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ANALYTICAL SOLUTIONS OF MHD FLOW AND HEAT TRANSFER OF A VISCOELASTIC FLUID OVER AN EXPONENTIALLY STRETCHING SHEET EMBEDDED IN A THERMALLY STRATIFIED MEDIUM

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ABSTRACT

MHD boundary layer flow and heat transfer of a viscoelastic fluid over an exponentially stretching sheet embedded in a thermally stratified medium subject to radiation are examined. Using similarity transformation the governing boundary layer non-linear partial differential equations are converted into non-linear ordinary differential equations. Homotopy analysis method (HAM) is applied to get series solution of obtained equaations. The convergence of the obtained series solution is discussed explicitly. It is found that the heat transfer rate at the surface increases in presence of thermal stratification. Fluid velocity decreases with increasing magnetic parameter. Temperature gradient increases considerably with increase of stratification parameter.

Keywords: Exponentially stretching sheet, HAM, MHD flow, thermally stratified medium.

I. INTRODUCTION

In recent years, the study of non-Newtonian fluids has achieved a lot success due to their practical applications in various fields like manufacturing of foods and papers, manufacturing of plastic sheets, etc. The study of boundary layer flow over a continuous solid surface moving with a constant speed was first studied by Sakiadis [1] in 1961. Later Crane [2] extended this problem to a stretching sheet whose surface velocity varies linearly with a certain distance from a fixed point. Chang [3] derived a closed form solution of the non-Newtonian flow problem of Rajgoplal et al. [4]. Char [5] discussed the effects of magnetic field and power law surface temperature on heat and mass transfer from a continuous flat surface. Heat and mass transfer characteristics in the presence of transverse magnetic field were obtained by Abel et al. [6]. In heat transfer process thermal radiation also place a vital role. Rapits [7], Abel and Gousia [8] analysed the viscoelastic fluid flow and heat transfer in the presence of thermal radiation under various physical conditions. Most of the researchers concentrated on the flow analysis caused by stretching the sheet linearly. Magyari and Keller [9] focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet. Elbashbeshy [10] examined the flow and heat transfer characteristics over an exponentially stretching continuous surface with suction. Bidin and Nazar [11] presented the numerical solutions for the problem of boundary layer flow over an exponentially stretching sheet in the presence of radiation. The flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated experimentally and analytically by Yang et al. [12]. Recently, Mukhopadhyay [13] analysed the MHD boundary layer flow and heat transfer towards an exponentially stretching sheet embedded in a thermally stratified medium by taking suction into account.

Hence, the aim of the present work is to study the characteristics of MHD boundary layer flow and heat transfer of a viscoelastic fluid over an exponentially stretching sheet embedded in a thermally stratified medium in the presence of radiation using HAM [14].

II. MATHEMATICAL FORMULATION

Consider the flow of an incompressible viscoelastic electrically conducting fluid past a flat heated sheet coinciding with the plane $y=0$ and the flow being confined to $y>0$. The flow is generated due to stretching the sheet exponentially by the application of two equal and opposite forces along the x-axis. So that the sheet is stretched keeping the origin fixed. A variable magnetic field of strength B is applied in the direction to normal the plate. The sheet is of temperature $T_w(x)$ and is embedded in a thermally stratified medium of variable temperature $T_\infty(x)$ where $T_w(x)>T_\infty(x)$. It is assumed that $T_w(x)=T_0+b\,e^{\frac{x}{2L}}$, $T_\infty(x)=T_0+c\,e^{\frac{x}{2L}}$ where T_0 is the reference temperature, $b>0, c\geq 0$ are constants.

The equations of continuity, momentum, energy and concentration for the flow of viscoelastic fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k_0\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B^2}{\rho}u, \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}, \tag{3}$$

where u and v are the velocity components in x and y directions, ν is the kinematic viscosity, k_0 is the elastic parameter, ρ is the fluid density, σ is the electrical conductivity of the fluid, T is the temperature in the boundary layer, k is the thermal conductivity, q_r is the radiative heat flux.

The boundary conditions are

$$\begin{aligned} &u=U_w=U_0\,e^{\frac{x}{L}},\quad v=0,\quad T=T_w, &&at\ \ y=0,\\ &u\to 0,\quad T\to T_\infty &&as\ \ y\to\infty. \end{aligned} \tag{4}$$

Here, the subscripts w,∞ refer to the surface and ambient conditions, respectively. U_w is the stretching velocity, U_0 is the reference velocity, L is the reference length.

