International Journal of Advance Research in Science and Engineering
Vol. No. 5, Issue No. 08 , August 2016
www.ijarse.com
IJARSE
ISSN 2319 - 8354

ALMOST γ-NORMAL AND MILDLY γ-NORMAL SPACES IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce and study two new classes of spaces, namely almost γ-normal and mildly γ-normal spaces are weaker forms of γ-normal spaces. We show that these normal spaces, namely almost γ-normal and mildly γ-normal spaces are regularly open hereditary. The relationships among normal, p-normal, γ-normal, β-normal, almost normal, almost p-normal, almost γ-normal, almost β-normal, quasi β-normal, mildly normal, mildly p-normal, mildly γ-normal, and mildly β-normal spaces are investigated. Moreover, we introduced some functions such as M-γ-open, M-γ-closed, almost γ-irresolute, rgγ-irresolute, rgγ-continuous and γ-rgγ-continuous. Further, utilizing gγ-closed and rgγ-closed sets, we obtain characterizations and preservation theorems for almost γ-normal and mildly γ-normal spaces.

Key words: γ-open sets, almost γ-normal, mildly γ-normal spaces, M-γ-open, M-γ-closed, almost γ-irresolute, rgγ-irresolute, rgγ-continuous and γ-rgγ-continuous functions.

2010 Mathematics subject classification: 54D10, 54D15, 54A05, 54C08.

I INTRODUCTION

Levine [**6**] initiated the study of so called generalized closed (briefly g-closed) sets in order to extend many of the most important properties of closed sets to a large family. Singal and Arya [**18**] introduced the concept of almost normal spaces. Various properties of new classes of topological spaces have been studied and the relations of these new concepts with the concepts of almost regularity have also been investigated. The notion of mildly normal space was introduced by Shchepin [**17**] and Singal and Singal [**19**] independently. Nour [**13**] introduced a weaker form of normality, called p-normality and obtained their properties. Mahmoud et al. [**7**] introduced the notion of β-normal spaces and obtained their characterizations and preservation theorems. E. Ekici [**3**] introduced a weaker form of normality, called γ-normality and obtained their properties. The relationships among normal, p-normal, s-normal and γ-normal spaces are investigated. The notion of quasi β-normal and mildly β–normal spaces were introduced by M. C. Sharma and Hamant Kumar [**16**]. The notion of almost p-normal and mildly p-normal spaces were introduced by G. B. Navalagi [**9**].The notion of almost β-normal space was introduced by Nidhi Sharma [**10**].

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www.ijarse.com
IJARSE
ISSN 2319 - 8354

II PRELIMINARIES

Throughout this paper, spaces (X, τ), (Y, σ), and (Z, γ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by **cl(A)** and **int(A)** respectively. A subset A is said to be **regular open** (resp. **regular closed**) if A = int(cl(A)) (resp. A = cl(int(A)). A is said to be **β-open** [**1**] if A ⊂ cl(int(cl(A))), **preopen** [**8**] (briefly **p-open**) if A ⊂ int(cl(A)), **γ-open [4]** if A ⊂ cl(int(A)) ∪ int(cl(A)). The complement of a β–open (resp. p-open) set is said to be **β–closed** [1] (resp. **p-closed** [**8**]). The intersection of all β–closed (resp. p-closed, γ-closed) sets containing A is called **β–closure** [**1**] (resp. **p-closure**, **γ-closure**) of A, and is denoted by **βcl(A)** (resp. **pcl(A)**, **γcl(A)**). The **γ-Interior** of A, denoted by **γint(A)**, is defined as union of all γ-open sets contained in A.

