



DISSIPATIVE ANALYSIS FOR RANDOM TIME VARYING DELAY SAMPLED-DATA CONTROLLER DESIGN FOR SINGULAR NETWORKED CASCADE CONTROL SYSTEM

¹M. Vijaya Kumar, ²S.Saravanakumar ³S.Nagarani

^{1,2,3} Department of Mathematics, Sri Ramakrishna Institute of Technology – Coimbatore (India)

ABSTRACT

In this paper we investigated the problem on dissipative analysis using sample data controller for a class of singular networked cascade control systems (NCCS) with random time varying delays and external disturbances. A new stochastic variable with a Bernoulli distribution has been introduced and the information of probability distribution of the time-varying delay is measured and transformed into a new deterministic time-varying delay. A new set of Lyapunov-Krasovskii functions are constructed to verify that the NCCS obeyed regular, impulse-free, stable and strictly (Q, S, R) - dissipative. The derived LKF conditions are framed by linear matrix inequalities (LMIs) and can be verified by using MATLAB LMI Toolbox. At the end a Numerical simulations are given to express the efficiency of the derived results.

Index Terms: Singular Systems; Networked Cascade Control; Sampled-Data; Convex Combination.

I. INTRODUCTION

Singular systems in other word descriptor systems have been studied by many researchers because of its valuable applications in various fields like aerospace systems, electrical circuits, power systems and mechanical systems and so on. In recent works number of standard state space are converted to singular systems. More preciously, the singular systems are more complicated to compare with other standard type systems. On recent days a large number of research article based on stability analysis are solved by many researchers.

In recent days number of improvements related to cascade control systems have been developed due its importance in many fields. [1] Developed the concept of H-infinity state feedback control for a class of networked cascade control systems with uncertain delay. [2] Investigated Cascade high gain predictors for a



class of nonlinear systems and Performance assessment of cascade control systems. Cascade control of superheated steam temperature with neuro-pid controller been developed by [3].

Robust dissipativity for delayed neural networks with random uncertainties has been modeled by [12]. Reliable dissipative control of discrete-time singular systems with time delays studied by [13]. Dissipative analysis for Markovian jump systems with time-varying delays, neural networks with time-varying delay and randomly occurring uncertainties, sliding mode control of switched stochastic systems and discrete-Time T-S fuzzy stochastic systems with time-varying delay been discussed by [14]-[16].

Motivated by the above consideration, this is the first attempt to consider the problem of dissipative analysis for singular NCCSs with random time varying delay and external disturbances via sampled-data cascade control. By implementing Wirtinger's inequality, a new set of sufficient conditions are developed which ensure that the resulting closed-loop system is admissible.

Notations: Throughout this paper, Superscripts " T " and " (-1) " stand for matrix transposition and matrix inverses respectively. R^n denotes the n -dimensional Euclidean space. Z_+ denotes the set of positive integers. $R^{n \times n}$ denotes the set of all $n \times n$ real matrices. $P > 0$ (respectively $P < 0$) means that P is positive definite (respectively, negative definite). I and 0 represent identity matrix and zero matrix with compatible dimension.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this section, we first present the system model for the NCCSs with a singular plant. The considered system is based on a class of singular plant with the data packets are transmitted via networks. The communication network are considered with the network-induced delay and packet loss simultaneously and the controllers depend on state vectors of the respective plants.

The cascade system under consideration have the primary plant given by the following equation:

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 y_2(t), \\ y_1(t) = C_1 x_1(t) + C_3 w(t), \end{cases} \quad (1)$$

where $x_1(t)$ and $y_1(t)$ are the state vector and output vector of the primary plant. A_1 , B_1 , C_1 and C_3 are known real constant matrices with appropriate dimensions.

Further, a continuous-time system secondary plant is formulated using a class of linear time invariant singular system with time-varying state delay and external disturbances, given by

$$\begin{cases} E \dot{x}_2(t) = A_2 x_2(t) + A_3 x_2(t - h(t)) + B_2 u_2(t) + B_3 w(t), \\ y_2(t) = C_2 x_2(t) + C_4 w(t), \\ x_2(t) = \phi(t), \quad t \in [-h, 0], \end{cases} \quad (2)$$

where $x_2(t)$ is the state vector; $u_2(t)$ is the control input vector; $w(t) \in L_2[0, \infty)$ is the disturbance with limited energy; $y_2(t)$ is the output of the secondary plant; matrix E may be singular and it is assumed that $\text{rank } E = r \leq n$; the function $\phi(t)$ is the initial condition defined on $[-h, 0]$. The constant matrices



A_2, A_3, B_2, B_3, C_2 and C_4 are the connection weights with appropriate dimensions. The time-varying delay in (2) is assumed to satisfy $0 \leq h(t) \leq h$ with $\dot{h}(t) = \mu < 1$, h is a positive constant representing maximum time delay.

The state feedback controller in secondary plant is described as;

$$u_2(t) = u_1(t) + K_2 x_2(t), \quad (3)$$

where $u_1(t)$ is the control input of the primary plant.

It is noted that, in networked cascade control system the data packets are transmitted via network, so there is a possibility of the control signals depend on the sampling communication and performing control operations according to the sampling results. Especially, the sampled data control problem will be formulated through an input delay approach. By considering this point, we consider the control input of primary plant of the form

$$u_1(t) = K_1 x_1(t_k), \quad t_k \leq t < t_{k+1}, \quad k = 0, 1, 2, \dots,$$

where K_1 is the gain matrix to be determined. The state variables of system are measured at time instants $\dots, t_k, t_{k+1}, \dots$, that is, only $x_1(t_k)$ are available for interval $t_k \leq t < t_{k+1}$. The interval between any two sampling instants is assumed to be bounded by τ for any $k \geq 0$, $t_{k+1} - t_k = \tau_k \leq \tau$ always hold, where τ is the maximum upper bound of the sampling interval. Usually, in existing literature, the input delay due to sampling is assumed as $\tau(t) = t - t_k$ then the sampling interval can be written as $t_k = t - (t - t_k) = t - \tau(t)$.

[A1:] Let the time delay $\tau(t)$ can be bounded by $0 \leq \tau(t) \leq \tau_2$ and its probability distribution is assumed as follows; Suppose $\tau(t)$ takes values in $[0 : \tau_1]$ or $(\tau_1, \tau_2]$ and $\text{prob}\{\tau(t) \in [0 : \tau_1]\} = \delta_0$ or $\text{prob}\{\tau(t) \in (\tau_1 : \tau_2]\} = 1 - \delta_0$ where τ_1, τ_2 are integers satisfying $0 \leq \tau(t) \leq \tau_2$ and $0 \leq \delta_0 \leq 1$. Further, in order to describe the probability distribution of the time delay, define two sets as follows;

$$D_1 = \{t/\tau(t) \in [0 : \tau_1]\} \text{ and } D_2 = \{t/\tau(t) \in (\tau_1 : \tau_2]\}. \quad (4)$$

[A2:] Time varying delays $\tau_1(t)$ and $\tau_2(t)$ satisfying the condition

$$0 \leq \tau_1(t) \leq \tau_1, \quad \tau_1(t) = 1 \text{ and } \tau_1 \leq \tau_2(t) \leq \tau_2, \quad \tau_2(t) = 1 \quad (5)$$

Further it follows from $D_1 \cup D_2 = Z > 0$, $D_1 \cap D_2 = \phi$, where ϕ is the empty set. It is easy to check that

$t \in D_1$ implies that event $\tau(t) \in [0 : \tau_1]$ occurs and $t \in D_2$ implies that event $\tau(t) \in (\tau_1 : \tau_2]$ occurs.

Define a Bernoulli random variable as;



$$\delta(t) = \begin{cases} 1, & t \in D_1, \\ 0, & t \in D_2. \end{cases} \quad (6)$$

Now to introduce time varying delays $\tau_1(t)$ and $\tau_2(t)$ such that

$$\tau_1(t) = \begin{cases} \tau(t), & t \in D_1 \\ 0, & \text{elsewhere} \end{cases}; \quad \tau_2(t) = \begin{cases} \tau(t), & t \in D_2 \\ \tau_1, & \text{elsewhere} \end{cases} \quad (7)$$

Remark 2.1: Under the Assumption II and above equation, it can be seen that $\delta(t)$ is a Bernoulli distributed white sequence with $P\{\delta(t) = 1\} = E[\delta(t)] = \delta_0$ and $P\{\delta(t) = 0\} = E[\delta(t)] = 1 - \delta_0$. Furthermore, we can show that $E[\delta(t) - \delta_0] = 0$ and $E[(\delta(t) - \delta_0)^2] = \delta_0(1 - \delta_0)$.