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It is assumed that the variable magnetic field $B(x)$ is of the form:

$$B(x) = B_0(x)e^{\frac{x}{2L}},$$

where B_0 is the constant magnetic field.

The equation of continuity is satisfied by the stream function $\psi(x,y)$ defined by

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x}.$$

Following Rosseland approximation, the radiative heat flux is

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y},$$

where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient. Further, we assume that the temperature difference within the flow is such that T^4 is expressed as a linear function of temperature. Hence, expanding T^4 in Taylor series about T_∞ and neglecting higher order terms, we obtain

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4,$$

Now, we introduce the following similarity transformations to convert the partial differential equations into ordinary differential equations:

$$\left.\begin{aligned} &u = U_0 e^{\frac{x}{L}} f'(\eta), \quad \eta = y\sqrt{\frac{U_0}{2L\nu}}e^{\frac{x}{2L}}, \qquad \psi(x,y) = \sqrt{2\nu L U_0}\, f(\eta)e^{\frac{x}{2L}}, \\ &v = -\sqrt{\frac{\nu U_0}{2L}}e^{\frac{x}{2L}}\left(f(\eta) + \eta f'(\eta)\right), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_0}, \end{aligned}\right\} \tag{5}$$

where η is the similarity variable, f is the dimensionless stream function, $\theta(\eta)$ is a dimensionless temperature of the fluid in the boundary layer region.

Substituting Equation (5) in Equations (2) to (4), we obtain

$$2f'^2 - ff'' = f''' - k_1\left(3f'f''' - \frac{1}{2}ff'''' - \frac{3}{2}f''^2\right) - M f', \tag{6}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Pr\left(f\theta' - f'\theta\right) - Pr\ St\ f' = 0, \tag{7}$$

where $k_1 = \dfrac{k_0 U_w}{L\nu}$ is the dimensionless viscoelastic parameter, $M = \dfrac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter, $R = \dfrac{4\sigma^* T_\infty^3}{k^* k}$ is the radiation parameter, $Pr = \dfrac{\nu \rho C_p}{k}$ is the Prandtl number, $St = \dfrac{c}{b}$ is the stratification parameter. $St > 0$ implies a stably stratified environment, while $St = 0$ corresponds to an unstratified environment.

The transformed boundary conditions are

$$
\begin{aligned}
&f = 0, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \quad at \quad \eta = 0,\\
&f' = 0, \quad \theta = 0, \quad \phi = 0 \quad as \quad \eta \to \infty.
\end{aligned} \tag{8}
$$

III. HAM

In this section, we employ HAM to solve the equations (6) and (7) subject to the boundary conditions (8). We choose the initial guesses f_0 and θ_0 of f and θ in the following form

$$
\begin{aligned}
f_0(\eta) &= 1 - e^{-\eta},\\
\theta_0(\eta) &= e^{-\eta}.
\end{aligned}
$$

The linear operators are selected as

$$
\begin{aligned}
L_1(f) &= f''' - f',\\
L_2(\theta) &= \theta'' - \theta,
\end{aligned}
$$

which have the following properties

$$
\begin{aligned}
L_1\left(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}\right) &= 0,\\
L_2\left(C_4 e^{\eta} + C_5 e^{-\eta}\right) &= 0,
\end{aligned}
$$

where C_1, C_2, C_3, C_4 *and* C_5 are the arbitrary constants.

If $p \in [0,1]$ denotes the embedding parameter and $\hbar_1$ and $\hbar_2$ are the non-zero auxiliary parameters then we construct the following zeroth-order deformation equations

$$(1-p)L_1\left(f(\eta;p) - f_0(\eta)\right) = p\hbar_1 N_1\left[f(\eta;p)\right], \tag{9}$$

$$(1-p)L_2\left(\theta(\eta;p) - \theta_0(\eta)\right) = p\hbar_2 N_2\left[f(\eta;p), \theta(\eta;p)\right], \tag{10}$$

subject to the boundary conditions

$$
\begin{aligned}
&f(0;p) = 0, \quad f'(0;p) = 1, \quad f'(\infty;p) = 0,\\
&\theta(0;p) = 1, \quad \theta(\infty;p) = 0.
\end{aligned} \tag{11}
$$