2.1. Definition. A subset A of a topological space X is said to be

1. **generalized closed** [**6**] (briefly **g-closed**) if cl(A) ⊂ U whenever A ⊂ U and U is open in X..
2. **regularly generalized** [**14**] (briefly **rg-closed**) if cl(A) ⊂ U whenever A ⊂ U and U is regularly open in X.
3. **generalized pre-closed** [**11**] (briefly **gp-closed**) if pcl(A) ⊂ U whenever A ⊂ U and U is open in X..
4. **generalized preregularly-closed** [**5**] (briefly **gpr-closed**) if pcl(A) ⊂ U whenever A ⊂ U and U is regularly open in X.
5. **generalized γ-closed** [**3**] (briefly **gγ-closed**) if γcl(A) ⊂ U whenever A ⊂ U and U is open in X.
6. **regularly generalized γ-closed** (briefly **rgγ-closed**) if γcl(A) ⊂ U whenever A ⊂ U and U is regularly open in X.
7. **generalized β-closed** [**2**] (briefly **gβ-closed**) if βcl(A) ⊂ U whenever A ⊂ U and U is open in X.
6. **regularly generalized β-closed** [**15**] (briefly **rgβ-closed**) if βcl(A) ⊂ U whenever A ⊂ U and U is regularly open in X.

The complement of g-closed, (resp. rg-closed, gp-closed, gpr-closed, gγ-closed, rgγ-closed, gβ-closed, rgβ-closed) sets is **g-open** (resp**. rg-open, gp-open**, **gpr-open**, **gγ-open**, **rgγ-open**, **gβ-open**, **rgβ-open**) sets.

By the definitions stated above and in preliminaries. We have the following diagram:

$$\begin{array}{ccccc}
\text{closed} & \Rightarrow & \text{g-closed} & \Rightarrow & \text{rg-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{p-closed} & \Rightarrow & \text{gp-closed} & \Rightarrow & \text{gpr-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\gamma\text{-closed} & \Rightarrow & \text{g}\gamma\text{-closed} & \Rightarrow & \text{rg}\gamma\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\gamma\text{-closed} & \Rightarrow & \text{g}\gamma\text{-closed} & \Rightarrow & \text{rg}\gamma\text{-closed}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

2.2. Example. Let $X = \{a, b, c, d\}$ and $\tau = \{ \phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$. Then $A = \{ a \}$ is gγ-closed but not closed.

2.3. Example. Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then $A = \{a, b\}$ is gγ-closed as well as rgγ-closed but not closed.

2.4. Example. Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{b\}, \{d\}, \{b, d\}, X\}$. Then $A = \{a, b, d\}$ is gγ-closed but not closed.

2.5. Example. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Then $A = \{b, c\}$ is gpr-closed as well as rgγ-closed, but not p-closed.

2.6. Example. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then $A = \{a\}$ gpr-closed as well as rgγ-closed, but not rg-closed.

2.7. Example. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\},\{a, b\}, X\}$. Then $A = \{a\}$ is gβ-closed. But it is neither closed nor gp-closed.

2.8. Example. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b, c\}, X\}$. Then $A = \{c\}$ is pre-closed but not closed.

2.9. Example. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then $A = \{b\}$ is g-closed but not closed.

2.10. Example. Let $X = \{a, b, c,\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then $A = \{b\}$ is gβ-closed but not g-closed.

III ALMOST γ-NORMAL SPACES

3.1. Definition. A topological space X is said to be **almost γ-normal (**resp. **almost normal** [18], **almost p-normal** [**9**], **almost β-normal** [**10**]) if for any two disjoint closed subsets A and B of X, one of which is regularly closed, there exist disjoint γ-open (resp. open, p-open, β-open) sets U and V of X such that $A \subset U$ and $B \subset V$.

3.2. Definition. A topological space X is said to be **γ-normal** [**3**] (resp. **p-normal [13]**, **β-normal** [7]) if for every pair of disjoint closed subsets A, B of X, there exist disjoint γ-open (resp. p-open, β-open) sets U, V of X such that $A \subset U$ and $B \subset V$.

