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# EVALUATING TRIPLE INTEGRALS WITH CYLINDRICAL AND SPHERICAL COORDINATES AND THEIR APPLICATIONS

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**DEFINITION**

**Triple Integral**

Let T be a transformation that maps a region S in uvw-space onto a region R in xyz-space by means of the equations

$$x = g(u.v,w) \quad y = h(u,v,w) \quad z = k(u,v,w)$$

The Jacobian of T is the following $3 \times 3$ determinat:

$$\begin{vmatrix} i & j & k \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & 0 \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & 0 \end{vmatrix} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} k = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} k$$

The determinant that arises in this calculation is called the Jacobian of transformation and is given a special notation.

Fubinis theorem for Triple integrals: If $f$ is continuous on the rectangular box $B = [a,b] \times [c,d] \times [r,s]$, then

$$\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)\, dxdydz$$

**Key words**: Triple Integrals ,double integrals , Applications, Transformation of cyclindercal coordinates,Sperical coordinates.

**Example 1:** Evaluate the triple integral $\iiint_B x,y,z^2 dV$, *where* B is the rectangular box given by

$B = \{(x,y,z): 0 \le x \le 1, -1 \le y \le 2, \; 0 \le z \le 3\}$

**Solution:** If we choose to integrate with respect to $x$, then $y$, and then z, we obtain

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$$\iiint_B xyz^2 dV = \int_0^3 \int_{-1}^2 \int_0^1 xyz^2 \ dxdydz = \int_0^3 \int_{-1}^2 \left[ \frac{x^2 yz^2}{2} \right]_{x=0}^{x=1} dydz$$

$$= \int_0^3 \int_{-1}^2 \frac{yz^2}{2} dydz = \int_0^3 \left[ \frac{y^2 z^2}{4} \right]_{y=-1}^{y=2} dz$$

$$= \int_0^3 \frac{3z^2}{4} dz = \frac{z^3}{4}\Big]_0^3 = \frac{27}{4}$$

Now we define the triple integral over a general bounded region E in three – dimensional space (a solid) by much the same procedure that we used for double integrals. We enclose E in a box B of the type $\{(x, y, z): a \le x \le b, c \le y \le d, \ r \le z \le s\}$. Then we define a function F so that it agrees with $f$ on E but is 0 for points in B that are outside E, by definition,

$$\iiint_E f(x, y, z) dV = \iiint_B F(x, y, z) dV$$

This integral exists if $f$ is continuous and the boundary of E is “reasonably smooth”.

A solid region E is said to be of type I if it lies between the graphs of two continuous functions of $x \ and \ y$, that is, $E = \{(x, y, z): (x,y) \in D, u1(x, y) \le \mathfrak{z} \le u_2(x, y)\} \cdots (*)$

where D is the projection of E onto xy – plane show in figure below. Notice that the boundary of the solid E is the surface with equation $z = u_2(x, y)$, while the lower boundary is the surface $z = u_1(x, y)$.

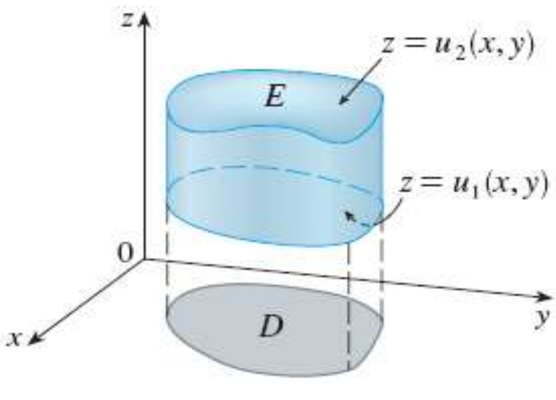



**FIGURE 2**
A type 1 solid region

If E is a type I region given by Equation $(*)$, then

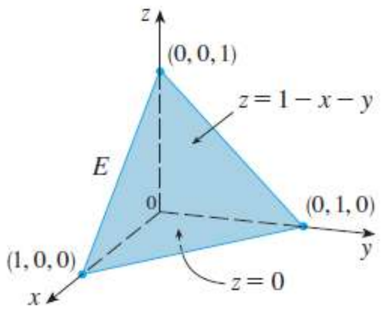
$$\iiint_E f(x, y, z) dV = \iint_D \left[ \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z) dz \right] dA$$

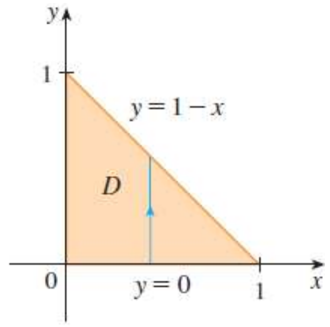
**Example 2**: Evaluate $\iiint_E zdz$ where E is the solid tetrahedron boundary by the four planes

$x = 0, y = 0, z = 0, and\ x + y + z = 1$.