Based on equations (6) and (7), we define the nonlinear operators N_1 and N_2 as

$$N_1[f(\eta;p)] = \frac{\partial^3 f(\eta;p)}{\partial\eta^3} - 2\left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2 + f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial\eta^2}$$
$$- k_1\left(3\frac{\partial f(\eta;p)}{\partial\eta}\frac{\partial^3 f(\eta;p)}{\partial\eta^3} - \frac{1}{2}f(\eta;p)\frac{\partial^4 f(\eta;p)}{\partial\eta^4} - \frac{3}{2}\left(\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right)^2\right)$$
$$- M\frac{\partial f(\eta;p)}{\partial\eta}, \tag{12}$$

$$N_2[f(\eta;p),\theta(\eta;p)] = \left(1+\frac{4}{3}R\right)\frac{\partial^2\theta(\eta;p)}{\partial\eta^2} + Pr\left(f(\eta;p)\frac{\partial\theta(\eta;p)}{\partial\eta} - \frac{\partial f(\eta;p)}{\partial\eta}\theta(\eta;p)\right)$$
$$- Pr\, St\left(\frac{\partial f(\eta;p)}{\partial\eta}\right), \tag{13}$$

When $p=0$ and $p=1$, we obtain

$$f(\eta;0) = f_0(\eta) \qquad f(\eta;1) = f(\eta),$$
$$\theta(\eta;0) = \theta_0(\eta) \qquad \theta(\eta;1) = \theta(\eta). \tag{14}$$

Thus, as p increases from 0 to 1 then $f(\eta;p)$ and $\theta(\eta;p)$ vary from initial approximations to the exact solutions of the original nonlinear differential equations.

Now, expanding $f(\eta;p)$ and $\theta(\eta;p)$ in Taylor's series w.r.to p, we have

$$f(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, \tag{15}$$

$$\theta(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)p^m, \tag{16}$$

where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial p^m}\bigg|_{p=0},$$
$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \theta(\eta;p)}{\partial p^m}\bigg|_{p=0}. \tag{17}$$

If the initial approximations, auxiliary linear operators and non-zero auxiliary parameters are chosen in such a way that the series (15) to (16) are convergent at $p=1$, then

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{18}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta). \tag{19}$$

Differentiating Equations (9) and (10) m times w.r.to p , setting $p=0,$ and finally dividing with $m!$, we get the mth-order deformation equations as follows

$$L_1\left(f_m(\eta) - \chi_m\, f_{m-1}(\eta)\right) = \hbar_1\, R_m^f(\eta), \tag{20}$$

$$L_2\left(\theta_m(\eta) - \chi_m\, \theta_{m-1}(\eta)\right) = \hbar_2\, R_m^\theta(\eta), \tag{21}$$

with the following boundary conditions

$$\begin{aligned} &f_m(0) = 0, \qquad f_m'(0) = 0, \qquad f_m'(\infty) = 0, \\ &\theta_m(0) = 0, \qquad\qquad\qquad \theta_m(\infty) = 0, \end{aligned} \tag{22}$$

where

$$\begin{aligned} R_m^f(\eta) = {} & f_{m-1}''' - 2\sum_{i=0}^{m-1} f_{m-1-i}'\, f_i' + \sum_{i=0}^{m-1} f_{m-1-i}\, f_i'' \\ & - k_1\left(3\sum_{i=0}^{m-1} f_{m-1-i}'\, f_i''' - \frac{1}{2}\sum_{i=0}^{m-1} f_{m-1-i}\, f_i'''' - \frac{3}{2}\sum_{i=0}^{m-1} f_{m-1-i}''\, f_i''\right) - M\, f_{m-1}', \end{aligned} \tag{23}$$

$$R_m^\theta(\eta) = \left(1 + \frac{4}{3}\right)\theta_{m-1}'' + Pr\left(\sum_{i=0}^{m-1} f_{m-1-i}\, \theta_i' - \sum_{i=0}^{m-1} f_{m-1-i}'\, \theta_i\right) - Pr\, St\, f_{m-1}', \tag{24}$$

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \tag{25}$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$ and $\phi_m^*(\eta)$ as the special solutions of mth order deformation equations, then the general solution is given by

$$\begin{aligned} f_m(\eta) &= f_m^*(\eta) + C_1 + C_2 e^{\eta} + C_3 e^{-\eta}, \\ \theta_m(\eta) &= \theta_m^*(\eta) + C_4 e^{\eta} + C_5 e^{-\eta}, \end{aligned} \tag{26}$$

where the integral constants $C_i\ (i = 1 \text{ to } 5)$ are determined using the boundary conditions.

