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AMMENSALISM WITH MORTAL ENEMY SPECIES-A SERIES SOLUTION

Dr.K.V.L.N.Acharyulu[1], Dr.N.Phani Kumar[2]

[1]*Associate Professor, Department of Mathematics, Bapatla Engineering College, Bapatla, (India)*

[2]*Professor, Dept. of Mathematics, College of Natural and Computational Science*

ABSTRACT

The paper deals to compute a series solution in an Ammensal Model. The Ammensal species has un limited resources with mortal enemy species. The model equations are made by a pair of non linear first order differential equations. Homotopy perturbation method is used for findling series solution.

Keywords: Ammensalism, Homotopy Analysis, Stability, Dominance Reversal Time

I. INTRODUCTION

Homotopy perturbation method is a better tool to derive a series solution for non linear differential equations. Abbasbandy,S [1] did constructive study in the concept of asymptotic techniques .Later Liao[5-8] suggested many ideas and developed Homotopy Perturbation Method (HPM) in 1992. Few other effective and efficient methods were developed by many Mathematicians [2,4].In the past two decades, the HPM concept has been utilized in some applications of Engineering and Sciences[3,9].

II. BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

Step (1): Let us consider nonlinear differential equation:

$$A(u)-f(r)=0,\ \ r\in\Omega \qquad \text{(I)}$$

With the boundary condition

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\qquad r\in\Gamma$$

Where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation among the normal drawn outwards from Ω .

Step (2): In general the operator A, is divided into two parts: a linear part L and a nonlinear part N. Therefore above differential equation(I) is expressed in the form of

$$L(u)-N(u)-f(r)=0 \qquad \text{(II)}$$

Step (3):

With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy $v(r,p):\Omega\times[0,1]\rightarrow R$ which satisfies

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$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0,\ p\in[0,1], r\in\Omega \qquad \text{(III)}$$

It is nothing but

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[A(v)-f(r)\right]=0 \qquad \text{(IV)}$$

Where $p\in[0,1]$ is named as an embedding parameter ,and u_0 is an initial approximation of equation(1),which satisfies the boundary conditions.

Step (4): Then equations (III), (IV) follow that

$$H(v,0)=L(v)-L(u_0)=0$$

And $H(v,1)=A(v)-f(r)=0$

Thus the changing process of p from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$.

Step (5): According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (III) and (IV) can be written as a power series in p :

$$v=v_0+pv_1+p^2v_2+p^3v_3+p^4v_4+\text{---------}$$

The approximate solution of equation (I) can be obtained as

$$u=\underset{p\to 1}{Lt}\, v=v_0+v_1+v_2+v_3+v_4+-----------$$

III. NOTATIONS ADOPTED

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) : The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{ii} : The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.

a_{12} :The inhibition coefficient of S_1 due to S_2 i.e The Commensal coefficient.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α , h_1, h_2 are assumed to be non-negative constants.

IV. BASIC EQUATIONS

$$\frac{dN_1}{dt}=a_1N_1-a_{11}N_1^2-a_{12}N_1N_2 \qquad (1)$$

$$\frac{dN_2}{dt}=-a_2N_2 \quad \text{With initial conditions N1 (0) =}c_1 \text{ and } N_2(0)=c_2 \qquad (2)$$

The following system can be constructed by the concept of homotopy as follows

$$v_1'-N_{10}'+p\left(N_{10}'-a_1v_1+a_{11}v_1^2-a_{12}v_1v_2\right)=0 \qquad (3)$$

$$v_2'-N_{20}'+p\left(N_{20}'+a_2v_2\right)=0 \qquad (4)$$

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The initial approximations are considered as

$$v_{1,0}(t) = N_{10}(t) = v_1(0) = c_1 \tag{5}$$

$$v_{2,0}(t) = N_{20}(t) = v_2(0) = c_2 \tag{6}$$

and $v_1(t) = v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots$ (7)

$$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots \tag{8}$$

Where $v_{i,J}(i = 1,2, J = 1,2,3 \ldots)$ *are to be computed by substituting* (5), (6), (7), (8) *in* (3), (4)

We get

$$v'_{1,0}(t) + pv'_{1,1}(t) + p^2v'_{1,2}(t) + p^3v'_{1,3}(t) + p^4v'_{1,4}(t) + p^5v'_{1,5}(t) + \cdots - N'_{10} +$$
$$p[N'_{10} - a_1(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots)$$
$$+a_{11}\left(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots\right)(v_{1,0}(t) + pv_{1,1}(t)$$
$$+p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots + a_{12}(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}$$
$$(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots)\left(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) +\right.$$
$$\left.p^5v_{2,5}(t) + \cdots\right)] = 0$$

(9)

From equation (4)

$$v'_{2,0}(t) + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - N'_{20}$$
$$+p[N'_{20} + a_2(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots$$
$$\left(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots\right)] \quad =0 \tag{10}$$

