COMMON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

In this paper, we prove common fixed point theorems for Semi compatible and Sub-sequentially continuous maps in intuitionistic fuzzy metric spaces. Our result is generalized the recent result of Chouhan and Kumar [4].

Keywords: Intuitionistic Fuzzy Metric Spaces, Sub-Sequentially Continuous Mappings, Semi-Compatible.

I INTRODUCTION

In 1986, Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets which is introduced by Zadeh [14]. In 2004, Park [10] defined the concept of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca [1] using the notion of intuitionistic fuzzy sets and defined the concept of intuitionistic fuzzy metric space with the help of continuous t–norms and continuous t-conorms as a generalization of fuzzy metric space which is introduced by Kramosil and Michalek [8]. Turkoglu [13] gave generalization of Jungck's [7] common fixed point theorem in intuitionistic fuzzy metric spaces.

Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [1] proved the well-known fixed point theorems of Banach [3] in the setting of intuitionistic fuzzy metric spaces. Later on,

Turkoglu et al. [13] proved Jungck's [7] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Turkoglu et al. [7] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [9]. Gregori et al. [6], Saadati and Park [11] studied the concept of intuitionistic fuzzy metric space and its applications.

II PRELIMINARIES

Definition 2.1 [12] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Examples of t-norm are $a * b=\min\{a,b\}$ and $a*b=ab$.

Definition 2.2.[12] A binary operation $\lozenge : [0, 1] \times[0, 1] \rightarrow [0, 1]$ is continuous t-conorm if $\lozenge$ satisfies the following conditions:

(i) $\lozenge$ is commutative and associative;

(ii) $\lozenge$ is continuous;

(iii) $a \lozenge 0 = a$ for all $a \in [0, 1]$;

(iv) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Examples of t-norm are $a\lozenge b=\min\{a,b\}$ and $a\lozenge b=ab$.

Definition 2.3. [8] A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-co-norm and M,N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi) for all $x, y \in X$, $M(x, y, \cdot) : [0,\infty) \rightarrow [0, 1]$ is left continuous;

(vii) $\lim_{n\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;

(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) for all $x, y \in X$, $N(x, y, \cdot) : [0,\infty) \rightarrow [0, 1]$ is right continuous;

(xiii) $\lim_{n\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

The functions M(x,y,t) and N(x,y,t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t respectively.

Remark 2.4. In intuitionistic fuzzy metric space $M(x,y,\cdot)$ is non decreasing and $N(x,y,\cdot)$ is non increasing for all $x,y\in X$.

Definition 2.6.[2] Let $(X,M,N,*,\lozenge)$ be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be

(i) Convergent to a point $x \in X$ if

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0 \text{ for all } t > 0,$$

(ii) Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0 \text{ for all } t > 0 \text{ and } p > 0.$$

Definition 2.7.[2] An intuitionistic fuzzy metric space $(X,M,N,*, \lozenge)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.8.[7] Let A and S be self-mappings of an intuitionistic fuzzy metric space $(X,M,N,*, \lozenge)$ Then a pair (A, S) is said to be commuting if $M(ASx, SAx, t) = 1$ and $N(ASx, SAx, t) = 0$.

Definition 2.5 [13] A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, * , \lozenge)$ is said to be compatible if

$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ for every

$t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

Definition 2.10.[13] Let A and S be self-mappings of an intuitionistic fuzzy metric space $(X,M,N,*, \lozenge)$.Then a pair (A, S) is said to be Sub-compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ and

$$\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0 \text{ for all } t > 0,$$

whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$ for some $u \in X$.

Lemma 2.13. [4] Let $\{u_n\}$ is a sequence in an intuitionistic fuzzy metric space $(X, M, N,*, \lozenge)$. If there exists a constant $k \in (0, 1)$ such that

$M(u_n, u_{n+1}, kt) \geq M(u_{n-1}, u_n, t)$ and $N(u_n, u_{n+1}, kt) \leq N(u_{n-1}, u_n, t)$

for $n = 1, 2, 3, \ldots$, then $\{u_n\}$ is a Cauchy sequence in X.

Lemma 2.14. [4] Let $(X, M, N,*, \lozenge)$.be an intuitionistic fuzzy metric space. If there exists a constant $k \in (0, 1)$ such that

$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$

for all $x, y \in X$ and $t > 0$, then $x = y$.