It is easy to solve the above linear homogeneous equations using MATHEMATICA one after other in the order $m = 1, 2, \ldots$.

IV. CONVERGENCE OF HAM

The convergence region and rate of approximation of series solutions obtained using HAM are mainly dependent on the non-zero auxiliary parameters $\hbar_1$ and $\hbar_2$. In order to find the appropriate values of $\hbar_1$ and $\hbar_2$, $\hbar$-curves are plotted in Fig. 1. From the figure, it is clear that the valid regions of $\hbar_1$ and $\hbar_2$ are about [-1.0, 0.0]. Our computations indicate that the series converge in the whole region of η when $\hbar_1 = \hbar_2 = -0.81$. The convergence of homotopy solution for different orders of approximations is given in Table 1.

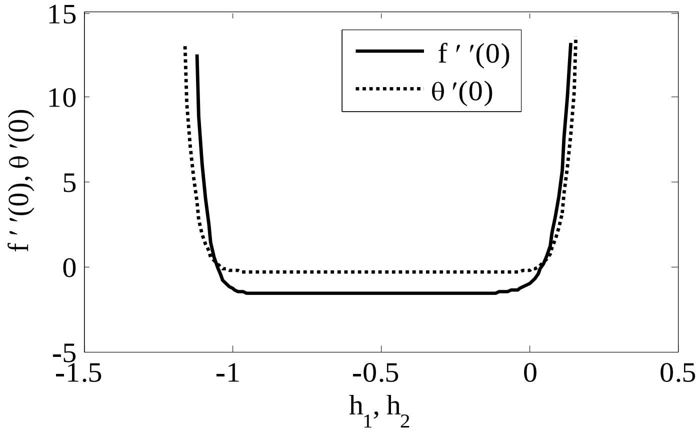

Fig. 1: $\hbar$-curves of $f''(0)$ and $\theta'(0)$ for 20th order approximation when

$k_1 = 0.1, M = 0.5, R = 0.5, Pr = 0.71, St = 0.8.$

Table 1. Convergence of HAM solution for different orders of approximations when

$k_1 = 0.1, M = 0.5, R = 0.5, Pr = 0.71, St = 0.8.$

Order	$-f''(0)$	$-\theta'(0)$
5	1.582727	0.299898
10	1.579617	0.299205
15	1.580412	0.299847
20	1.580186	0.300006
25	1.580261	0.300163
30	1.580234	0.300202
35	1.580244	0.300210
40	1.580244	0.300212
45	1.580244	0.300212

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V. RESULTS AND DISCUSSION

To ensure the accuracy of the applied method, the values of heat transfer rate $-\theta'(0)$ are compared with the available results in the literature and are presented in Table 2.

Table 2. Comparison of $-\theta'(0)$ for different values of M, R, Pr when $k_1 = St = 0.0$.

M	R	Pr	Bidin and Nazar [11]	HAM
0.0	0.0	1.0	0.9547	0.954783
0.0	0.0	2.0	1.4714	1.471460
0.0	0.0	3.0	1.8691	1.869067
0.0	1.0	1.0	0.5315	0.531503
1.0	0.0	1.0	0.8611	0.861427

In the present study, the following default parameter values are adopted for computations:

$k_1 = 0.1, M = 0.5, R = 0.5, Pr = 0.71, St = 0.2.$

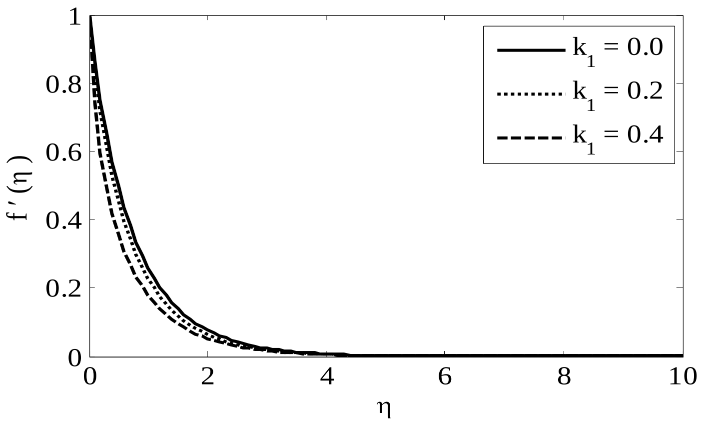

Fig. 2. Velocity $f'(\eta)$ for different values of k_1.