By the definitions stated above and in preliminaries. We have the following diagram:

$$\begin{array}{ccc}
\text{normal} & \Rightarrow & \text{almost normal} \\
\Downarrow & & \Downarrow \\
\text{p-normality} & \Rightarrow & \text{almost p-normality} \\
\Downarrow & & \Downarrow \\
\gamma\text{-normality} & \Rightarrow & \text{almost } \gamma\text{-normality} \\
\Downarrow & & \Downarrow \\
\beta\text{-normality} & \Rightarrow & \text{almost } \beta\text{-normality}
\end{array}$$

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www.ijarse.com
IJARSE
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Where none of the implications is reversible as can be seen from the following examples:

3.3 Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {b, d}, {a, b, d}, {b, c, d}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {c}. Also U = {a, b} and V = {c, d} are p-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is p-normal as well as β-normal but not normal, since U and V are not open sets.

3.4 Example. Let X = {a, b, c, d} and τ = {ϕ, {b, d}, {a, b, d}, {b, c, d}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {c}. Also U = {a, b} and V = {c, d} are β-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is β-normal but not normal, since U and V are not open sets.

3.5 Example. Let X = {a, b, c, d} and τ = {ϕ, {b}, {d}, {b, d}, {a, b, d}, {b, c, d}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {c}. Also U = {a, b} and V = {c, d} are β-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is β-normal. But it is neither p-normal nor normal, since U and V are neither p-open nor open sets.

3.6 Example. Let X = {a, b, c, d, e} and τ = {ϕ, {a}, {c}, {a, c}, {b, c}, {c, e}, {a, b, c}, {a, c, e}, {b, c, d}, {b, c, e}, {a, b, c, d}, {a, b, c, e}, {b, c, d, e}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {d}. Also U = {a} and V = {b, c, d} are open sets such that $A \subset U$ and $B \subset V$. Hence the space X is normal as well as β-normal, since U and V are also β-open sets.

3.7 Example. Let X = {a, b, c,} and τ = {ϕ, {a}, {b, c}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {b, c}. Also U = {a} and V = {b, c} are open sets such that $A \subset U$ and $B \subset V$. Hence the space X is normal as well as almost normal.

3.8 Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X}. The pair of disjoint closed subsets of X are A = {a, b, c} and B = {d}. Also U = {a, b, c} and V = {d} are open sets such that $A \subset U$ and $B \subset V$. Hence the space X is normal as well as γ-normal.

3.9. Theorem. $A \subset X$ is rgγ-open if and only if $F \subset \gamma\text{int}(A)$ whenever F is regularly closed and $F \subset A$.

Proof. Let A be rgγ-open. Let F be regularly closed and $F \subset A$. Then $X – A \subset X – F$. $X – F$ is regularly open. $X – A$ is rgγ-closed. $\gamma cl(X – A) \subset X – F$. $X – \gamma\text{int}(A) \subset X – F$. So, $F \subset \gamma\text{int}(A)$. Let F regularly closed and $F \subset A$ imply $F \subset \gamma\text{int}(A)$. Let $X – A \subset U$, where U is regularly open $X – U \subset A$, where $X – U$ is regularly closed. Hence $X – U \subset \gamma\text{int}(A)$. So $X – \gamma\text{int}(A) \subset U$. That is $\gamma cl(X – A) \subset U$. Hence $X – A$ is rgγ-closed. This implies A is rgγ-open.

3.10. Theorem. For a topological space X, the following are equivalent:

(a) X is almost γ-normal.

(b) For every closed set A and every regularly open set B containing A, there is a γ-open set U such that

$A \subset U \subset \gamma cl(U) \subset B$.

(c) For every regularly closed set A and every open set B containing A, there is a γ-open set U such that

$A \subset U \subset \gamma cl(U) \subset B$.

(d) For every pair of disjoint sets A and B of X, one of which is closed and other is regularly closed, there exist

γ-open sets U and V such that $A \subset U$, $B \subset V$ and $\gamma cl(U) \cap \gamma cl(V) = \phi$.

Proof. (a) ⇒ (b). Let A be a closed set and let B be a regularly open set containing A. Thus $A \cap (X - B) = \phi$, where A is closed and $X - B$ is regularly closed. Therefore, there exist γ-open sets U and V such that $A \subset U$, $X - B \subset V$ and $U \cap V = \phi$. Thus, $A \subset U \subset X - V \subset B$. Now, $X - V$ is closed. Therefore, $A \subset U \subset \gamma cl(U) \subset B$.