**Solution:** When we set up a triple integral it's wise to draw two diagrams one of the solid

region $E$ and one of its projection $D$ on the xy – plane.




The lower boundary of the tetrahedron is the plane $z = 0$ and the upper boundary is the

plane $x + y + z = 1\ (or\ z = 1 - x - y)$, so we use $u_1(x, y) = 0\ and\ u_2(x, y) = 1 - x - y$.

Notice that the planes $x + y + z = 1\ and\ z = 0$ intersect in the line $x + y = 1$ or $y = 1 - x$

in the $xy$ – plane. So the project of E is the triangular region shown in figure 3, and we have

$\mathrm{E} = \{(x, y, \mathfrak{z}): 0 \leq x \leq 1, 0 \leq y \leq 1 - x, 0 \leq z \leq 1 - x - y\}$

This description of E as a type 1 region enables us to evaluate the integral as follows:

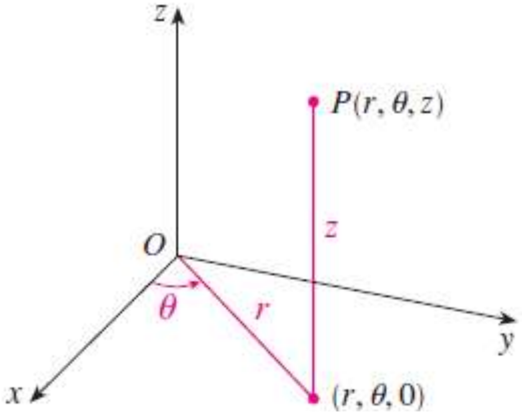
$$\iiint_{\mathrm{E}} z\, dV = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} z\, dz\, dy\, dx = \int_0^1 \int_0^{1-x} \left[ \frac{z^2}{2} \right]_{x=0}^{z=1-x-y} dydx$$

$$= \frac{1}{2} \int_0^1 \int_0^{1-x} (1 - x - y)^2\, dydx = \frac{1}{2} \int_0^1 \left[ -\frac{(1 - x - y)^3}{3} \right]_{y=0}^{y=1-x} dx$$

$$= \frac{1}{6} \int (1 - x)^3\, dx = \frac{1}{6} \left[ -\frac{(1 - x)^4}{4} \right]_0^1 = \frac{1}{24}$$

**Evaluating Triple Integrals with Cylindrical Coordinates**

In the cylindrical coordinate system, a point P in three dimensional space is represented by the ordered triple $(r, \theta, z)$, were r and $\theta$ are polar coordinates of the projection of P onto the xy – plane and z is the directed distance from the xy – plane to P.




Suppose that $E$ is a type 1 region whose $D$ on the $xy$–plane is conveniently described in polar coordinates. In particular, suppose that $f$ is continuous and

$E = \{(x, y, z): (x, y) \in D, u_1(x, y) \leq z \leq u_2(x, y)\}$ where $D$ is given in polar coordinates by

$D = \{(r, \theta): \alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta)\}$

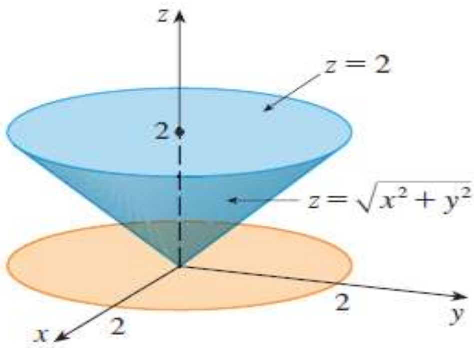
We know that $\iiint\limits_{E} f(x, y, z)dV = \iint\limits_{D} \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)dz\right]dA$

By changing x and y to polar coordinates, we obtain

$$\iiint\limits_{E} f(x, y, z)dV = \int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)}\int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta,\ r\sin\theta, z)r\, d\, z\, d\, r\, d\, \theta$$

**Example 3:** Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2 + y^2)dzdydx$

**Solution:**



This iterated integral is a triple integral over the solid region

$\mathrm{E} = \{(x, y, z): -2 \leq x \leq 2, -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}, \sqrt{x^2+y^2} \leq z \leq 2\}$

and the projection of E onto the xy – plane is the disk $(x^2+y^2) \leq 4$. The lower surface of E is the cone $z = \sqrt{x^2+y^2}$ and its upper surface is the plane $z = 2$. This region has a much simpler description in cylindrical coordinates.