From (9),

$$0 + pv'_{1,1}(t) + p^2v'_{1,2}(t) + p^3v'_{1,3}(t) + p^4v'_{1,4}(t) + p^5v'_{1,5}(t) + \cdots - 0$$
$$+p[0 - a_1v_{1,0}(t) - a_1pv_{1,1}(t) - a_1p^2v_{1,2}(t) - a_1p^3v_{1,3}(t) - a_1p^4v_{1,4}(t) - a_1p^5v_{1,5}(t) - \ldots$$
$$+a_{11}\, v_{1,0}^2(t) + a_{11}pv_{1,0}(t)v_{1,1}(t) + a_{11}p^2v_{1,0}(t)v_{1,2}(t) + a_{11}p^3v_{1,0}(t)v_{1,3}(t)$$
$$+a_{11}p^4v_{1,0}(t)v_{1,4}(t) + \cdots + \; a_{11}pv_{1,0}(t)v_{1,1}(t) + a_{11}p^2v_{1,1}^2(t) + a_{11}p^3v_{1,1}(t)v_{1,2}(t)$$
$$+a_{11}p^4v_{1,1}(t)v_{1,3}(t) \; + a_{11}p^5v_{1,1}(t)v_{1,4}(t) + \cdots + a_{11}p^2v_{1,0}(t)v_{1,2}(t) + a_{11}p^3v_{1,1}(t)v_{1,2}(t)$$
$$+a_{11}p^4v_{1,2}^2(t) + a_{11}p^5v_{1,2}(t)v_{1,3}(t) + \cdots + a_{11}p^3v_{1,0}(t)v_{1,3}(t) + a_{11}p^4v_{1,1}(t)v_{1,3}(t)$$
$$+a_{11}p^5v_{1,2}(t)v_{1,3}(t) + \cdots \; + a_{11}p^4v_{1,0}(t)v_{1,4}(t) + a_{11}p^5v_{1,1}(t)v_{1,4}(t) + \cdots$$
$$+a_{11}p^5v_{1,0}(t)v_{1,5}(t) + \cdots + a_{12}\; v_{1,0}(t)v_{2,0}(t) + a_{12}pv_{1,0}(t)v_{2,1}(t) + a_{12}p^2v_{1,0}(t)v_{2,2}(t)$$
$$+a_{12}p^3v_{1,0}(t)v_{2,3}(t) + a_{12}p^4v_{10}(t)v_{2,4}(t)\ldots\ldots + a_{12}\; pv_{1,1}(t)v_{2,0}(t) + a_{12}p^2v_{1,1}(t)v_{2,1}(t)$$
$$+a_{12}p^3v_{1,1}(t)v_{2,2}(t) + a_{12}p^4v_{1,1}(t)v_{2,3}(t).\ldots + a_{12}p^2v_{2,0}(t)v_{1,2}(t) + a_{12}p^3v_{1,2}(t)v_{2,1}(t)$$

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$+a_{12}p^4v_{1,2}(t)v_{2,2}(t)\ldots+a_{12}p^3v_{1,3}(t)v_{2,0}(t)+a_{12}p^4v_{1,3}(t)v_{2,1}(t).\ldots+a_{12}p^4v_{1,4}(t)v_{2,0}(t)$

$\ldots.] = 0$ (11)

From (10),

$0+pv'_{2,1}(t)+p^2v'_{2,2}(t)+p^3v'_{2,3}(t)+p^4v'_{2,4}(t)+p^5v'_{2,5}(t)+\cdots-0+p[0+a_2v_{2,0}(t)$

$+a_2pv_{2,1}(t)+a_2p^2v_{2,2}(t)+a_2p^3v_{2,3}(t)+a_2p^4v_{2,4}(t)+a_2p^5v_{2,5}(t)]=0$ (12)

Now comparing the coefficient of various powers of p in (11) & (12),we obtain

The coefficient of P^1:

$v'_{1,1}(t)-a_1v_{1,0}(t)+a_{11}v^2_{1,0}(t)+a_{12}v_{1,0}(t)v_{2,0}(t)=0$

$v'_{2,1}(t)+a_2v_{2,0}(t)=0$

The coefficient of P2:

$v'_{1,2}(t)-a_1v_{1,1}(t)+a_{11}v_{1,0}(t)v_{1,1}(t)+a_{11}v_{1,0}(t)v_{1,1}(t)+\ a_{12}v_{1,0}(t)v_{2,1}(t)$

$+a_{12}v_{1,1}(t)v_{2,0}(t)=0$

$v'_{2,2}(t)+a_2v_{2,1}(t)=0$

The coefficient of P3:

$v'_{1,3}(t)-a_1v_{1,2}(t)+a_{11}v_{1,0}(t)v_{1,2}(t)+a_{11}v^2_{1,1}(t)+a_{11}v_{1,0}(t)v_{1,2}(t)+a_{12}v_{1,0}(t)v_{2,2}(t)-$
$a_{12}v_{1,1}(t)v_{2,1}(t)+\ a_{12}v_{2,0}(t)v_{1,2}(t)$
$=0$

$v'_{2,3}(t)+a_2v_{2,2}(t)=0$

The coefficient of P4:

$v'_{1,4}(t)-a_1v_{1,3}(t)+a_{11}v_{1,0}(t)v_{1,3}(t)+a_{11}v_{1,1}(t)v_{1,2}(t)+a_{11}v_{1,1}(t)v_{1,2}(t)$

$+\ \ a_{11}v_{1,0}(t)v_{1,3}(t)+a_{12}v_{1,0}(t)v_{2,3}(t)+a_{12}v_{1,1}(t)v_{2,2}(t)+a_{12}v_{2,1}(t)v_{1,2}(t)$

$+a_{12}v_{2,0}(t)v_{1,3}(t)=0$

$v'_{2,4}(t)+a_2v_{2,3}(t)=0$

Now $v_1(0)\ =c_{1,}v_2(0)=c_2$

$$v_{1,1}(t)=a_1\int_0^t v_{1,0}(t)dt-a_{11}\int_0^t v^2_{1,0}(t)dt-a_{12}\int_0^t v_{1,0}(t)v_{2,0}(t)dt$$

$$=c_1a_1t-a_{11}c_1^2t+a_{12}c_1c_2t$$

$\therefore v_{1,1}(t)\ =(a_1-a_{11}c_1-a_{12}c_2)c_1t$

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$$v_{2,1}(t) = -a_2 \int_0^t v_{2,0}(t)dt = -a_2c_2t$$

$$\therefore v_{2,1}(t) = -a_2c_2t$$

$$v_{1,2}(t) = a_1 \int_0^t v_{1,1}(t)dt - 2a_{11} \int_0^t v_{1,0}(t)v_{1,1}(t)dt - a_{12} \int_0^t v_{1,0}(t)v_{2,1}(t)dt$$

$$-a_{12} \int_0^t v_{1,1}(t)v_{2,0}(t)dt$$

$$= a_1(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2} - 2a_{11}c_1(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2}$$

$$+c_1a_2c_2\frac{t^2}{2} - a_{12}c_2(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2}$$

$$\therefore v_{1,2}(t) = [(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 + a_2a_{12}c_1c_2]\frac{t^2}{2}$$

$$v_{2,2}(t) = -a_2 \int_0^t v_{2,1}(t)dt$$

$$= [a_2{}^2c_2]\frac{t^2}{2}$$

$$\therefore v_{2,2}(t) = [a_2{}^2c_2]\frac{t^2}{2}$$

$$v_{1,3}(t) = a_1 \int_0^t v_{1,2}(t)dt - 2a_{11}c_1 \int_0^t v_{1,2}(t)dt - a_{11} \int_0^t v_{1,1}^2(t)dt - a_{12}c_1 \int_0^t v_{2,2}(t)dt$$

$$-a_{12}c_2 \int_0^t v_{1,2}(t)dt - a_{12} \int_0^t v_{1,1}(t)v_{2,1}(t)dt$$

$$= (a_1 - 2a_{11}c_1 - a_{12}c_2)\{(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1$$

$$-a_{12}c_1c_2(a_2 - a_{22}c_2)\}\frac{t^3}{6} - a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1^2\frac{t^3}{3}$$

$$-a_{12}c_1(a_2{}^2c_2{}^2)\frac{t^3}{6} - a_{12}a_2c_{2_2}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^3}{3}$$

$$\therefore v_{1,3}(t) = [(a_1 - 2a_{11}c_1 - a_{12}c_2)\{(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1$$

$$+a_{12}a_2c_1c_2\ \} + (a_1 - a_{11}c_1 - a_{12}c_2)c_1\{2a_{12}c_2(-a_2) - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\}$$

$$-a_{12}c_1c_2\{(-a_2)(-a_2)\}]\frac{t^3}{6}$$

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$$v_{2,3}(t) = -a_2 \int_0^t v_{2,2}(t)dt$$

$$= {a_2}^3 c_2 \frac{t^3}{6}$$

$$\therefore v_{2,3}(t) = {a_2}^3 c_2 \frac{t^3}{6}$$

$$v_{1,4}(t) = (a_1 - 2a_{11}c_1 - a_{12}c_2) \int_0^t v_{1,3}(t)dt - 2a_{11} \int_0^t v_{1,1}(t)v_{1,2}(t)dt - a_{12} \int_0^t v_{1,1}(t)v_{2,2}(t)dt$$