III MAIN RESULT

Theorem 2.1. Let L, A, B, P, Q and R be self mappings of intuitionistic fuzzy metric space $(X, M, N,*, \lozenge)$ with continuous t-norm and continuous t-co-norm $\lozenge$ defined by $t*t \geq t$ and $(1-t)\lozenge(1-t) \leq (1-t)$ for all $a,b \in [0,1]$.If the pairs (L,QR) and (A, BP) are semi-compatible and sub-sequentially continuous mappings, then

(a) The pair (L, QR) has a coincidence point.

(b) The pair (A, BP) has a coincidence point.

(c) Further, the mapping L, A, B, P, Q and R have a unique common fixed point in X provided the involved maps satisfy the inequality

$$M^2(Lx,Ay,t)*[M(QRx,Lx,t)*M(BPy,Ay,t)]$$
$$\geq [pM(QRx,Lx,t)+qM(QRx,BPy,t)]M(QRx,Ay,t)$$

and

$$N^2(Lx,Ay,t)\Diamond[N(QRx,Lx,t)\Diamond N(BPy,Ay,t)]$$
$$\leq [pN(QRx,Lx,t)+qN(QRx,BPy,t)]N(QRx,Ay,t) \quad \textbf{(1.1)}$$

for all x, y $\in$ X and t $>$ 0, where $0< p, q < 1$ and p+q=1.

Proof:- The pairs (L,QR) and (A,BP) are Semi-compatible and Sub-sequentially continuous mappings, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} QRx_n = z \text{ for some } z \in X$$

and $\lim_{n\to\infty} M(L(QR)x_n, (QR)Lx_n, t) = 1$, for all $t< 0$.

and $\lim_{n\to\infty} M(Lz,QRz,t)=1$ **(1.2)**

then we have Lz=QRz,

Similarly

$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} BPy_n = w \in X$

$\lim_{n\to\infty} M(A(BP)y_n, (BP)Ay_n, t) = 1$, for all $t< 0$

and $\lim_{n\to\infty} M(Aw,BPw,t)=1$ **(1.3)**

Hence z is coincidence point of L, QR and w is coincidence point of A, BP, then we get

Lz=QRz. **(1.4)**

Aw=BPw. **(1.5)**

<u>Step1</u>:- First we prove that z = w. Putting $x = x_n$, $y = y_n$ in inequality (1.1) we have

$$M^2(Lx_n,Ay_n,t)*[M(QRx_n,Lx_n,t).M(BPy_n,Ay_n,t)] \geq [pM(QRx_n,Lx_n,t)+qM(QRx_n,BPy_n,t)]M(QRx_n,Ay_n,t)$$

and

$$N^2(Lx_n,Ay_n,t)\Diamond[N(QRx_n,Lx_n,t)\Diamond N(BPy_n,Ay_n,t)] \leq [pN(QRx_n,Lx_n,t)+qN(QRx_n,BPy_n,t)]N(QRx_n,Ay_n,t)$$

Now

$$M^2(z,w,t)*[M(z,z,t).M(w,w,t)] \geq pM[(z,z,t)+qM(z,w,t)]M(z,w,t)$$

and

$$N^2(z,w,t)\Diamond[N(z,z,t)\Diamond N(w,w,t)] \leq pN[(z,z,t)+qN(z,w,t)]N(z,w,t)$$

$$M^2(z,w,t)\geq [p+qM(z,w,t)]M(z,w,t)$$

and

$$N^2(z,w,t)\leq [p+qN(z,w,t)]N(z,w,t)$$

$M(z,w,t)\geq \frac{p}{1-q}$ and

$$N(z,w,t)\leq \frac{p}{1-q}$$

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$M(z,w,t)=1$ and $N(z,w,t)=0$. **(1.7)**

Thus we have $z = w$

Step2:-Now we prove that $Lz = z$, Putting $x=z$ and $y=y_n$ in (1.1)

$$M^2(Lz,Ay_n,t)*[M(QRz,Lz,t).M(BPy_n,Ay_n,t)] \geq [pM(QRz,Lz,t)+qM(QRz, BPy_n,t)]M(QRz, Ay_n, t)$$

and

$$N^2(Lz,Ay_n,t)\Diamond[N(QRz,Lz,t)\Diamond N(BPy_n,Ay_n,t)] \leq [pN(QRz,Lz,t)+qN(QRz,BPy_n,t)]N(QRz, Ay_n, t)$$