$$\mathrm{E} = \{(r, \theta, z): 0 \leq \theta \leq 2\pi, 0 \leq r \leq 2, r \leq z \leq 2\}$$

Therefore, we have

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx = \iiint_{\mathrm{E}}(x^2+y^2)\,dV = \int_0^{2\pi}\int_0^2\int_r^2 r^2\,r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^2 r^3(2-r)\,dr = 2\pi\left[\frac{1}{2}r^4 - \frac{1}{5}r^5\right]_0^2 = \frac{16}{5}\pi$$

**Evaluating Triple Integrals with Spherical Coordinates**

The spherical coordinates $(\rho, \theta, \emptyset)$ of a point P in space are shown in Figure 1, where $\rho = |\mathrm{OP}|$ is the distance from the origin to $P, \theta$ is the angle as in cylindrical coordinates, and $\emptyset$ is the angle between the positive $z$ - axis and the line segment OP.

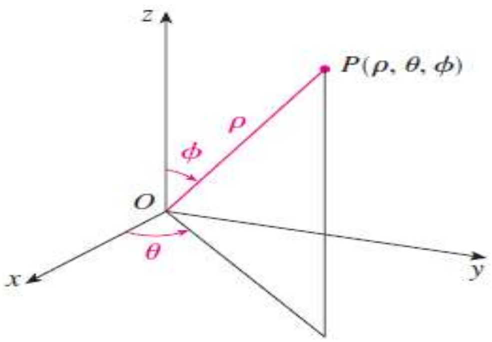



**Figure 1: Spherical coordinate of points**

Here $z = \rho\cos\emptyset \text{ and } r = \rho\sin\emptyset$. But $x = r\cos\theta \text{ and } y = r\sin\theta$. So $x = \rho\sin\emptyset\cos\theta,\ y = \rho\sin\emptyset\sin\theta,\ z = \rho\cos\emptyset$. Also the distance formula shows that $\rho^2 = x^2 + y^2 + z^2$.

**Theorem:** (Formula for triple integration in spherical coordinates)

$$\iiint_{\mathrm{E}} f(x,y,z)\,dV = \int_c^d\int_\alpha^\beta\int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

Were E is a spherical wedge given by

$$\mathrm{E} = \{(\rho, \theta, \emptyset): a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \emptyset \leq d\}$$

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**Example 4:** Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$, where B is the unit ball:

$B = \{(x,y,z): x^2 + y^2 + z^2 \leq 1\}$

**Solution:** Since the boundary of B is a sphere, we use spherical coordinates

$$B = \{(\rho,\theta,\emptyset): 0 \leq \rho \leq 1, 0 \leq \theta \leq 2\pi, 0 \leq \emptyset \leq \pi\}$$

In addition, spherical coordinates are appropriate because $x^2 + y^2 + z^2 = \rho^2$.

Thus, $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV = \int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{(\rho^2)^{3/2}} \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

$$= \int_0^{\pi} \sin\emptyset\, d\emptyset \int_0^{2\pi} d\theta \int_0^1 \rho^2 e^{\rho^3}\, d\rho$$

$$= [-\cos\emptyset]_0^{\pi}\,(2\pi)\left[\frac{1}{3}e^{\rho^3}\right]_0^1 = \frac{4}{3}\pi(e-1)$$

**Applications : Volume, Center of mass of Solid Region**

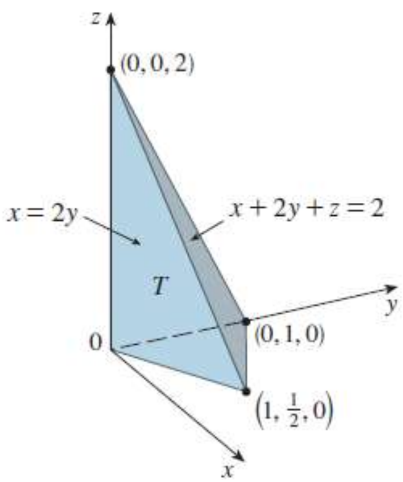
**Volume**

If $f(x,y,z) = 1$ for all points in E, then the triple integral does represent the volume of $E$.

$$V(E) = \iiint_E dV$$

**Example 1:** Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2, x = 2y, and\ z = 0$.

**Solution:** The tetrahedron T and its projection D on the $xy$ – plane are shown in figures 1 and 2. The lower boundary of T is the plane $z = 0$and the upper boundary is the plane $x + 2y + z = 2$, that is, $z = 2 - x - 2y$.