(b) ⇒ (c). Let A be regularly closed set and let B be an open set containing A. Then, $X - B \subset X - A$, whence $X - A$ is a regularly open set containing the closed set $X - B$. Therefore, there is a γ-open set M such that $X - B \subset M \subset \gamma cl(M) \subset X - A$. Thus, $A \subset X - \gamma cl(M) \subset X - M \subset B$. Let $X - \gamma cl(M) = U$. Then U is γ-open and $A \subset U \subset \gamma cl(U) \subset B$.

(c) ⇒ (d). Let A be a regularly closed set and B be a closed set such that $A \cap B = \phi$. Then, $A \subset X - B$ which is open. Therefore, there exists a γ-open set M such that $A \subset M \subset \gamma cl(M) \subset X - B$. Again, M is a γ-open set containing the regularly closed set A. Therefore, there is a γ-open set U such that $A \subset U \subset \gamma cl(U) \subset M$. Let $X - \gamma cl(M) = V$. Then, $A \subset U$, $B \subset V$ and $\gamma cl(U) \cap \gamma cl(V) = \phi$.

(d) ⇒ (a) is obvious.

3.11 Theorem. For a topological space X**,** the following are equivalent:

(a) X is almost γ-normal.

(b) For every closed set A and every regularly closed set B, there exist disjoint gγ-open sets U and V such that $A \subset U$ and $B \subset V$.

(c) For every closed set A and every regularly closed set B, there exist disjoint rgγ-open sets U and V such that $A \subset U$ and $B \subset V$.

(d) For every closed set A and every regularly open set B containing A, there exists a gγ-open set U of X such that $A \subset U \subset \gamma\text{-}cl(U) \subset B$.

(e) For every closed set A and every regularly open set B containing A, there exists a rgγ-open set U of X such that $A \subset U \subset \gamma\text{-}cl(U) \subset B$.

(f) For every pair of disjoint sets A and B of X , one of which is closed and other is regularly closed, there exist γ-open sets U and V such that $A \subset U$ and $B \subset V$ and $U \cap V = \phi$.

Proof. (a) ⇒ (b), (b) ⇒ (c), (d) ⇒ (e), (c) ⇒ (d), (e) ⇒ (f) and (f) ⇒ (a).

(a) $\Rightarrow$ (b). Let X be almost γ-normal. Let A be a closed and B be a regularly closed sets in X. By assumption, there exist disjoint γ-open sets U and V such that $A \subset U$ and $B \subset V$. Since every γ-open set is gγ-open set, U, V are gγ-open sets such that $A \subset U$ and $B \subset V$.

(b) $\Rightarrow$ (c). Let A be a closed and B be a regularly closed sets in X. By assumption, there exist disjoint gγ-open sets U and V such that $A \subset U$ and $B \subset V$. Since every gγ-open set is rgγ-open set, U, V are rgγ-open sets such that $A \subset U$ and $B \subset V$.

(d) $\Rightarrow$ (e). Let A be any closed set and B be any regularly open set containing A. By assumption, there exists a gγ-open set U of X such that $A \subset U \subset \gamma cl(U) \subset B$. Since every g$\gamma$-open set is rg$\gamma$-open set, there exists a rgγ-open set U of X such that $A \subset U \subset \gamma cl(U) \subset B$.

(c) $\Rightarrow$ (d). Let A be any closed set and B be a regularly open set containing A. By assumption, there exist disjoint rgγ-open sets U and W such that $A \subset U$ and $X - B \subset W$. By **Theorem 3.9,** we get, $X - B \subset \gamma int(W)$ and $\gamma cl(U) \cap \gamma int(W) = \phi$. Hence $A \subset U \subset \gamma cl(U) \subset X - \gamma\, int(W) \subset B$.