The effect of viscoelastic parameter k_1 on velocity and temperature is shown in Figs. 2 and 3. As shown in the figures velocity is decreasing while temperature is increasing with the increase of k_1. This is due to the tensile stress introduced by viscoelasticity which causes transverse contraction of the boundary layer.

Figs. 4 and 5 illustrate the influence of magnetic parameter M on velocity and temperature profiles. As M increases, the Lorentz force which has the tendency to slow down the motion of the fluid also increases. Hence, the velocity of the fluid decreases whereas temperature increases.

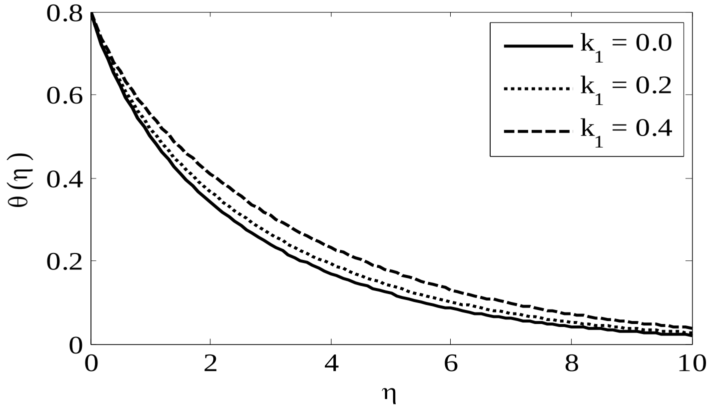

Fig. 3. Temperature $\theta(\eta)$ for different values of k_1.

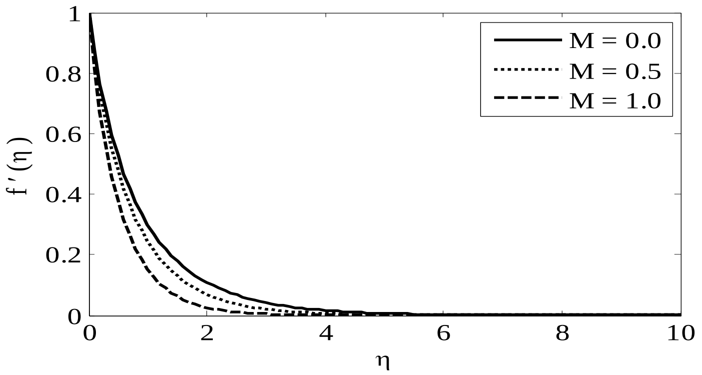

Fig. 4. Horizontal velocity $f'(\eta)$ for different values of M.

Fig. 6 is the graphical representation of temperature profiles $\theta(\eta)$ for several values of stratification parameter St. It is found that the temperature decreases as the stratification parameter St increases.

The temperature gradient increases considerably with an increase in stratification parameter St as shown in the Fig. 7.

In case of higher Prandtl values the diffusion of heat away from the heated surface is very slow when compared to the smaller Prandtl values. Hence temperature decreases with the increase of Prandtl number Pr as shown in the Fig. 8.

The effect of radiation parameter R on temperature is displayed in Fig. 9. It is noticed that the temperature increases with the increase of R. This is due to the fact that the thermal boundary layer thickness increases with the increase of radiation parameter.

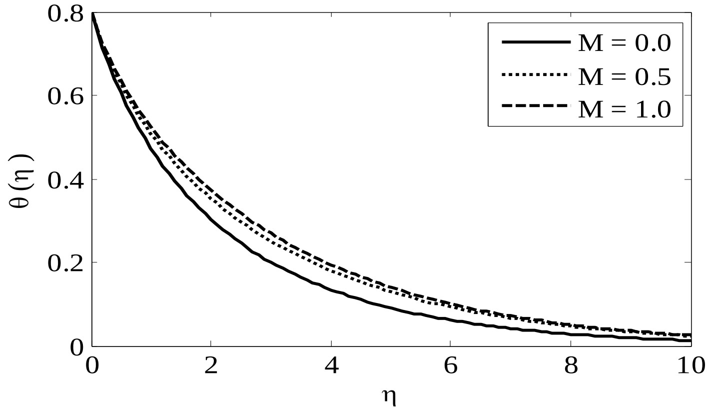

Fig. 5. Temperature $\theta(\eta)$ for different values of M .

