frac{D_{3i}}{D_{0i}} \qquad i = 1,2,3,4$

$,\beta_{1i} = -\frac{\delta_3(m_i^2 - k^2)}{(m_i^2 - k^2) + \delta_4} \qquad i = 5,6$

3.1 Boundary Condition

We consider normal and tangential force acting on the surface $x_3 = 0$ along with vanishing of couple stress, micro stress and temperature gradient with insulated and impermeable boundary at $x_3 = 0$. Mathematically this can be written as:

$$t_{33} = -F_1 e^{-(kx_1 - \omega t)},$$

$$t_{31} = -F_2 e^{-(kx_1 - \omega t)},$$

$$m_{32} = 0, \lambda_3 = 0\ , h_1 \frac{\partial T}{\partial x_3} + h_2 T = 0\ , h_3 \frac{\partial C}{\partial x_3} + h_3 C = 0, \qquad (4.1)$$

Where F_1 and F_2 are the magnitude of the applied force.Here, $h_2, h_4 \rightarrow 0$ corresponds to insulated impermeable boundaries. Similarly, $h_1, h_3 \rightarrow 0$ corresponds to isothermal and isoconcentrated boundaries. F_1 and F_2 are the magnitude of the applied force.

Using these boundary conditions and solving the linear equations formed, we obtain:

$$t_{33} = \sum_{i=1}^{6} G_{1i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, i = 1,2, \dots ,6 \qquad (4.2)$$

$$t_{31} = \sum_{i=1}^{6} G_{2i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, i = 1,2, \dots ,6 \qquad (4.3)$$

$$m_{32} = \sum_{i=1}^{6} G_{3i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, i = 1,2, \dots ,6 \qquad (4.4)$$

$$\lambda_3 = \sum_{i=1}^{6} G_{4i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, i = 1,2, \dots ,6 \qquad (4.5)$$

$$u_1 = \sum_{i=1}^{6} G_{5i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}, i = 1,2, \dots ,6 \qquad (4.6)$$

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

$$u_3 = \sum_{i=1}^{6} G_{6i} e^{-m_i x_3} e^{-(kx_1-\omega t)}, i = 1,2,\dots,6 \qquad (4.7)$$

$$T = \sum_{i=1}^{6} G_{7i} e^{-m_i x_3} e^{-(kx_1-\omega t)}, i = 1,2,\dots,6 \qquad (4.8)$$

$$C = \sum_{i=1}^{6} G_{8i} e^{-m_i x_3} e^{-(kx_1-\omega t)}, i = 1,2,\dots,6 \qquad (4.9)$$

Here $G_{ij}, i = 1,2,\dots,6,\ j = 1,2,\dots,8$ are the constants.

IV. APPLICATIONS

4.1 Inclined Line Load

For an inclined line load F_0 we have:

$$F_1 = F_0 cos\theta,$$

$$F_2 = F_0 sin\theta \quad (4.10)$$

Making use of (4.7) in (4.2)-(4.6), we obtain the corresponding expressions of normal stress, tangential stress, couple stress, temperature distribution and mass concentration due to inclined load.

4.2 Particular cases

If we take $\tau_1 = \tau^1 = 0,\ \varepsilon = 1,\ \gamma_1 = \tau_0$, in Eqs. (4.2)- (4.9), we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.

(i) If we take $\varepsilon = 0,\ \gamma_1 = \tau^0$ in Eqs. (4.2)- (4.9), the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.

(ii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = \gamma_1 = 0$ in Eqs. (4.2) - (4.9), yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

4.3 Special cases

(a) **Microstretch Thermoelastic Solid**

If we neglect the diffusion effect in Equations (4.2) - (4.9), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) **Micropolar Thermoelastic Diffusive Solid**

If we neglect the microstretch effect in Equations (4.2) - (4.9), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.

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ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

V. CONCLUSIONS

The problem is useful for geophysical mechanics where the interest is the phenomenon in earth quake and measuring of displacement in certain sources. Finally we conclude:

- The normal mode analysis technique is used to derive the components of normal stress, shear stress, couple stress, micro stress, temperature distribution and the mass concentration.
- Values of displacement components, stress components are close to each other due to LS, GL and CT theories.