Combining (1), (2), and (3), the closed-loop model of the singular networked cascade control system with time

Varying delay and external disturbance can be described as;

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 C_2 x_2(t) + B_1 C_4 w(t), \\ E \dot{x}_2(t) = A_2 x_2(t) + A_3 x_2(t - h(t)) + \delta(t) B_2 K_1 x_1(t - \tau_1(t)) + (1 - \delta(t)) B_2 K_1 x_1(t - \tau_2(t)) \\ + B_2 K_2 x_2(t) + B_3 w(t), \\ y_1(t) = C_1 x_1(t) + C_3 w(t). \end{cases} \quad (8)$$

Definition 2.2: 1. The pair (E, A_2) is said to be regular if $\det(sE - A_2)$ is not identically zero. 2. The pair (E, A_2) is said to be impulse free if $\deg(\det(sE - A_2)) = \text{rank}(E)$.

3. The unforced singular system is said to be regular and impulse free, if the pair (E, A_2) is regular and impulse free.

Definition 2.3: Given scalar $\theta > 0$, matrices Q, R and S with Q and R real symmetric, system (8) is strictly

(Q, S, R) dissipative if for $t > 0$ under zero initial state, the following condition is satisfied:

$$\langle z, Qz \rangle_t + 2\langle z, Sw \rangle_t + \langle w, Rw \rangle_t \geq \theta \langle w, w \rangle_t.$$

Without loss of generality, we assume that the matrix $Q \leq 0$ and $Q = -Q$.

Remark 2.4: From the above definition, it can be seen that the strict (Q, S, R) dissipativeness includes H_1 and



passivity as special cases, which are stated as follows, **1.** If $\mathcal{Q} = -I$, $\mathcal{S} = 0$ and $\mathcal{R} = \gamma^2 I$, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative reduces to a H_∞ performance constraint. **2.** If $\mathcal{Q} = 0$, $\mathcal{S} = I$ and $\mathcal{R} = 0$, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative reduces to a passivity performance. **3.** If $\mathcal{R} = \theta_1 \gamma^2 I$, $\theta_1 \in [0, 1]$ or $\mathcal{Q} = -\gamma^{-1} \theta_1 I$, $\mathcal{S} = (1 - \theta_1)I$ and $\mathcal{R} = \gamma \theta_1 I$, where $\theta_1 \in [0, 1]$ be a given scalar weight representing a trade off between H_∞ and passivity performance, then $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipative reduces to the mixed H_∞ and positive real performance.

Lemma 2.5: Given constant matrices Ξ_1 , Ξ_2 and Ξ_3 with appropriate dimensions, where $\Xi_1 = \Xi_1^T > 0$ and $\Xi_2 = \Xi_2^T > 0$ then $\Xi_1 + \Xi_3^T \Xi_2^{-1} \Xi_3 < 0$ if and only if $\begin{bmatrix} \Xi_1 & \Xi_3^T \\ * & -\Xi_2 \end{bmatrix} < 0$.

Lemma 2.6: For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t)$, $[a, b] \rightarrow \mathbb{R}^n$ such that the integrals concerned are well defined, then the following holds

$$\left[\int_a^b x(s) ds \right]^T M \left[\int_a^b x(s) ds \right] \leq (b-a) \int_a^b x^T(s) M x(s) ds.$$

Lemma 2.7: For any matrix $R \in \mathbb{R}^{n \times m}$, $R = R^T > 0$, any differentiable function ω in $[a, b] \rightarrow \mathbb{R}^n$ the following inequalities holds:

$$\int_a^b \dot{\omega}^T(s) R \dot{\omega}(s) ds \geq \frac{\varsigma^T \left[W_1^T R W_1 + \pi^2 W_2^T R W_2 \right] \varsigma}{b-a},$$

where $\varsigma = [\omega^T(b) \ \omega^T(a) \ \int_a^b \omega^T(s)/(b-a) ds]^T$, $W_1 = [I - I \ 0]$, $W_2 = [I/2 \ I/2 \ -I]$.

Lemma 2.8: For any positive definite matrix R_1 and matrix W with appropriate dimensions, vector function $\xi(t)$ and $x_1(t)$, scalars τ_1 and τ_2 satisfying $\tau_2 > \tau_1$ such that the integrations concerned are well defined, we have

$$-\int_{\tau_1}^{\tau_2} \dot{x}_1^T(s) R_1 \dot{x}_1(s) ds \leq (\tau_2 - \tau_1) \eta^T(t) W R_1^{-1} W^T \eta(t) + 2\eta(t) W \int_{\tau_1}^{\tau_2} x(s) ds.$$

III. MAIN RESULTS

In this section, we investigate the dissipative sampled data control design for singular NCCS (1) and (2) with time-varying differentiable delay. Further the sufficient conditions for admissibility of considered singular NCCS is proved, which is a key result in studying the singular system.

Theorem 3.1: For given scalars $\theta > 0$, $\tau_1 > 0$, $\tau_2 > 0$ and the actuator fault matrix G is known, matrices $\mathcal{Q} = \mathcal{Q}^T$, $\mathcal{R} = \mathcal{R}^T$ and $\mathcal{S}$ with the time-varying delay $\tau_1(t)$ and $\tau_2(t)$ the system (8) is admissible and strictly dissipative, if there exist symmetric positive definite matrices $\hat{S}_i$, $i = 1, 2, 3$, $\hat{Q}_i$, $i = 1, 2$, $\hat{R}_i$, $i = 1, 2$, $\hat{L}_i$, $\hat{M}_i$, $\hat{N}_i$ and $\hat{H}_i$, $i = 1, 2, 3, 4$ and real matrices Y_1 and Y_2 such that the following inequalities hold;

$$X_2^T E^T = E X_2 \geq 0,$$

$$X_2^T E^T = E X_2 \geq 0,$$

$$\begin{bmatrix} \hat{\Psi} & \sqrt{\tau_1} \hat{L} \\ * & X_3 - 2X_1 \end{bmatrix} < 0, \quad \begin{bmatrix} \hat{\Psi} & \sqrt{\tau_1} \hat{M} \\ * & X_3 - 2X_1 \end{bmatrix} < 0, \quad \begin{bmatrix} \hat{\Psi} & \sqrt{\tau_2 - \tau_1} \hat{N} \\ * & X_4 - 2X_1 \end{bmatrix} < 0, \quad \begin{bmatrix} \hat{\Psi} & \sqrt{\tau_2 - \tau_1} \hat{H} \\ * & X_4 - 2X_1 \end{bmatrix} < 0$$

$$\text{where } \hat{\Psi} = \begin{bmatrix} [\hat{\Omega}_{l,m}]_{15 \times 15} & \hat{\Omega}_1 & \hat{\Omega}_2 & \hat{\Omega}_3 & \hat{\Omega}_4 \\ * & -R_3 & 0 & 0 & 0 \\ * & * & -R_4 & 0 & 0 \\ * & * & * & -S_4 & 0 \\ * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned} \hat{\Omega}_{1,1} &= A_1 X_1 + \hat{Q}_1 + \tau_1 \hat{R}_1 + (\tau_2 - \tau_1) \hat{R}_2 + \hat{M}_1, \hat{\Omega}_{1,2} = B_1 C_2 X_2, \hat{\Omega}_{1,5} = \hat{L}_1 - \hat{M}_1, \hat{\Omega}_{1,6} = \hat{N}_1 - \hat{H}_1, \\ \hat{\Omega}_{1,7} &= -\hat{L}_1 + \hat{H}_1, \hat{\Omega}_{1,8} = -\hat{N}_1, \hat{\Omega}_{1,15} = B_1 C_4, \hat{\Omega}_{2,2} = A_2 X_2 + B_2 Y_2 + \hat{S}_1 + \hat{S}_2 + h \hat{S}_3 + \frac{1}{h} E^T (X_5 - 2X_2) E + \\ &\frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \hat{\Omega}_{2,3} = A_3 X_2 + \frac{1}{h} E^T (-X_5 + 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \hat{\Omega}_{2,5} = B_2 Y_1 \delta_1, \hat{\Omega}_{2,6} = B_2 Y_1 \delta_2, \\ \hat{\Omega}_{2,11} &= \frac{\pi^2}{2h^2} E^T (X_5 - 2X_2) E, \hat{\Omega}_{2,15} = B_3, \hat{\Omega}_{3,3} = -(1 - \mu) \hat{S}_1 + \frac{1}{h} E^T (X_5 - 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \\ \hat{\Omega}_{3,4} &= \frac{1}{h} E^T (X_5 - 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \hat{\Omega}_{3,10} = \frac{\pi^2}{2h^2} E^T (X_5 - 2X_2) E, \hat{\Omega}_{3,11} = \frac{\pi^2}{2h^2} E^T (X_5 - 2X_2) E, \\ \hat{\Omega}_{4,4} &= -\hat{S}_2 + \frac{1}{h} E^T (X_5 - 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \hat{\Omega}_{4,10} = \frac{\pi^2}{2h^2} E^T (2X_2 - X_5) E, \hat{\Omega}_{5,5} = \hat{L}_2 - \hat{M}_2, \\ \hat{\Omega}_{5,6} &= -\hat{H}_2, \hat{\Omega}_{5,7} = -\hat{L}_2 + \hat{H}_2, \hat{\Omega}_{6,6} = \hat{N}_2 - \hat{H}_3, \hat{\Omega}_{6,7} = -\hat{L}_3 + \hat{H}_3, \hat{\Omega}_{6,8} = -\hat{N}_2, \hat{\Omega}_{7,7} = -\hat{Q}_1 + \hat{Q}_2 - \hat{L}_4 + \hat{H}_4, \\ \hat{\Omega}_{6,8} &= -\hat{N}_3, \hat{\Omega}_{8,8} = -\hat{Q}_2 - \hat{N}_4, \hat{\Omega}_{9,9} = -\frac{\hat{R}_2}{\tau_2 - \tau_1}, \hat{\Omega}_{10,10} = \frac{\hat{S}_3}{h} + \frac{\pi^2}{h^3} E^T (X_5 - 2X_2) E, \hat{\Omega}_{11,11} = -\frac{\hat{S}_3}{h} + \\ &\frac{\pi^2}{h^3} E^T (X_5 - 2X_2) E, \hat{\Omega}_{12,12} = -\frac{\hat{R}_1}{\tau_1}, \hat{\Omega}_{13,13} = -\frac{\hat{R}_1}{\tau_1}, \hat{\Omega}_{14,14} = -\frac{\hat{R}_2}{\tau_2 - \tau_1}, \hat{\Omega}_{15,15} = -C_1 Q - C_3 S - R + \theta I, \\ \hat{\Omega}_1 &= \sqrt{\tau_1} \begin{bmatrix} A_1 X_1^T & B_1 C_2 X_2^T & 0_{12n} & B_1 C_4 \end{bmatrix}^T, \hat{\Omega}_2 = \sqrt{\tau_2 - \tau_1} \begin{bmatrix} A_1 X_1^T & B_1 C_2 X_2^T & 0_{12n} & B_1 C_4 \end{bmatrix}^T, \\ \hat{\Omega}_3 &= \sqrt{h} \begin{bmatrix} 0 & A_2 X_2^T + B_2 Y_2 & A_3 X_2^T & 0_n & \delta_1 B_2 Y_1 & \delta_2 B_2 Y_1 & 0_{8n} & B_3 \end{bmatrix}^T, \hat{\Omega}_4 = \begin{bmatrix} \bar{Q} C_1 X_1^T & 0_{13n} & \bar{Q} C_3 \end{bmatrix}^T. \end{aligned}$$