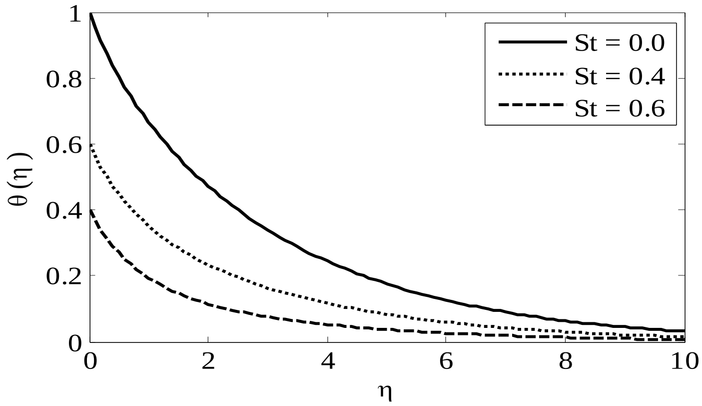

Fig. 6. Temperature $\theta(\eta)$ for different values of St .

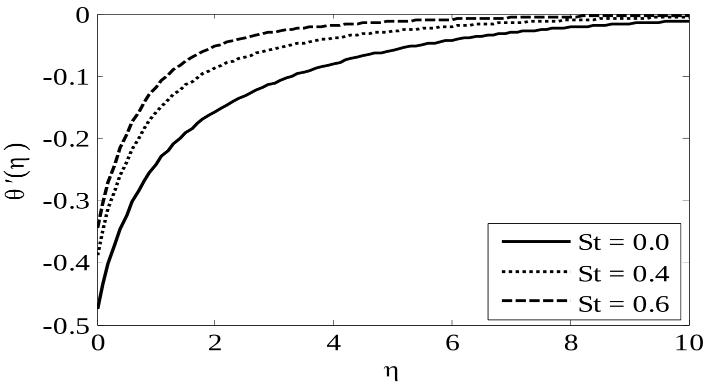

Fig. 7. Temperature gradient $\theta'(\eta)$ for different values of St .

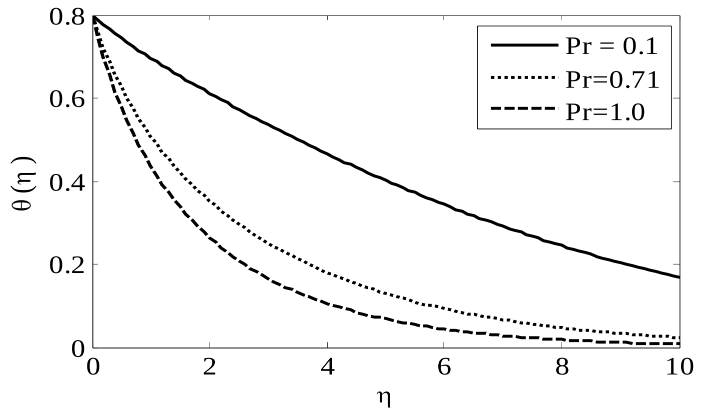

Fig. 8. Temperature $\theta(\eta)$ for different values of Pr .

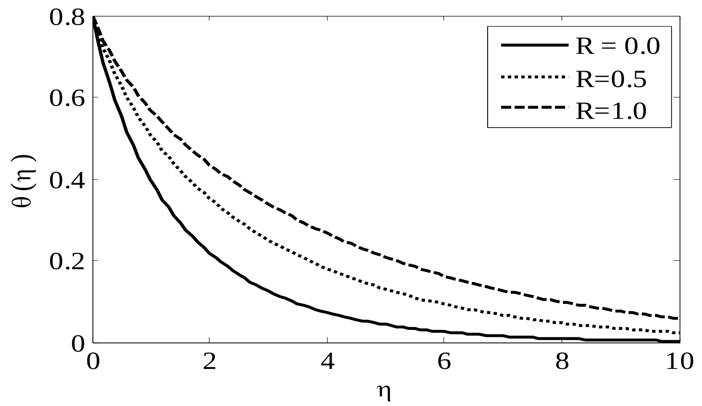

Fig. 9. Temperature $\theta(\eta)$ for different values of R .

VI. CONCLUSIONS

In the present analysis MHD boundary layer flow and heat transfer towards an exponentially stretching sheet embedded in a thermally stratified medium subject to radiation are described. The effect of magnetic parameter on a viscoelastic incompressible fluid is to suppress the velocity field which in turn causes the enhancement of the skin-friction coefficient. Rate of transport is reduced with the increasing magnetic field. The temperature decreases with increasing values of the stratification parameter.

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