(e) $\Rightarrow$ (f). For any closed set A and any regularly open set B containing A. Then $A \subset X - B$ and $X - B$ is a regularly closed set. By assumption, there exists a rgγ-open set G of X such that $A \subset G \subset \gamma cl(G) \subset X - B$. Put $U = \gamma int(G)$, $V = X - \gamma cl(G)$. Then U and V are disjoint γ-open sets of X such that $A \subset U$ and $B \subset V$.

(f) $\Rightarrow$ (a) is obvious.

IV PRESERVATION THEOREMS

4.1 Definition. A function $f : X \to Y$ is called

1. **M-γ-open** if $f(U) \in \gamma O(Y)$ for each $U \in \gamma O(X)$.
2. **M-γ-closed** if $f(U) \in \gamma C(Y)$ for each $U \in \gamma C(X)$.
3. **almost γ-irresolute** if for each $x \in X$ and each γ-neighbourhood V of $f(x)$, γ-cl $(f^{-1}(V))$ is a γ-neighbourhood of x.

4.2 Definition. A topological space X is called

1. **weakly γ-regular** if for each point x and a regularly open set U containing $\{x\}$, there is a γ-open set V such that $x \in V \subset \gamma\text{-}cl(V) \subset V$.
2. **almost γ-regular** if for every regularly closed set F and each point $x \notin F$, there exist disjoint γ-open sets U and V such that $x \in V$ and $F \subset V$.

4.3 Remark. One can prove that almost γ-normality is also regularly closed hereditary.

4.4 Remark. Almost γ-normality does not imply almost γ-regular.

Next, we prove the invariant of almost γ-normality in the following.

4.5 Theorem. If $f : X \to Y$ is continuous M-γ-open rc-continuous and almost γ-irresolute surjection from an almost γ-normal space X onto a space Y, then Y is almost γ-normal.

Proof. Let A be a closed set and B be a regularly open set containing A. Then by rc-continuity of f, $f^{-1}(A)$ is a closed set contained in the regularly open set $f^{-1}(B)$. Since X is almost γ-normal, there exist a γ-open set V in X such that $f^{-1}(A) \subset V \subset \gamma\text{-cl}(V) \subset f^{-1}(B)$ by **Theorem 3.11**. Then, $f(f^{-1}(A)) \subset f(V) \subset f(\gamma\text{-cl}(V))\ f(f^{-1}(B))$. Since f is M-$\gamma$-open and almost γ-irresolute surjection, we obtain a $A \subset f(V) \subset \gamma\text{-cl}(f(V)) \subset B$. Then again by **Theorem 3.11**, Y is almost γ-normal.

4.6 Theorem. If $f : X \to Y$ is rc-continuous M- γ-closed map from an almost γ-normal space X onto a space Y, then Y is almost γ-normal.

Proof. Easy to verify.

V MILDLY γ-NORMAL SPACES

5.1 Definition. A topological space X is said to be **mildly γ-normal** (resp. **mildly normal** [**17**, **19**], **mildly p-normal** [**9**], **mildly β-normal** [**16**]) if for any two disjoint regularly closed subsets A and B of X, there exist two disjoint γ-open (resp. open, p-open, β-open) subsets U and V of X such that $A \subset U$ and $B \subset V$.

By the definitions stated above and in preliminaries. We have the following diagram:

$$\begin{array}{ccccc}
\text{normal} & \Rightarrow & \text{almost normal} & \Rightarrow & \text{mildly normal} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{p-normal} & \Rightarrow & \text{almost p-normal} & \Rightarrow & \text{mildly p-normal} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\gamma\text{-normal} & \Rightarrow & \text{almost } \gamma\text{-normal} & \Rightarrow & \text{mildly } \gamma\text{-normal} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\beta\text{-normal} & \Rightarrow & \text{almost } \beta\text{-normal} & \Rightarrow & \text{mildly } \beta\text{-normal}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

5.2 Example. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$. The pair of disjoint closed subsets of X are $A = \{a\}$ and $B = \{b, e\}$. Also $U = \{a\}$ and $V = \{b, c, d, e\}$ are open sets such that $A \subset U$ and $B \subset V$. Hence the space X is normal as well as γ-normal.