Moreover, the gain matrices of the feedback controller (3) is given by $K_1 = Y_1 X_1^{-1}$ and $K_2 = Y_2 X_2^{-1}$.

Proof: To prove the NCCS system (8) is admissible, first we need to prove that the system (8) is regular and impulse free. It follows from (3.1)

$$\begin{bmatrix} \Gamma_{2,2} & \Gamma_{2,3} & \Gamma_{2,11} \\ * & \Gamma_{3,3} & \Gamma_{3,11} \\ * & * & \Gamma_{11,11} \end{bmatrix} < 0, \quad (9)$$

$$\begin{aligned} \text{where } \Gamma_{2,2} &= \frac{1}{h} E^T (X_5 - 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \Gamma_{2,3} = \frac{1}{h} E^T (-X_5 + 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \\ \Gamma_{2,11} &= \frac{\pi^2}{2h^2} E^T (X_5 - 2X_2) E, \Gamma_{3,3} = \frac{1}{h} E^T (X_5 - 2X_2) E + \frac{\pi^2}{4h} E^T (X_5 - 2X_2) E, \Gamma_{3,11} = \frac{\pi^2}{2h^2} E^T (X_5 - 2X_2) E, \\ \Gamma_{11,11} &= -\frac{\hat{S}_3}{h} + \frac{\pi^2}{h^3} E^T (X_5 - 2X_2) E. \end{aligned}$$

$$\text{Let } V = \begin{bmatrix} I & 0 & A_2 \\ 0 & I & A_3 \end{bmatrix}, \text{ pre and post multiplying (9) by } V \text{ and } V^T, \text{ respectively, we get } \Phi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_3 \end{bmatrix}, \text{ where}$$

$$\begin{aligned} \Phi_1 &= \Gamma_{2,2} + A_2^T \Gamma_{2,11}^T + \Gamma_{2,11} A_2 + \text{sym}(A_2^T \Gamma_{11,11}^T A_2), \Phi_2 = \Gamma_{2,3} + \Gamma_{2,11} A_3 + A_2^T \Gamma_{11,11}^T A_3, \Phi_3 = \Gamma_{3,3} + \Gamma_{3,11} A_2 + \\ &A_3^T \Gamma_{11,11}^T A_3. \end{aligned}$$



Since $\text{rank}(E) = r \leq n$, there must exist two invertible matrices G and $H \in \mathbb{R}^{n \times n}$, such that

$$\bar{E} = GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \quad (10)$$

Now, the matrix R can be parameterized as $R = G^T \begin{bmatrix} 0 \\ \varphi \end{bmatrix}$, where $\varphi \in \mathbb{R}^{(n-r) \times (n-r)}$ is any non-singular matrix.

$$\text{Similar to (10), we define } \bar{A} = GAH = \begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix}, \bar{P} = G^{-T}PG^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{13} & P_{14} \end{bmatrix}, \bar{F} = H^TF = \begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix}.$$

$$\text{Pre and post multiplying both sides of } \Phi < 0 \text{ by } H^T \text{ and } H, \text{ we have } \hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} \\ \hat{\Phi}_{21} & A_{14}^T \varphi^T F_{12}^T + F_{12} \varphi^T A_{14} \end{bmatrix}.$$

Then, from (8), it is easy to see that $A_{14}^T \varphi^T F_{12}^T + F_{12} \varphi^T A_{14} < 0$. Suppose that A_{14} is singular, there must exist a non-zero vector $\rho \in \mathbb{R}^{n-r}$, which ensures that $A_{14}\rho = 0$.

Therefore, we can conclude that $\rho^T (A_{14}^T \varphi^T F_{12}^T + F_{12} \varphi^T A_{14}) \rho = 0$. Hence A_{14} is non-singular. Then, it is easy to check $\det(s\hat{E} - \hat{A}_2) = \det(sE - A_2)$ which implies that $\det(sE - A_2)$ is not identically zero and $\det(sE - A_2) = r = \text{rank}(E)$. Thus, the pair (E, A_2) is regular and impulse free.

Now, we need to prove the singular NCCS (8) is stable when $w(t) = 0$. Let us define the Lyapunov-Krasovskii function for the system (8);

$$V(x(t)) = \sum_{m=1}^6 V_m(x(t)), \quad (11)$$

where

$$\begin{aligned} V_1(x(t)) &= x_1^T(t)P_1x_1(t) + x_2^T(t)E^TP_2x_2(t), \\ V_2(x(t)) &= \int_{t-\tau_1}^t x_1^T(s)Q_1x_1(s)ds + \int_{t-\tau_2}^{t-\tau_1} x_1^T(s)Q_2x_1(s)ds, \\ V_3(x(t)) &= \int_{-\tau_1}^0 \int_{t+\theta}^t x_1^T(s)R_1x_1(s)dsd\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t x_1^T(s)R_2x_1(s)dsd\theta, \\ V_4(x(t)) &= \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}_1^T(s)R_3\dot{x}_1(s)dsd\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}_1^T(s)R_4\dot{x}_1(s)dsd\theta, \\ V_5(x(t)) &= \int_{t-h(t)}^t x_2^T(s)S_1x_2(s)ds + \int_{t-h}^t x_2^T(s)S_2x_2(s)ds, \\ V_6(x(t)) &= \int_{-h}^0 \int_{t+\theta}^t x_2^T(s)S_3x_2(s)dsd\theta + \int_{-h}^0 \int_{t+\theta}^t \dot{x}_2^T(s)E^TS_4E\dot{x}_2(s)dsd\theta, \end{aligned}$$