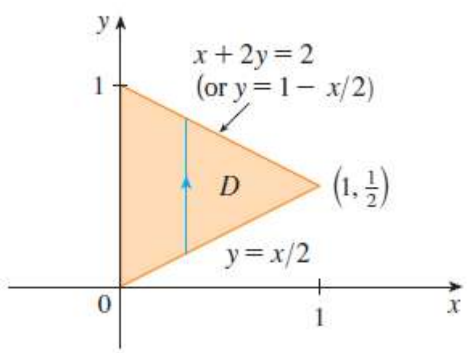



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Figure 1 Figure 2

Therefore we have

$$V\ (\mathrm{T}) = \iiint_{\mathrm{T}} dV = \int_0^1 \int_{x/2}^{1-x/2} \int_0^{2-x-2y} dz\,dy\,dx = \int_0^1 \int_{x/2}^{1-x/2} (2-x-2y)\,dy\,dx = \frac{1}{3}$$

**Center of Mass**

If the density function of a solid object that occupies the region E is $\rho(x, y, z)$, in units of mass per unit volume at any point $(x, y, z)$, then its mass is

$$m = \iiint_{\mathrm{E}} \rho\ (x, y, z)\ dV$$

and its moments about the three coordinate planes are $\mathrm{M}_{yz} = \iiint_{\mathrm{E}} x\rho\ (x, y, z)\ dV$, $\mathrm{M}_{xz} = \iiint_{\mathrm{E}} y\rho\ (x, y, z)\ dV$, $\mathrm{M}_{xy} = \iiint_{\mathrm{E}} z\rho\ (x, y, z)\ dV$

The center of mass is located at the point $(\bar{x}, \bar{y}, \bar{z})$ where $\bar{x} = \frac{\mathrm{M}yz}{m}$ , $\bar{y} = \frac{\mathrm{M}xz}{m}$ , $\bar{z} = \frac{\mathrm{M}xy}{m}$.

**Example 2**: Find the center of mass of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z, z = 0, and\ x = 1$.

**Solution:**

The lower and upper surfaces of E are the planes $\mathrm{z} = 0\ \ and\ \mathrm{z} = x$, so we describe E as a type 1 region:

$$\mathrm{E} = \{(x, y, z): -1 \le y \le 1,\ y^2 \le x \le 1,\ 0 \le z \le x\}$$

Then, if the density is $\rho\ (x, y, z) = \rho$, the mass is

$$m = \iiint_{\mathrm{E}} \rho dv = \int_{-1}^{1} \int_{y^2}^{1} \int_0^x \rho dz dx dy$$

$$= \rho \int_{-1}^{1} \int_{y^2}^{1} x dx dy = \rho \int_{-1}^{1} \left[\frac{x^2}{2}\right]_{x=y^2}^{x=1} dy$$

$$= \frac{\rho}{2} \int_{-1}^{1} (1 - y^4) dy = \rho \int_0^1 (1 - y^4) dy$$

$$= \rho \left[y - \frac{y^5}{5}\right]_0^1 = \frac{4\rho}{5}$$

Because of the symmetry of E and $\rho$ about $xz$ - plane, we can immediately say that $\mathrm{M}xz = 0$ and therefore $\bar{y} = 0$,

The other moments are

$$\mathrm{M}_{yz} = \iiint_{\mathrm{E}} x\rho\, dv = \int_{-1}^{1} \int_{y^2}^{1} x\rho dz dx dy = \rho \int_{-1}^{1} \int_{y^2}^{1} x^2 dx dy = \rho \int_{-1}^{1} \left[\frac{x^3}{3}\right]_{x=y^2}^{x=1} dy$$

$$= \frac{2\rho}{3}\int_0^1 (1-y^6)dy = \frac{2\rho}{3}\left[y-\frac{y^7}{7}\right]_0^1 = \frac{4\rho}{7}$$

$$Mxy = \iiint_E z\rho\, dv = \int_{-1}^1\int_{y^2}^1 z\rho dz dx dy = \rho\int_{-1}^1\int_{y^2}^1\left[\frac{z^2}{2}\right]_{z=0}^{z=x} dxdy$$

$$= \frac{\rho}{2}\int_{-1}^1\int_{y^2}^1 x^2\, dxdy = \frac{\rho}{3}\int_0^1(1-y^6)\, dy = \frac{2\rho}{7}$$

Therefore the center of mass is

$$(\bar{x},\bar{y},\bar{z}) = \left(\frac{Myz}{m}\ ,\ \frac{Mxz}{m}\ ,\ \frac{Mxy}{m}\right) = \left(\frac{5}{7},0,\frac{5}{14}\right).$$

**Change of Variables in Multiple Integral**

Consider a transformation T from the $uv$-plane to $xy$-plane defined by $T(u,v) = (x,y)$ where x and y are related to u and v by the equations

$x = g(u,v) \qquad y = h(u,v)$

or, as we sometimes write, $x = x(u,v), y = y(u,v)$.