$$M^2(Lz,w,t)*[M(QRz,Lz,t).M(w,w,t)] \geq [pM(QRz,Lz,t)+qM(QRz,w,t)]M(QRz, w, t)$$

and

$$N^2(Lz,w,t)\Diamond[N(QRz,Lz,t)\Diamond N(w,w,t)] \leq [pN(QRz,Lz,t)+qM(QRz,w,t)]N(QRz, w, t)$$

$$M^2(Lz,w,t)*[M(Lz,Lz,t).M(w,w,t)] \geq [pM(Lz,Lz,t)+qM(Lz,w,t)]M(Lz, w, t)$$

and

$$N^2(Lz,w,t)\Diamond[N(Lz,Lz,t)\Diamond N(w,w,t)] \leq [pN(Lz,Lz,t)+qN(Lz,w,t)]N(Lz, w, t)$$

$$M^2(Lz,w,t) \geq [p+qM(Lz,w,t)]M(Lz,w,t)$$

and

$$N^2(Lz,w,t) \leq [p+qN(Lz,w,t)]N(Lz,w,t)$$

$$M(Lz,w,t) \geq \frac{p}{1-q} \text{ and } N(Lz,w,t) \leq \frac{p}{1-q}$$

$$M(Lz,w,t) =1 \text{ and } N(Lz,w,t)=0$$

Hence $Lz = w = z$.

Step 3:-Again we prove that $Bz = z$

Then we have $x = x_n$, $y = z$ in (1.1)

$$M^2(Lx_n,Az,t)*[M(QRx_n,Lx_n,t)M(BPz,Az,t)] \geq [pM(QRx_n,Lx_n,t)+qM(QRx_n,BPz, t)] M(QRx_n,Az, t)$$

and

$$N^2(Lx_n,Az,t)\Diamond[N(QRx_n,Lx_n,t)\Diamond N(BPz,Az,t)] \leq [pN(QRx_n,Lx_n,t)+qN(QRx_n,BPz, t)] N(QRx_n,Az, t)$$

$$M^2(z,Az,t)*[M(Az,Az,t).M(z,z,t)] \geq [pM(Az,Az,t)+qM(z,Az,t)]M(z, Az, t)$$

and

$$N^2(z,Az,t)\Diamond[N(Az,Az,t)\Diamond N(z,z,t)] \leq [pN(Az,Az,t)+qN(z,Az,t)]N(z, Az, t)$$

$$M^2(z,Az,t) \geq [p+qM(z,Az,t)] M(z,Az,t)$$

and

$$N^2(z,Az,t) \leq [p+qN(z,Az,t)] N(z,Az,t)$$

$$M(z,Az,t) \geq \frac{p}{1-q} \text{ and } N(z,Az,t) \leq \frac{p}{1-q}$$

$$M(z,Az,t) =1 \text{ and } N(z,Az,t)=0$$

we get $z=Az$ **(1.8)**

Step4:-Again we claim that $Pz=z$,

putting $x=Pz$ and $y=z$ in (1.1)

$$M^2(LPz,Az,t)*[M(QR(Pz),Lz,t).M(BPz,Az,t)] \geq [pM(QR(Pz),Lz,t)+qM(QR(Pz),BPz, t)] M(QR(Pz), Az, t)$$

and

$N^2(LPz,Az,t)\Diamond[N(QR(Pz),Lz,t)\Diamond N(BPz,Az,t)] \leq [pN(QR(Pz),Lz,t)+qN(QR(Pz),BPz,t)]\ N(QR(Pz),Az,t)$

$M^2(LPz,z,t)*[M(QR(Pz),Lz,t).M(BPz,Az,t)] \geq [pMQz,ATz,t)+qM(TPQz,Bz,t)]\ M(TPQz,Bz,t)$

and

$N^2(LPz,z,t)\Diamond[N(QR(Pz),Lz,t)\Diamond N(BPz,Az,t)] \leq [pNQz,ATz,t)+qN(TPQz,Bz,t)]\ N(TPQz,Bz,t)$

$M^2(Pz,z,t)*[M(Pz,Pz,t).M(z,z,t)] \geq [pM(Pz,Pz,t)+qM(Pz,z,t)]\ M(Pz,z,t)$

$M^2(Pz,z,t) \geq [p+qM(Pz,z,t)]\ M(Pz,z,t)$

and

$N^2(Pz,z,t) \Diamond [p+qN(Pz,z,t)]\ N(Pz,z,t)$

$M(Pz,z,t) \geq \frac{p}{1-q}$ and $N(Pz,z,t) \leq \frac{p}{1-q}$

$M(Pz,z,t) = 1$ and $N(Pz,z,t)=0$

we get $Tz = z$ **(1.9)**

Step5:- Again we show that $Bz = z$,

putting $x=Bz$ and $y=z$ in (1.1)