$$-a_{12} \int_0^t v_{1,2}(t)v_{2,1}(t)dt - a_{12}c_1 \int_0^t v_{2,3}(t)dt$$

$$= [(a_1 - 2a_{11}c_1 - a_{12}c_2)\{(a_1 - 2a_{11}c_1 - c_2)\{(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)\,c_1$$

$$+a_{12}c_1c_2a_2\}+(a_1 - a_{11}c_1 - a_{12}c_2)c_1\{2a_{12}c_2(-a_2)$$
$$-2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\} - a_{12}c_1c_2{a_2}^2]\frac{t^4}{24} - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\{(a_1 - 2a_{11}c_1 -$$
$$a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 + a_{12}c_1a_2c_2\}\frac{t^4}{8} + a_{12}c_1\{(a_2$$
$$)[(-a_2)c_2(-a_2)]\}\frac{t^4}{24}$$
$$-a_{12}c_1(a_1 - a_{11}c_1 - a_{12}c_2)[{a_2}^3]\frac{t^4}{8} - a_{12}a_2c_2[(a_1 - 2a_{11}c_1 - a_{12}c_2)\,(a_1 - a_{11}c_1 - a_{12}c_2)c_1]\frac{t^4}{8}$$

$$\therefore v_{1,4}(t) = [(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 - a_2a_{12}c_1c_{2_2}$$
$$[(a_1 - 2a_{11}c_1 - a_{12}c_2)^2 - c_1 6a_{11}\,(a_1 - a_{11}c_1 - a_{12}c_2)] + (a_1 - 2a_{11}c_1 - a_{12}c_2)$$
$$\{2(a_1 - a_{11}c_1 - a_{12}c_1)c_1[a_{12}c_2a_2 - a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1] - a_{12}c_1c_2{a_2}^2\} + [{a_2}^2c_2]$$
$$[a_{12}c_1 - 3a_{12}c_1(a_1 - a_{11}c_1 - a_{12}c_2)] - a_2c_2\{[(a_1 - a_{11}c_1 - a_{12}c_2)c_1[3a_{12}(2a_{11}c_1 - a_1 -$$
$$a_{12}c_2)] - a_2c_2 3a_{12}^2c_1\}\frac{t^4}{24}$$

$$v_{2,4}(t) = -a_2 \int_0^t v_{2,3}(t)dt$$

$$= {a_2}^4 c_2 \frac{t^4}{24}$$

$$\therefore v_{2,4}(t) = {a_2}^4 c_2 \frac{t^4}{24}$$

Up to the terms which contain maximum the power of four, we obtain

$$N_1(t) = \lim_{p \to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + v_{1,1}(t) + v_{1,2}(t) + v_{1,3}(t) + v_{1,4}(t)$$

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$$N_1(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + v_{2,1}(t) + v_{2,2}(t) + v_{2,3}(t) + v_{2,4}(t)$$

The solutions by Homotopy Perturbation Method are derived as

$$N_1(t) = c_1 + [(a_1 - a_{11}c_1 - a_{12}c_2)c_1]t$$

$$+[(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 + a_2a_{12}c_1c_2]\frac{t^2}{2}$$

$$+\{(a_1 - 2a_{11}c_1 - a_{12}c_2)[(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 + a_2a_{12}c_1c_2]$$

$$+(a_1 - a_{11}c_1 - a_{12}c_2)c_1[-2a_2a_{12}c_2 - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1]$$

$$-a_{12}c_1c_2{a_2}^2\frac{t^3}{6}\{[(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1$$

$$+a_2a_{12}c_1c_2][(a_1 - 2a_{11}c_1 - a_{12}c_2)^2 - 6a_{11}c_1(a_1 - a_{11}c_1 - a_{12}c_2)]$$

$$+(a_1 - 2a_{11}c_1 - a_{12}c_2)\{2(a_1 - a_{11}c_1 - a_{12}c_2)c_1a_2a_{12} - a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1$$

$$-a_{12}c_1c_2{a_2}^2\} + {a_2}^2[a_{12}c_1a_2 - 3a_{12}c_1(a_1 - a_{11}c_1 - a_{12}c_2)]$$

$$-a_2c_2\{(a_1 - a_{11}c_1 - a_{12}c_2)c_1[3(a_1 - 2a_{11}c_1 - a_{12}c_2)a_{12}] - 3a_{12}^2c_1a_2c_2\}\frac{t^4}{24}$$

$$\therefore\ N_2(t) = c_2 - a_2t + {a_2}^2c_2\frac{t^2}{2} + {a_2}^3c_2\frac{t^3}{6} + {a_2}^3\ c_2\frac{t^4}{24}$$

V. CONCLUSIONS

A mathematical model of Ecological Model of Ammensalism with mortal Enemy species is constituted by a couple of first order nonlinear differential equations. A series solution of this Ammensalism is computed by Homotopy Perturbation Method.

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