Calculating the derivatives $\dot{V}(x(t))$ along the trajectories of the system (8), we have

$$\begin{aligned}\dot{V}_1(x(t)) = & 2x_1^T(t)P_1[A_1x_1(t) + B_1C_2x_2(t)] + 2x_2^T(t)P_2[A_2x_2(t) + A_3x_2(t-h(t)) \\ & + \delta(t)B_2K_1x_1(t-\tau_1(1)) + (1-\delta(t))B_2K_1x_1(t-\tau_2(t)) + B_2K_2x_2(t)],\end{aligned}\quad (12)$$

$$\begin{aligned}\dot{V}_2(x(t)) = & x_1^T(t)Q_1x_1(t) - x_1^T(t-\tau_1)Q_1x_1(t-\tau_1) + x_1^T(t-\tau_1)Q_2x_1(t-\tau_1) \\ & - x_1^T(t-\tau_2)Q_2x_1(t-\tau_2)\end{aligned}\quad (13)$$

$$\begin{aligned}\dot{V}_3(x(t)) = & \tau_1x_1^T(t)R_1x_1(t) - \int_{t-\tau_1}^t x_1^T(s)R_1x_1(s)ds + (\tau_2-\tau_1)x_1^T(t)R_2x_1(t) \\ & - \int_{t-\tau_2}^{t-\tau_1} x_1^T(s)R_2x_1(s)ds,\end{aligned}\quad (14)$$

$$\begin{aligned}\dot{V}_4(x(t)) = & \tau_1\dot{x}_1^T(t)R_3\dot{x}_1(t) - \int_{t-\tau_1}^t \dot{x}_1^T(s)R_3\dot{x}_1(s)ds + (\tau_2-\tau_1)\dot{x}_1^T(t)R_4\dot{x}_1(t) \\ & - \int_{t-\tau_2}^{t-\tau_1} \dot{x}_1^T(s)R_4\dot{x}_1(s)ds,\end{aligned}\quad (15)$$

$$\dot{V}_5(x(t)) = x_2^T(t)S_1x_2(t) - (1-\mu)x_2^T(t-h(t))S_1x_2(t-h(t)) + x_2^T(t)S_2x_2(t) - x_2^T(t-h)S_2x_2(t-h)\quad (16)$$

$$\dot{V}_6(x(t)) = hx_2^T(t)S_3x_2(t) - \int_{t-h}^t x_2^T(s)S_3x_2(s)ds + h\dot{x}_2^T(t)E^T S_4 E \dot{x}_2(t) - \int_{t-h}^t \dot{x}_2^T(s)E^T S_4 E \dot{x}_2(s)ds \quad (17)$$

By applying Jensen inequality Lemma 2.6 for the integral terms in (14), (17) and the time delay interval, the integrations in the above equations can be written as

$$\begin{aligned}- \int_{t-\tau_1}^t x_1^T(s)R_1x_1(s)ds &= - \int_{t-\tau_1}^{t-\tau_1(t)} x_1^T(s)R_1x_1(s)ds - \int_{t-\tau_1(t)}^t x_1^T(s)R_1x_1(s)ds \\ - \int_{t-\tau_1}^{t-\tau_1(t)} x_1^T(s)R_1x_1(s)ds &\leq - \frac{1}{\tau_1} \left[\int_{t-\tau_1}^{t-\tau_1(t)} x_1(s)ds \right]^T R_1 \left[\int_{t-\tau_1}^{t-\tau_1(t)} x_1(s)ds \right]\end{aligned}\quad (18)$$

$$- \int_{t-\tau_1(t)}^t x_1^T(s)R_1x_1(s)ds \leq - \frac{1}{\tau_1} \left[\int_{t-\tau_1(t)}^t x_1(s)ds \right]^T R_1 \left[\int_{t-\tau_1(t)}^t x_1(s)ds \right]\quad (19)$$

$$\begin{aligned}- \int_{t-\tau_2}^{t-\tau_1} x_1^T(s)R_2x_1(s)ds &= - \int_{t-\tau_2}^{t-\tau_2(t)} x_1^T(s)R_2x_1(s)ds - \int_{t-\tau_2(t)}^{t-\tau_1} x_1^T(s)R_2x_1(s)ds \\ - \int_{t-\tau_2}^{t-\tau_2(t)} x_1^T(s)R_2x_1(s)ds &\leq - \frac{1}{\tau_2-\tau_1} \left[\int_{t-\tau_2}^{t-\tau_2(t)} x_1(s)ds \right]^T R_2 \left[\int_{t-\tau_2}^{t-\tau_2(t)} x_1(s)ds \right]\end{aligned}\quad (20)$$

$$- \int_{t-\tau_2(t)}^{t-\tau_1} x_1^T(s)R_2x_1(s)ds \leq - \frac{1}{\tau_2-\tau_1} \left[\int_{t-\tau_2(t)}^{t-\tau_1} x_1(s)ds \right]^T R_2 \left[\int_{t-\tau_2(t)}^{t-\tau_1} x_1(s)ds \right]\quad (21)$$

$$\begin{aligned}- \int_{t-h}^t x_2^T(s)S_3x_2(s)ds &= - \int_{t-h}^{t-h(t)} x_2^T(s)S_3x_2(s)ds - \int_{t-h(t)}^t x_2^T(s)S_3x_2(s)ds \\ - \int_{t-h}^{t-h(t)} x_2^T(s)S_3x_2(s)ds &\leq - \frac{1}{h} \left[\int_{t-h}^{t-h(t)} x_2(s)ds \right]^T S_3 \left[\int_{t-h}^{t-h(t)} x_2(s)ds \right]\end{aligned}\quad (22)$$

$$- \int_{t-h(t)}^t x_2^T(s)S_3x_2(s)ds \leq - \frac{1}{h} \left[\int_{t-h(t)}^t x_2(s)ds \right]^T S_3 \left[\int_{t-h(t)}^t x_2(s)ds \right]\quad (23)$$



By applying Wirtinger-based inequality lemma 2.7 for the integral term in (17), we can get the following inequalities;

$$-\int_{t-h}^t \dot{x}_2^T(s) E^T S_4 E \dot{x}_2(s) ds = -\int_{t-h}^{t-h(t)} \dot{x}_2^T(s) E^T S_4 E \dot{x}_2(s) ds - \int_{t-h(t)}^t \dot{x}_2^T(s) E^T S_4 E \dot{x}_2(s) ds$$

$$-\int_{t-h}^{t-h(t)} \dot{x}_2^T(s) E^T S_4 E \dot{x}_2(s) ds \leq \frac{-1}{h} \begin{bmatrix} x_2(t-h(t)) \\ x_2(t-h) \\ \frac{1}{h} \int_{t-h}^{t-h(t)} x_2(s) ds \end{bmatrix}^T W \begin{bmatrix} x_2(t-h(t)) \\ x_2(t-h) \\ \frac{1}{h} \int_{t-h}^{t-h(t)} x_2(s) ds \end{bmatrix}, \quad (24)$$

$$-\int_{t-h(t)}^t \dot{x}_2^T(s) E^T S_4 E \dot{x}_2(s) ds \leq \frac{-1}{h} \begin{bmatrix} x_2(t) \\ x_2(t-h(t)) \\ \frac{1}{h} \int_{t-h(t)}^t x_2(s) ds \end{bmatrix}^T W \begin{bmatrix} x_2(t) \\ x_2(t-h(t)) \\ \frac{1}{h} \int_{t-h(t)}^t x_2(s) ds \end{bmatrix} \quad (25)$$

$$\text{where } W = \begin{bmatrix} E^T S_4 E & -E^T S_4 E & 0 \\ * & E^T S_4 E & 0 \\ * & * & 0 \end{bmatrix} + \begin{bmatrix} \frac{\pi^2}{4} E^T S_4 E & \frac{\pi^2}{4} E^T S_4 E & -\frac{\pi^2}{2} E^T S_4 E \\ * & \frac{\pi^2}{4} E^T S_4 E & -\frac{\pi^2}{2} E^T S_4 E \\ * & * & \pi^2 E^T S_4 E \end{bmatrix}.$$

By using Lemma 2.8, for any arbitrary matrices L, M, N and H , with compatible dimensions, the following inequalities hold;

$$-\int_{t-\tau_1}^t \dot{x}_1^T(s) R_3 \dot{x}_1(s) ds = -\int_{t-\tau_1}^{t-\tau_1(t)} \dot{x}_1^T(s) R_3 \dot{x}_1(s) ds - \int_{t-\tau_1(t)}^t \dot{x}_1^T(s) R_3 \dot{x}_1(s) ds \quad (26)$$

$$-\int_{t-\tau_1}^{t-\tau_1(t)} \dot{x}_1^T(s) R_3 \dot{x}_1(s) ds \leq (\tau_1 - \tau_1(t)) \eta^T(t) L R_3^{-1} L^T \eta(t) + 2\eta(t) L (x_1(t - \tau_1(t)) - x_1(t - \tau_1)) \quad (27)$$

$$-\int_{t-\tau_1(t)}^t \dot{x}_1^T(s) R_3 \dot{x}_1(s) ds \leq \tau_1(t) \eta^T(t) M R_3^{-1} M^T \eta(t) + 2\eta(t) M (x_1(t) - x_1(t - \tau_1(t))) \quad (28)$$

$$-\int_{t-\tau_2}^{t-\tau_1} \dot{x}_1^T(s) R_4 \dot{x}_1(s) ds = -\int_{t-\tau_2}^{t-\tau_2(t)} \dot{x}_1^T(s) R_4 \dot{x}_1(s) ds - \int_{t-\tau_2(t)}^{t-\tau_1} \dot{x}_1^T(s) R_4 \dot{x}_1(s) ds \quad (29)$$