$M^2(LBz,Az,t)*[M(QRBz,Lz,t).M(BPz,Az,t)] \geq [pM(QRBz,LBz,t)+qM(QRBz,BPz,t)]\ M(QRBz,Az,t)$

and

$N^2(LBz,Az,t)\Diamond[N(QRBz,Lz,t)\Diamond N(BPz,Az,t)] \leq [pN(QRBz,LBz,t)+qN(QRBz,BPz,t)]\ N(QRBz,Az,t)$

$M^2(Bz,z,t)*[M(Lz,Lz,t).M(z,z,t)] \geq [pM(Lz,Lz,t)+qM(Bz,z,t)]\ M(Bz,z,t)$

and

$N^2(Bz,z,t)\Diamond[N(Lz,Lz,t)\Diamond N(z,z,t)] \leq [pN(Lz,Lz,t)+qN(Bz,z,t)]\ N(Bz,z,t)$

$M^2(Bz,z,t) \geq [p+qM(Bz,z,t)]\ M(Bz,z,t)$

and

$N^2(Bz,z,t) \leq [p+qN(Bz,z,t)]\ N(Bz,z,t)$

$M(Bz,z,t) \geq \frac{p}{1-q}$ and $N(Bz,z,t) \leq \frac{p}{1-q}$

$M(Bz,z,t) = 1$ and $N(Bz,z,t)=0$

We get $Bz=z$ **(1.10)**

Step6:- Again we prove that $Rz=z$,

putting $x=Rz$ and $y=z$ in (1.1)

$M^2(LRz,z,t)*[M(QR(Rz),Lz,t).M(BP(Rz),Az,t)] \geq [pM(QR(Rz),L(Rz),t)+qM(QR(Rz),BPz,t)]M(QR(Rz),Az,t)$

and

$N^2(LRz,z,t)\Diamond[N(QR(Rz),Lz,t)\Diamond N(BP(Rz),Az,t)] \leq [pN(QR(Rz),L(Rz),t)+qN(QR(Rz),BPz,t)]N(QR(Rz),Az,t)$

$M^2(Rz,z,t)*[M(Rz,Rz,t).M(Az,Az,t)] \geq [pM(Rz,Rz,t)+qM(Rz,z,t)]\ M(Rz,z,t)$

and

$N^2(Rz,z,t)\Diamond[N(Rz,Rz,t)\Diamond N(Az,Az,t)] \leq [pN(Rz,Rz,t)+qN(Rz,z,t)]N(Rz,z,t)$

$M^2(Rz,z,t) \geq [p+qM(Rz,z,t)]\ M(Rz,z,t)$

and

$N^2(Rz,z,t) \lozenge [p+qN(Rz,z,t)]\ N(Rz,z,t)$

$M(Rz,z,t) \geq \frac{p}{1-q}$ and $N(Rz,z,t) \leq \frac{p}{1-q}$

$M(Rz,z,t) = 1$ and $N(Rz,z,t)=0$

We get Rz=z **(1.11)**

Step7:- Again we prove that Qz=z, putting x=Qz and y=z in (1.1)

$M^2(LQz,Az,t)*[M(QQz,Lz,t).M(BPz,Az,t)] \leq [pM(QRQz,Lz,t)+qM(QRQz,BPz,t)]\ M(QRQz,Az,t)$

and

$N^2(LQz,Az,t)\lozenge[N(QQz,Lz,t).N(BPz,Az,t)] \leq [pN(QRQz,Lz,t)+qN(QRQz,BPz,t)]\ N(QRQz,Az,t)$

$M^2(Qz,z,t)*[M(Qz,Qz,t).M(z,z,t)] \geq [pM(Qz,Qz,t)+qM(Qz,z,t)]\ M(Qz,z,t)$

and

$N^2(Qz,z,t)\lozenge[N(Qz,Qz,t)\lozenge N(z,z,t)] \leq [pN(Qz,Qz,t)+qN(Qz,z,t)]\ N(Qz,z,t)$

$M^2(Qz,z,t) \geq [p+qM(Qz,z,t)]\ M(Qz,z,t)$

and

$N^2(Qz,z,t) \leq [p+qN(Qz,z,t)]\ N(Qz,z,t)$

$M(Qz,z,t) \geq \frac{p}{1-q}$ and $N(Qz,z,t) \leq \frac{p}{1-q}$

$M(Qz,z,t) = 1$ and $N(Qz,z,t)=0$

We get Qz=z **(1.12)**

i.e. Lz=Az=Bz=Pz=Qz=Rz=z

Hence z is a common fixed point of L, A, B, P,R and Q.