5.3. Example. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a, b\}, \{c\}, X\}$. The pair of disjoint regularly closed subsets of X are $A = \{a, b\}$ and $B = \{c\}$. Also $U = \{a, b\}$ and $V = \{c\}$ are open sets such that $A \subset U$ and $B \subset V$. Hence the space X is mildly normal as well as mildly γ-normal, since U and V are also γ-open sets.

5.4 Example. Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, d\}, \{a, b, c\}, X\}$. The pair of disjoint regularly closed subsets of X are $A = \{c\}$ and $B = \{d\}$. Also $U = \{b, c\}$ and $V = \{a, d\}$ are β-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is mildly β-normal. But the space X is neither mildly normal nor normal, since U and V are not open sets.

5.5 Theorem. For a topological space X the following are equivalent:

(a) X is mildly γ-normal.

(b) For every pair of regularly open sets U and V of X whose union is X, there exist γ-closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$

(c) For every regularly closed set F and every regularly open set G containing F, there exists a γ-open set U such that $F \subset U \subset \gamma cl(U) \subset G$.

Proof. (a) $\Rightarrow$ (b). Let U and V be a pair of regularly open sets in a mildly γ-normal space X such that $X = U \cup V$. Then $X - U$, $X - V$ are disjoint regularly closed sets. Since X is mildly γ-normal, there exist disjoint γ-open sets U_1 and V_1 such that $X - U \subset U_1$ and $X - V \subset V_1$. Let $A = X - U_1$, $B = X - V_1$. Then A and B are γ-closed sets such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(b) $\Rightarrow$ (c). Let F be a regularly closed set and G be a regularly open set containing F. Then $X - F$ and G are regularly open sets whose union is X. Then by (b), there exist γ-closed sets W_1 and W_2 such that $W_1 \subset X - F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F \subset X - W_1$, $X - G \subset X - W_2$ and $(X - W_1) \cap (X - W2) = \phi$. Let $U = X - W_1$ and $V = X - W_2$. Then U and V are disjoint γ-open sets such that $F \subset U \subset X - V \subset G$. As $X - V$ is γ-closed set, we have $\gamma cl(U) \subset X - V$ and $F \subset U \subset \gamma cl(U) \subset G$.

(c) $\Rightarrow$ (a). Let F_1 and F_2 be any two disjoint regularly closed sets of X. Put $G = X - F_2$, then $F_1 \cap G = \phi$. $F_1 \subset G$, where G is a regularly open set. Then by (c), there exists a γ-open set U of X such that $F_1 \subset U \subset \gamma cl(U) \subset G$. It follows that $F_2 \subset X - \gamma cl(U) = V$, says, then V is γ-open and $U \cap V = \phi$. Hence F_1 and F_2 are separated by γ-open sets U and V. Therefore X is mildly γ-normal.

5.6 Theorem. For a topological space X, the following are equivalent:

(a) X is mildly γ-normal.

(b) For any disjoint regularly closed sets A and B of X, there exist disjoint gγ-open sets U and V such that $A \subset U$ and $B \subset V$.

(c) For any disjoint regularly closed sets A and B of X, there exist disjoint rgγ-open sets U and V such that $A \subset U$ and $B \subset V$.

(d) For any regularly closed set A and any regularly open set V containing A, there exists a gγ-open set U of X such that $H \subset U \subset \gamma cl(U) \subset V$.

(e) For any regularly closed set A and any regularly open set V containing A, there exists a rgγ-open set U of X such that $A \subset U \subset \gamma cl(U) \subset V$.

Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (c) $\Rightarrow$ (d), (d) $\Rightarrow$ (e) and (e) $\Rightarrow$ (a).

(a) $\Rightarrow$ (b). Let X be mildly γ-normal space. Let A, B be disjoint regularly closed sets of X. By assumption, there exist disjoint γ-open sets U, V such that $A \subset U$ and $B \subset V$. Since every γ-open set is gγ-open, so, U and V are gγ-open sets such that $H \subset U$ and $K \subset V$.