$$-\int_{t-\tau_2}^{t-\tau_2(t)} \dot{x}_1^T(s) R_4 \dot{x}_1(s) ds \leq (\tau_2 - \tau_2(t)) \eta^T(t) N R_4^{-1} N^T \eta(t) + 2\eta(t) N (x_1(t - \tau_2(t)) - x_1(t - \tau_2)) \quad (30)$$

$$-\int_{t-\tau_2(t)}^{t-\tau_1} \dot{x}_1^T(s) R_4 \dot{x}_1(s) ds \leq (\tau_2(t) - \tau_1) \eta^T(t) H R_4^{-1} H^T \eta(t) + 2\eta(t) H (x_1(t - \tau_1) - x_1(t - \tau_2(t))) \quad (31)$$

where $L = [L_1^T \ 0_{3n} \ L_2^T \ L_3^T \ L_4^T \ 0_{8n}]^T$, $M = [M_1^T \ 0_{3n} \ M_2^T \ M_3^T \ M_4^T \ 0_{8n}]^T$, $N = [N_1^T \ 0_{4n} \ N_2^T \ N_3^T \ N_4^T \ 0_{7n}]^T$ and $H = [H_1^T \ 0_{3n} \ H_2^T \ H_3^T \ H_4^T \ 0_{8n}]^T$ and $\eta(t) = [x_1^T(t) \ x_2^T(t) \ x_2^T(t-h(t)) \ x_2^T(t-h) \ x_1^T(t-\tau_1(t)) \ x_1^T(t-\tau_2(t)) \ x_1^T(t-\tau_1) \ x_1^T(t-\tau_2) \ \int_{t-\tau_2(t)}^{t-\tau_1} x_1^T(s) ds \ \int_{t-h}^{t-h(t)} x_2^T(s) ds \ \int_{t-h(t)}^t x_2^T(s) ds \ \int_{t-\tau_1}^{t-\tau_1(t)} x_1^T(s) ds \ \int_{t-\tau_1(t)}^t x_1^T(s) ds \ \int_{t-\tau_2}^{t-\tau_2(t)} x_1^T(s) ds]^T$.

Combaining and applying Schur complement lemma we get

$$\dot{V}(x(t)) \leq \eta^T(t) \psi \eta(t) \quad (32)$$



where $\psi = \Theta + (\tau_1 - \tau_1(t))LR_3^{-1}L^T + \tau_1(t)MR_3^{-1}M^T + (\tau_2 - \tau_2(t))NR_4^{-1}N^T + (\tau_2(t) - \tau_1)HR_4^{-1}H^T$ with

$$\Theta = \begin{bmatrix} [\Omega_{l,m}]_{14 \times 14} & \Omega_1 & \Omega_2 & \Omega_3 \\ * & -R_3 & 0 & 0 \\ * & * & -R_4 & 0 \\ * & * & * & -S_4 \end{bmatrix},$$

where

$$\begin{aligned} \Omega_{1,1} &= P_1A_1 + Q_1 + \tau_1R_1 + (\tau_2 - \tau_1)R_2 + M_1, \Omega_{1,2} = P_1B_1C_2, \Omega_{1,5} = L_1 - M_1, \Omega_{1,6} = N_1 - H_1, \\ \Omega_{1,7} &= -L_1 + H_1, \Omega_{1,8} = -N_1, \Omega_{2,2} = P_2A_2 + P_2B_2K_2 + S_1 + S_2 + S_3 - \frac{1}{h}E^TS_4E - \frac{\pi^2}{4h}E^TS_4E, \Omega_{2,3} = \\ P_2A_3 &+ \frac{1}{h}E^TS_4E - \frac{\pi^2}{4h}E^TS_4E, \Omega_{2,5} = P_2B_2K_1\delta_1, \Omega_{2,6} = P_2B_2K_1\delta_2, \Omega_{2,11} = \frac{\pi^2}{4h^2}E^TS_4E, \Omega_{3,3} = -(1 - \\ \mu)S_1 &- \frac{1}{h}E^TS_4E + \frac{\pi^2}{4h}E^TS_4E, \Omega_{3,4} = \frac{1}{h}E^TS_4E - \frac{\pi^2}{4h}E^TS_4E, \Omega_{3,10} = \frac{\pi^2}{2h^2}E^TS_4E, \Omega_{3,11} = \frac{\pi^2}{2h^2}E^TS_4E, \Omega_{4,4} = \\ -S_2 &- \frac{1}{h}E^TS_4E - \frac{\pi^2}{4h}E^TS_4E, \Omega_{4,10} = \frac{\pi^2}{2h^2}E^TS_4E, \Omega_{5,5} = L_2 - M_2, \Omega_{5,6} = -H_2, \Omega_{5,7} = -L_2 + H_2, \\ \Omega_{6,6} &= N_2 - H_3, \Omega_{6,7} = -L_3 + H_3, \Omega_{6,8} = -N_2, \Omega_{7,7} = Q_1 + Q_2 - L_4 + H_4, \Omega_{7,8} = -N_3, \Omega_{8,8} = -Q_2 - N_4, \\ \Omega_{9,9} &= -\frac{R_2}{\tau_2 - \tau_1}, \Omega_{10,10} = -\frac{S_3}{h} + \frac{\pi^2}{2h^3}E^TS_4E, \Omega_{11,11} = -\frac{S_3}{h} + \frac{\pi^2}{h^3}E^TS_4E, \Omega_{12,12} = -\frac{R_1}{\tau_1}, \Omega_{13,13} = -\frac{R_1}{\tau_1}, \\ \Omega_{14,14} &= -\frac{R_2}{\tau_2 - \tau_1}, \Omega_1 = \sqrt{\tau_1} \begin{bmatrix} R_3^TA_1 & R_3^TB_1C_2 & 0_{12n} \end{bmatrix}^T, \Omega_2 = \sqrt{\tau_2 - \tau_1} \begin{bmatrix} R_4^TA_1 & R_4^TB_1C_2 & 0_{12n} \end{bmatrix}^T, \Omega_3 = \\ \sqrt{h} \begin{bmatrix} S_4^TA_2 + S_4^TB_2K_2 & S_4^TA_3 & 0_{2n} & S_4^TB_2K_1\delta_1 & S_4^TB_2K_1\delta_2 & 0_{8n} \end{bmatrix}^T. \end{aligned}$$

Then, by applying the idea of convex combination, for the interval $0 < \tau_1(t) < \tau_1$ and $\tau_1 < \tau_2(t) < \tau_2$, the term Ψ in (32) can be changed equivalently into the form

$$\Psi_1 = \Theta + \tau_1LR_3^{-1}L^T \quad (33)$$

$$\Psi_2 = \Theta + \tau_1MR_3^{-1}M^T \quad (34)$$

$$\Psi_3 = \Theta + (\tau_2 - \tau_1)NR_4^{-1}N^T \quad (35)$$

$$\Psi_4 = \Theta + (\tau_2 - \tau_1)HR_4^{-1}H^T \quad (36)$$

By using Schur complement lemma and in view of (3.1), (3.1), (3.1) and (3.1) we conclude that the system (8) is asymptotically stable.

Next, we study about the dissipativity of system (8) with nonzero $w(t)$ and given disturbance attenuation level $\theta > 0$, for this we introduce the following relation:

$$\begin{aligned} J_t &= -\langle z, Qz \rangle_t - 2\langle z, Sw \rangle_t - \langle \omega, R\omega \rangle_t + \theta\langle \omega, \omega \rangle_t \\ &= \int_0^t [-z(s)^T Qz(s) - 2z^T(s)Sw(s) - \omega^T(s)R\omega(s) + \omega^T(s)\theta\omega(s)] ds. \end{aligned} \quad (37)$$