Uniqueness: - Let v be another common fixed point of L, A, B, P, R and Q. Suppose $z \neq v$.

Putting x=z, y= v in (1.1)

$M^2(Lz,Av,t)*[M(QRz,Lz,t).M(BPv,Av,t)] \geq [pM(QRz,Lz,t)+qM(QRz,BPv,t)]M(QRz,Av,t)$

and

$N^2(Lz,Av,t)\lozenge[N(QRz,Lz,t).N(BPv,Av,t)] \leq [pN(QRz,Lz,t)+qN(QRz,BPv,t)]N(QRz,Av,t)$

$M^2(Lz,Av,t)*[M(Lz,Lz,t).M(Av,Av,t)] \geq [pM(Lz,Lz,t)+qM(Lz,Av,t)]M(Lz,Av,t)$

and

$N^2(Lz,Av,t)\lozenge[N(Lz,Lz,t).N(Av,Av,t)] \leq [pN(Lz,Lz,t)+qN(Lz,Av,t)]M(Lz,Az,t)$

$M^2(z,v,t) \geq [p+qM(z,v,t)]M(z,v,t)$

and

$N^2(z,v,t) \leq [p+qN(z,v,t)]N(z,v,t)$

$M(z,v,t) \geq \frac{p}{1-q}$ and $N(z,v,t) \leq \frac{p}{1-q}$

$M(z,v,t) = 1$ and $N(z,v,t)=0$

We get z=v.

REFERENCES

[1] Alaca C., Turkoglu D. and Yildiz C., Fixed points in intuitionistic fuzzy metric spaces, Smallerit Choas, Solitons & Fractals, 29(5)(2006), 1073-1078.

[2] Atanassov K., Intuitionistic Fuzzy sets, Fuzzy sets and system, 20(1986) 87- 96.

[3] Banach S., "Theorie Les Operations Lineaires, Manograie Mathematyezne Warsaw Poland," In French, Z Subwencji Funduszu Kultury Narodowej, New York, 1932.

[4] Chauhan,S.,and Kumar S.,Common fixed point theorms for compatible and subsequentially continuous mappings in fuzzy metric space,Kragujevac J. math. 36(2)(2012), pp.225-235.

[5] Coker D., "An Introduction to Intuitionistic Fuzzy Topological Spaces," Fuzzy Sets and System, Vol. 88, No. 1, 1997, pp. 81-89.

[6] Gregori V., Romaguera S., and Veeramani P., "A Note on Intuitionistic Fuzzy Metric Spaces," Chaos, Solitons and Fractals, Vol. 28, No. 4, 2006, pp. 902-905.

[7] Jungck G., Commuting mappings and fixed points, Amer. Math. Monthly, 83(1976), 261-263.

[8] Kramosil O., and Michalek J., Fuzzy metric and statistical metric spaces, Kybernetika, 11(1975).

[9] Pant R. P., "Common Fixed Points of Noncommuting Mappings," Journal of Mathematical Analysis and Applications, Vol. 188, No. 2, 1994, pp. 436-440.

[10] Park J. H., Intuitionistic fuzzy metric spaces, chaos, Solitions & Fractals 22(2004), 1039- 1046.

[11] Saadati R., and Park J. H., "On the Intuitionistic Fuzzy Topological Spaces," Chaos, Solitons and Fractals, Vol. 27, No. 2, 2006, pp. 331-344.

[12] Schweizer B., and Sklar A., "Statistical Metric Spaces," Pacific Journal Mathematic, Vol. 10, 1960, pp. 314-334.

[13] Turkoglu D., Alaca C., Cho Y.J., and Yildiz C., "Common Fixed Point Theorems in Intuitionistic Fuzzy Metric spaces," Journal of Applied Mathematics and Computing, Vol. 22, No. 1-2, 2006, pp. 411-424.

[14] Zadeh L.A., Fuzzy sets, Inform. and Control 8(1965), 338-353.