(b) $\Rightarrow$ (c). Let A, B be two disjoint regularly closed sets. By assumption, there exist gγ-open sets U and V such that $A \subset U$ and $B \subset V$. Since every gγ-open set is rgγ-open, so, U and V are rgγ-open such that $H \subset U$ and $K \subset V$.

(c) $\Rightarrow$ (d). Let A be any regularly closed set and V be any regularly open set containing A. By assumption, there exist rgγ-open sets U and W such that $A \subset U$ and $X - V \subset W$. By **Theorem 3.9**, we get $X - V \subset \gamma int(W)$ and $U \cap \gamma int\ (W) = \phi$. Therefore, we obtain $\gamma cl(U) \cap \gamma int(W) = \phi$ and hence $A \subset U \subset \gamma cl(U) \subset X - \gamma int(W) \subset V$.

(d) $\Rightarrow$ (e). Let A be any regularly closed set and V be any regularly open set containing A. By assumption, there exist gγ-open set U of X such that $H \subset U \subset \gamma cl(U) \subset V$. Since, every g$\gamma$-open set is rg$\gamma$-open, there exist rg$\gamma$-open sets U of X such that $H \subset U \subset \gamma cl(U) \subset V$.

(e) $\Rightarrow$ (a). Let A, B be any two disjoint regularly closed sets of X. Then $H \subset X - K$ and $X - B$ is regularly open. By assumption, there exists rgγ-open set G of X such that $A \subset G \subset \gamma cl(G) \subset X - B$. Put $U = \gamma int(G)$, $V = K - \gamma cl(G)$. Then U and V are disjoint γ-open sets of X such that $A \subset U$ and $B \subset V$.

Using **Theorem 5.6,** it is easy to show the following theorem, which is a Urysohn's Lemma version for mild γ-normality. A proof can be established by a similar way of the normal case.

5.7 Theorem. A space X is mildly γ-normal if and only if for every pair of disjoint regularly closed sets A and B of X, there exists a continuous function f on X into [0, 1], with its usual topology, such that f(A) = {0} and f(B) = {1}.

5.8 Theorem. Let X is a mildly γ-normal space and $f : X \to Y$ is an open continuous injective function. Then f(X) is a mildly γ-normal space.

Proof. Let A and B be any two regularly closed subset of f(X) such that $A \cap B = \phi$. Then $f^{-1}(A)$ and $f^{-1}(B)$ regularly closed sets of X. Since X is mildly γ-normal, there are two disjoint γ-open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is one-one and open, result follows.

5.9 Corollary. Mild γ-normality is a topological property.

5.10 Definition. A function $f : X \to Y$ is said to be

1. **rgγ-continuous** if $f^{-1}(F)$ is rgγ-closed in X for every closed set F of Y.
2. **γ-rgγ-continuous** if $f^{-1}(F)$ is rgγ-closed in X for every γ-closed set F of Y.
3. **rgγ-irresolute** if $f^{-1}(F)$ is rgγ-closed in X for every rgγ-closed set F of Y.

5.11 Theorem. If $f : X \to Y$ is a γ-rgγ-continuous, rc-preserving injection and Y is mildly γ-normal then X is mildly γ-normal.

Proof. Let A and B be any disjoint regularly closed sets of X. Since f is an rc-preserving injection, f(A) and f(B) are disjoint regularly closed sets of Y. By mild γ-normality of Y, there exist disjoint γ-open sets U and V of Y such that $f(A) \subset U$ and $f(B) \subset V$. Since f is γ-rgγ-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint rgγ-open sets containing A and B respectively. Hence by **Theorem 5.6,** X is mildly γ-normal.

5.12 Theorem. If $f : X \to Y$ is a γ-rgγ-continuous, almost closed surjection and Y is γ-normal space, then X is mildly γ-normal.

Proof. Similar to previous one.

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