In simple terms, we write

$$[\dot{V}(x(t)) + \int_0^t (-z(s)^T Q z(s) - 2z^T(s) S w(s) - \omega^T(s) R \omega(s) + \omega^T(s) \theta \omega(s)) ds] \leq \eta_1^T(t) \hat{\Psi} \eta_1(t), \quad (38)$$

where $\eta_1(t) = [x_1^T(t) \ x_2^T(t) \ x_2^T(t-h(t)) \ x_2^T(t-h) \ x_1^T(t-\tau_1(t)) \ x_1^T(t-\tau_2(t)) \ x_1^T(t-\tau_1) \ x_1^T(t-\tau_2) \ \int_{t-\tau_2(t)}^{t-\tau_1} x_1^T(s) ds \ \int_{t-h}^{t-h(t)} x_2^T(s) ds \ \int_{t-h(t)}^t x_2^T(s) ds \ \int_{t-\tau_1}^{t-\tau_1(t)} x_1^T(s) ds \ \int_{t-\tau_1(t)}^t x_1^T(s) ds \ \int_{t-\tau_2}^{t-\tau_2(t)} x_1^T(s) ds \ w^T(t)]$ with

$$\hat{\Psi} = \begin{bmatrix} [\Omega_{l,m}]_{15 \times 15} & \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 \\ * & -R_3 & 0 & 0 & 0 \\ * & * & -R_4 & 0 & 0 \\ * & * & * & -S_4 & 0 \\ * & * & * & * & -I \end{bmatrix},$$

where $\Omega_{1,1} = P_1 A_1 + Q_1 + \tau_1 R_1 + (\tau_2 - \tau_1) R_2 + M_1$, $\Omega_{1,2} = P_1 B_1 C_2$, $\Omega_{1,5} = M_1$, $\Omega_{1,6} = N_1 - H_1$, $\Omega_{1,7} = -L_1 + H_1$, $\Omega_{1,8} = -N_1$, $\Omega_{1,15} = P_1 B_1 C_4$, $\Omega_{2,2} = P_2 A_2 + P_2 B_2 K_2 + S_1 + S_2 + h S_3 - \frac{1}{h} E^T S_4 E - \frac{\pi^2}{4h} E^T S_4 E$, $\Omega_{2,3} = P_2 A_3 + \frac{1}{h} E^T S_4 E - \frac{\pi^2}{4h} E^T S_4 E$, $\Omega_{2,5} = P_2 B_2 K_1 \delta_1$, $\Omega_{2,6} = P_2 B_2 K_1 \delta_2$, $\Omega_{2,11} = \frac{\pi^2}{2h^2} E^T S_4 E$, $\Omega_{2,15} = P_2 B_3$, $\Omega_{3,3} = -(1-\mu) S_1 - \frac{1}{h} E^T S_4 E + \frac{\pi^2}{4\tau} E^T S_4 E$, $\Omega_{3,4} = \frac{1}{h} E^T S_4 E - \frac{\pi^2}{4h} E^T S_4 E$, $\Omega_{3,10} = \frac{\pi^2}{2h^2} E^T S_4 E$, $\Omega_{3,11} = \frac{\pi^2}{2h^2} E^T S_4 E$, $\Omega_{4,4} = -S_2 - \frac{1}{h} E^T S_4 E - \frac{\pi^2}{4h} E^T S_4 E$, $\Omega_{4,10} = \frac{\pi^2}{2h^2} E^T S_4 E$, $\Omega_{5,5} = L_2 - M_2$, $\Omega_{5,6} = -H_5$, $\Omega_{5,7} = -L_2 + H_2$, $\Omega_{6,6} = N_2 - H_3$, $\Omega_{6,7} = -L_3 + H_3$, $\Omega_{6,8} = -N_2$, $\Omega_{7,7} = -Q_1 + Q_2 - L_4 + H_4$, $\Omega_{7,8} = -N_3$, $\Omega_{8,8} = -Q_2 - N_4$, $\Omega_{9,9} = -\frac{R_2}{\tau_2 - \tau_1}$, $\Omega_{10,10} = \frac{S_3}{h} - \frac{\pi^2}{h^3} E^T S_4 E$, $\Omega_{11,11} = \frac{S_3}{h} - \frac{\pi^2}{h^3} E^T S_4 E$, $\Omega_{12,12} = -\frac{R_1}{\tau_1}$, $\Omega_{13,13} = -\frac{R_1}{\tau_1}$, $\Omega_{14,14} = -\frac{R_2}{\tau_2 - \tau_1}$, $\Omega_{15,15} = -C_1 Q - C_3 S - R + \theta I$, $\Omega_1 = \sqrt{\tau_1} \begin{bmatrix} R_3^T A_1 & R_3^T B_1 C_2 & 0_{12n} & R_3^T B_1 C_4 \end{bmatrix}^T$, $\Omega_2 = \sqrt{\tau_2 - \tau_1} \begin{bmatrix} R_4^T A_1 & R_4^T B_1 C_2 & 0_{12n} & R_4^T B_1 C_4 \end{bmatrix}^T$, $\Omega_3 = \sqrt{h} \begin{bmatrix} 0_n & S_4^T A_2 + S_4^T B_2 K_2 & S_4^T A_3 & 0_n & S_4^T B_2 K_1 \delta_1 & S_4^T B_2 K_1 \delta_2 & 0_{8n} & S_4^T B_3 \end{bmatrix}^T$, $\Omega_4 = \begin{bmatrix} \bar{Q} C_1 & 0_{13n} & \bar{Q} C_3 \end{bmatrix}^T$. Further the equations (33), (34), (35) and (36) reduces to

$$\begin{bmatrix} \hat{\Psi} & \sqrt{\tau_1 - \tau_0} L \\ * & -R_3 \end{bmatrix}, \begin{bmatrix} \hat{\Psi} & \sqrt{\tau_0} M \\ * & -R_3 \end{bmatrix}, \begin{bmatrix} \hat{\Psi} & \sqrt{\tau_2 - \tau_1} N \\ * & -R_4 \end{bmatrix}, \begin{bmatrix} \hat{\Psi} & \sqrt{\tau_2 - \tau_1} H \\ * & -R_4 \end{bmatrix}.$$

To complete the proof, pre and post-multiplying (III), (III), (III) and (III) by $\text{diag}\{X_1, X_2, X_2, X_2, X_1, X_1, X_1, X_1, X_1, X_2, X_2, X_1, X_1, X_1, I, X_3, X_4, X_5, I, X_1\}$ and by letting $X_2^T E^T = E X_2$, $X_1 = P_1^{-1}$, $X_2 = P_2^{-1}$, $X_3 = R_3^{-1}$, $X_4 = R_4^{-1}$, $X_5 = S_4^{-1}$, $\hat{S}_i = X_2 S_i X_2$, $i = 1, 2, 3$, $\hat{Q}_i = X_1 Q_i X_1$, $i = 1, 2$, $\hat{R}_i = X_1 R_i X_1$, $i = 1, 2$, $\hat{L}_i = X_1 L_i X_1$, $i = 1, 2, 3, 4$, $\hat{M}_i = X_1 M_i X_1$, $i = 1, 2, 3, 4$, $\hat{N}_i = X_1 N_i X_1$, $i = 1, 2, 3, 4$, $\hat{H}_i = X_1 H_i X_1$, $i = 1, 2, 3, 4$.

By considering (37) and (38), we can find that the LMIs (III), (III), (III) and (III) are equivalent to the LMIs (3.1),

(3.1), (3.1) and (3.1). Then, we have

$$J_t \leq \left[\int_0^t [-z(s)^T Q z(s) - 2z^T(s) \mathcal{S} w(s) - \omega^T(s)(\mathcal{R} + \theta I) \omega(s)] ds + \dot{V}(x(t)) \right] < 0$$

for any non-zero $w(t) \in L_2[0, \infty)$. This implies

$$\left[\int_0^t (z(s)^T Q z(s) + 2z^T(s) \mathcal{S} w(s) + \omega^T(s) \omega(s)) ds \right] > \theta \int_0^t \omega(s)^T \omega(s) ds. \quad (39)$$

Thus, we conclude the system (8) is strictly $(Q, \mathcal{S}, \mathcal{R})$ dissipative according to Definition 2.3. Hence the proof is completed.

IV. NUMERICAL SIMULATION

In this section, we provide a simulation example to illustrate the effectiveness and applicability of the proposed method. The example is constructed by a heating furnace model (power plant boiler-turbine system), which can be described by a singular networked cascade control system with time varying delay and external disturbance.

Here we consider the heating furnace of power transmission system, it is an important part of the steam generation process. In this process there are two sensors associated with the furnace, the primary one transforms the information of the primary plant and the secondary one transforms the information of the secondary plant. The information of the primary and/or secondary plant include the temperature, disturbance and so on of the furnace. The secondary controller can control the valve of the furnace, thus the quantity of the fuel material can be modulated. The disturbance is mainly in the inner loop which can be controlled rapidly. So, the cascade control is an effective one for the temperature control of the heated material.

Let the state-space representation of the primary plant is described by

$$\begin{cases} \dot{x}_1(t) = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix} y_2(t), \\ y_1(t) = \begin{bmatrix} 0 & 0.1 \end{bmatrix} x_1(t) + 0.2w(t). \end{cases} \quad (40)$$

The state-space representation of the secondary plant is described by

$$\begin{cases} E \dot{x}_2(t) = \begin{bmatrix} 1.3 & 1 \\ 0.2 & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 1 \end{bmatrix} x_2(t-h(t)) + \begin{bmatrix} 0.2 \\ 1 \end{bmatrix} u_2(t) + \begin{bmatrix} -0.4 \\ 0.1 \end{bmatrix} w(t), \\ y_2(t) = \begin{bmatrix} -0.3 & 0.1 \end{bmatrix} x_2(t) + 0.1w(t). \end{cases} \quad (41)$$

The state feedback control for the singular NCCS with state delay and disturbance will be designed using Theorem 3.1.

Dissipative case: If we set the state time delay bound $\tau_1 = 0.1$ and assuming $h = 1.21$, $Q = -1$, $R = 0.1$, $S = -0.01$ and $\mu = 0.2$ by solving the LMIs given in Theorem 3.1 using the Matlab LMI toolbox, we can obtain that the upper bound on the samplings $\tau_2 = 1.12$. In this case, the corresponding gain matrices can be obtained as

$$K_1 = \begin{bmatrix} -0.0312 & 0.2764 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -13.4245 & -6.1215 \end{bmatrix}.$$

H_∞ case: If we set the state time delay bound $\tau_1 = 0.1$ and assuming $h = 1.21$, $Q = -1$, $R = \gamma^2 I$, $S = 0$ and $\mu = 0.2$ by solving the LMIs given in Theorem 3.1 using the Matlab LMI toolbox, we can obtain that the upper bound on the samplings $\tau_2 = 1.15$. In this case, the corresponding gain matrices can be obtained as

$$K_1 = \begin{bmatrix} -0.012 & 0.0481 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -14.3918 & -6.6387 \end{bmatrix}.$$

Passivity based control design: If we set the state time delay bound $\tau_1 = 0.1$ and assuming $h = 1.21$, $Q = 0$, $R = 0$, $S = I$ and $\mu = 0.2$ by solving the LMIs given in Theorem 3.1 using the Matlab LMI toolbox, we can obtain that the upper bound on the samplings $\tau_2 = 1.1$. In this case, the corresponding gain matrices can be obtained as

$$K_1 = 10^{-3} \begin{bmatrix} 0.3726 & 0.5178 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -17.6868 & -8.4108 \end{bmatrix}.$$

When the we consider delay bound is differentiable, the desired systems can be obtained by Theorem 3.1.

Moreover, to reflect the effectiveness of the developed design scheme simulation results are presented in Figs 1-8. For this, it is assumed the disturbance signal

$$w(t) = \begin{cases} 0.05 * \exp(0.5t), & 0.005 \leq t \leq 3, \\ 0, & \text{Otherwise.} \end{cases}$$

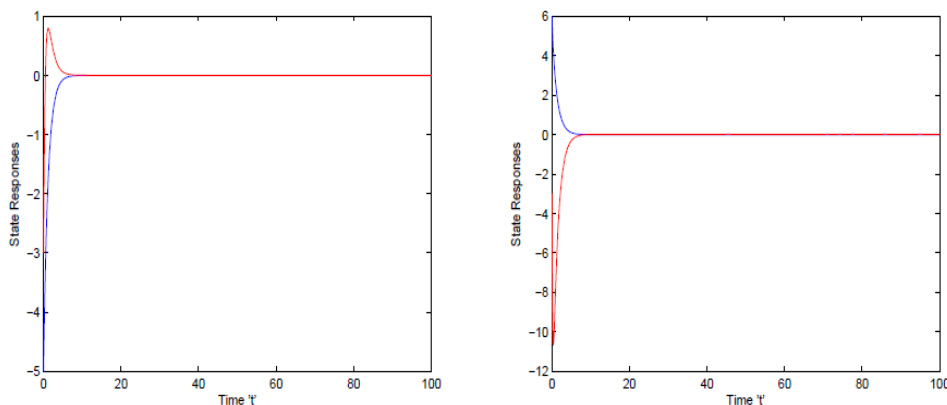


Fig. 1: The state responses for dissipative case primary and secondary plant

Simulation results for state responses of singular NCCSs (8) for dissipative performance, $\mathcal{H}_\infty$ performance,

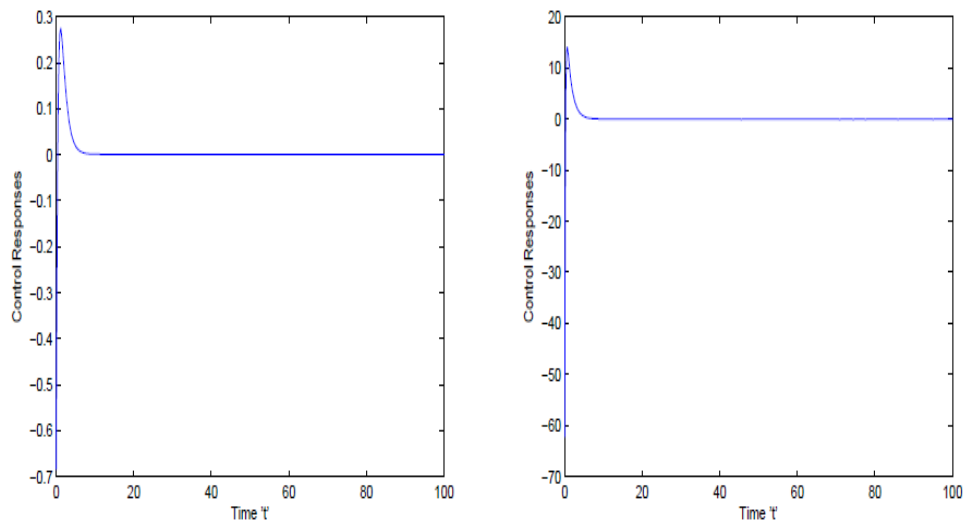


Fig. 2: The response for dissipative case primary and secondary control signal $u(t)$.

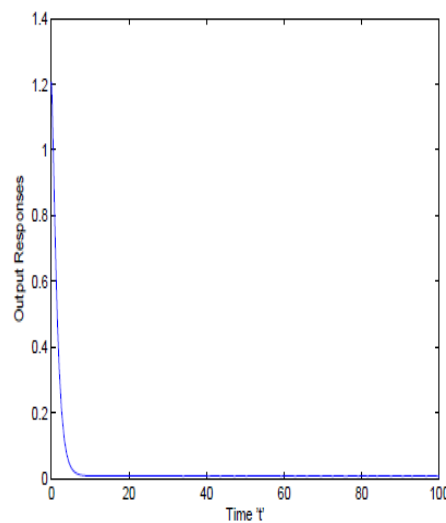


Fig. 3: The response for dissipative case output signal.

Passive cases are shown in Figs 1, 7 and 4, respectively. Further, the corresponding controller performance are shown in Figs 2, 5 and 8, respectively. From the simulation results, it is easy to see that the obtained controller designs are suitable to make sure the state trajectories are converging well. Also, it is noticed that the optimized minimum gamma obtained through the results shows that the LMI based conditions in Theorems are worked well over the network imperfections and external disturbance.

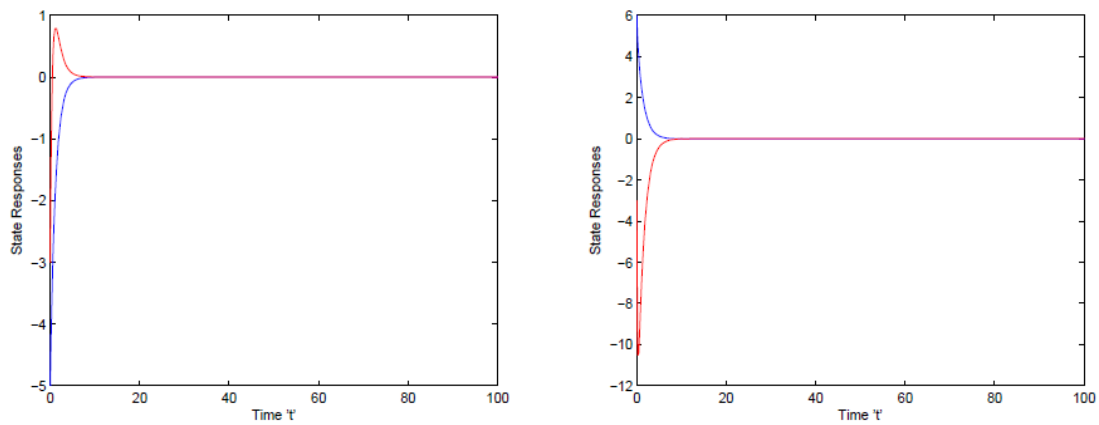


Fig. 4: The state responses for H_∞ case primary and secondary plant

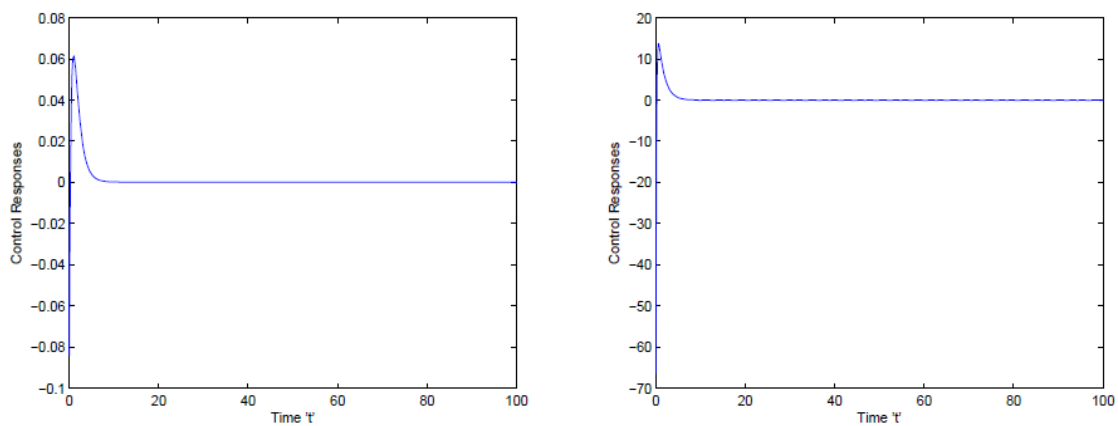


Fig. 5: The response for H_∞ case primary and secondary plant control signal $u(t)$.

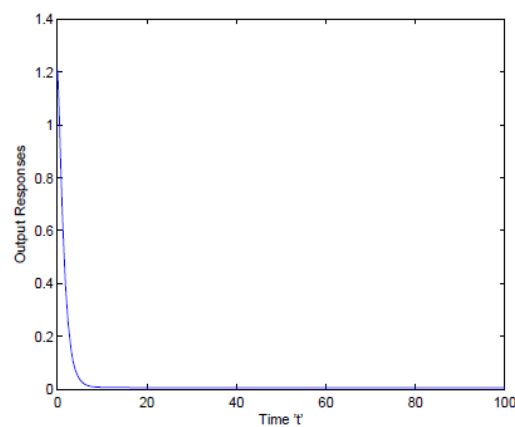


Fig. 6: The response for H_∞ case output signal.

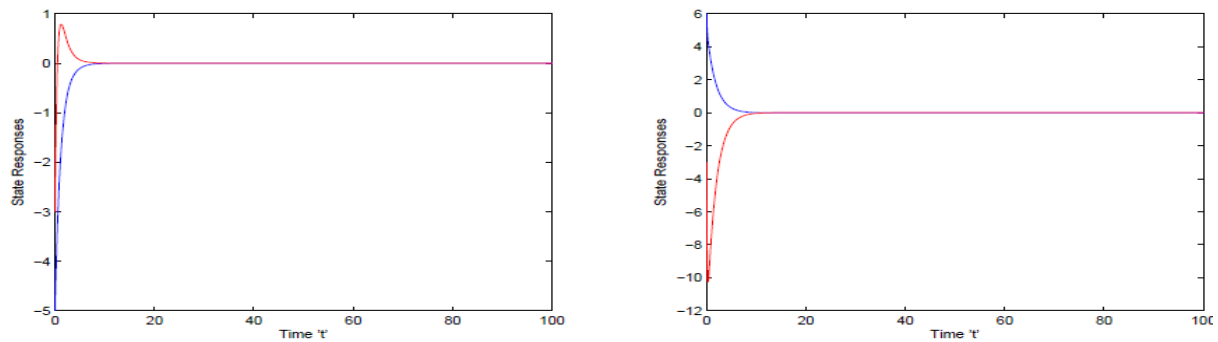


Fig. 7: The state responses for passive case primary and secondary plant

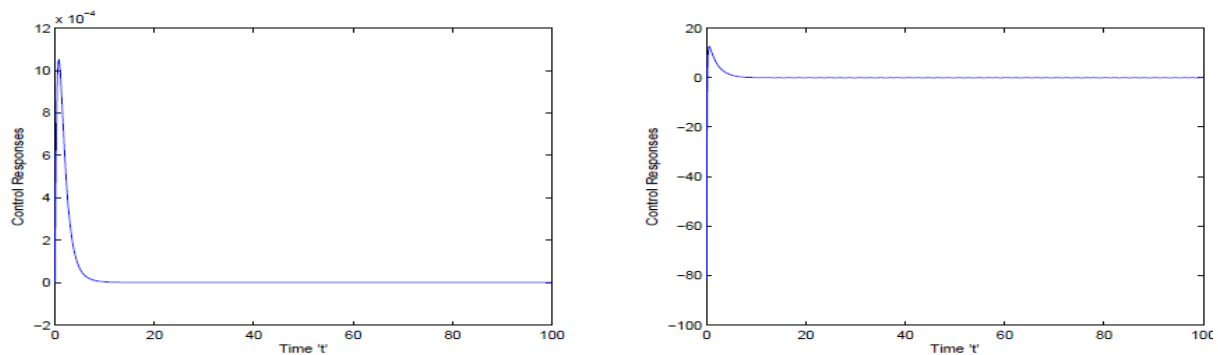


Fig. 8: The response for passive case primary and secondary plant control signal $u(t)$.

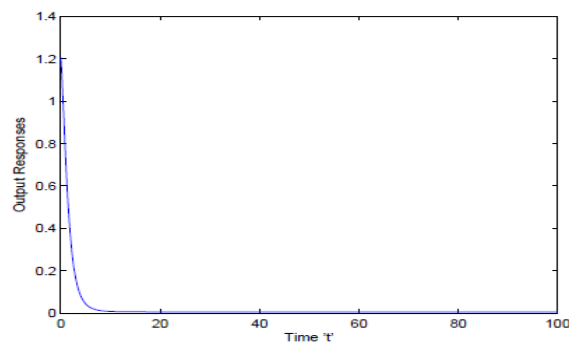


Fig. 9: The response for passive case output signal.

V. CONCLUSION

The problems of stability and dissipative analysis of NCCS have been investigated. In this paper for a time varying random delay technique, some novel LyapunovKrasovskii functional candidates were introduced for admissibility and dissipative of NCCS. The derived results are tabulated. At the end, a numerical example were given to demonstrate the modeling and guarantee the effectiveness of the developed approaches.

**REFERENCES**

- [1] C. Huang, Y. Bai and X. Liu, H-infinity state feedback control for a class of networked cascade control systems with uncertain delay, *IEEE Transactions on Industrial Informatics*, 6 (2010) 62-72.
- [2] T. Teo, S. Lakshminarayanan and G. Rangaiah, Performance assessment of cascade control systems, *Journal of The Institution of Engineers, Singapore*, 45 (2005) 27-38.
- [3] J. Zhang, F. Zhang, M. Ren, G. Hou and F. Fang, Cascade control of superheated steam temperature with neuro-pid controller, *ISA Transactions*, 51 (2012) 778-785.
- [4] Z. Du, D. Yue and Songlin Hu, H-Infinity stabilization for singular networked cascade control systems with state delay and disturbances, *IEEE Transactions on Industrial Informatics*, 10 (2013) 882-894.
- [5] Y. Feng and M. Yagoubi, On state feedback h-infinity control for discrete-time singular systems, *IEEE Transactions on Automatic Control*, 58 (2013) 2674-2679.
- [6] H. Wong, B. Zhou, R. Lu and A. Xue, new stability and stabilization criteria for a class of fuzzy singular systems with time varying delay, *Journal of Franklin institute*, 351 (2014) 3766-3781.
- [7] Z.G. Wu, H. Su, P. Shi and J. Chu, *Analysis and synthesis of singular systems with time-Delays, Computational Intelligence and Complexity (Springer)*, 2013.
- [8] X. Zhu, Y. Wang and Y. Gan, H_1 filtering for continuous-time singular systems with time-varying delay, *International Journal of Adaptive Control and Signal Processing*, 49 (2011) 137-154.
- [9] M. Luo, S. Zhong, R. Wang and W. Kang, Robust stability analysis for discrete-time stochastic neural networks systems with time-varying delays, *Applied Mathematics and Computation*, 209 (2009) 305-313.
- [10] Q. Song, Z. Zhao and Y. Liu, Stability analysis of complex-valued neural networks with probabilistic time-varying delays, *Neurocomputing*, 159 (2015) 96-104.
- [11] R. Yang, H. Gao, J. Lam and P. Shi, New stability criteria for neural networks with distributed and probabilistic delays, *Circuits Systems. Signal Process*, 28 (2009) 505-522.
- [12] J. Wang, J. H. Park, H. Shen and J. Wang, Delay-dependent robust dissipativity conditions for delayed neural networks with random uncertainties, *Applied Mathematics and Computation*, 221 (2013) 710-719.
- [13] J. Lin, Y. Shi, S. Fei and Z. Gao, Reliable dissipative control of discrete-time switched singular systems with mixed time delays and stochastic actuator failures, *IET Control Theory and Applications*, 7 (2013) 1447-1462.
- [14] S. Magdi, Mahmoud, A. Abdul-Wahid and A. Saif, Dissipativity analysis and design for uncertain Markovian jump systems with time-varying delays, *Applied Mathematics and Computation* 219 (2013) 9681-9695.
- [15] L. Wu, W. Zheng and H. Gao, Dissipativity-based sliding mode control of switched stochastic systems, *IEEE Transactions, Automata Control*, 58 (2013) 785-793.
- [16] L. Xu, T. Zhang and Y. Yi, Robust dissipative control for time-delay stochastic jump systems, *Journal of Systems Engineering and Electronics*, 22 (2011) 314-321.