Assume the transformation T is a single valued continuous and has continuous partial derivatives. If $(u_1,v_1) = (x_1,y_1)$ , then the point $(x_1,y_1)$ is called the image of the point $(u_1,v_1)$. If no two points have the some image, T is one-to-one. Figure 1 shows the effect of a transformation T on a region S in the $uv$-plane. T transforms S

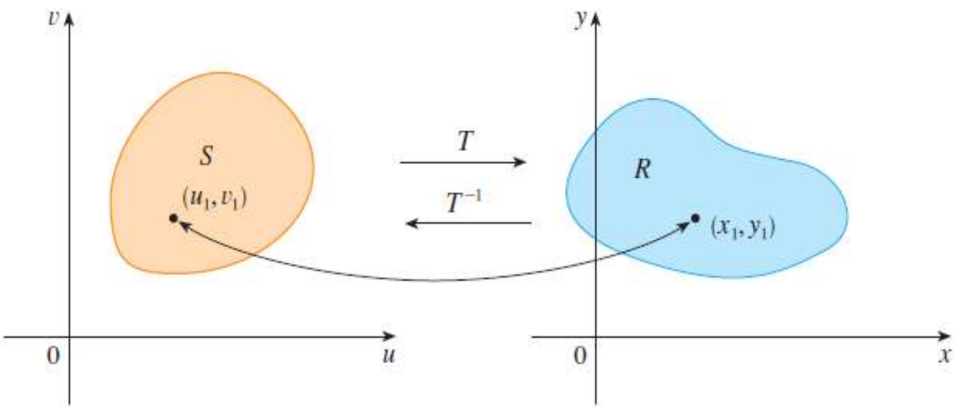


**Figure 1**

into a region R in the $xy$-palne called the image of S, consisting of the images of all points in S.

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If $T$ is one-to-one transformation, then it has an inverse transformation $T^{-1}$ from the $xy$-plane to the uv-plane and it is possible to solve for u and v in terms of $x$ and $y$:

$$u = G(x, y) \quad and \quad v = H(x, y)$$

Now let us see how a change in variables affects a double integral. We start with a small rectangle S in uv-plane whose lower left corner is the point $(u_{0,} v_0)$ and whose dimensions are $\Delta u$ and $\Delta v$. (See Figure 2)

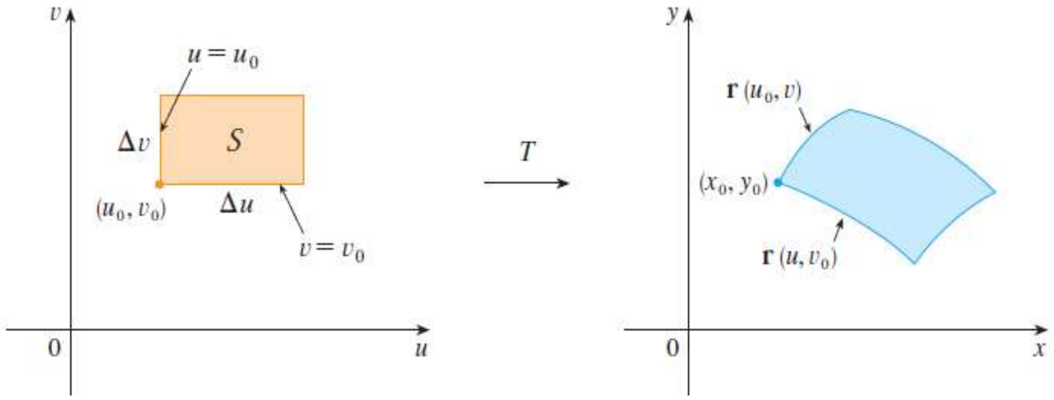


**Figure 2**

The image of S is a region R in the $xy$ −plane, one of whose boundary points is

$$T(u_{0,} v_0) = (x_{0,} y_0).$$

The vector $r(u, v) = g(u, v)i + h(u, v)j$ is the position vector of the image of the point $(u, v)$. The equation of the lower side of S is $v = v_0$, whose image curve is given by the vector function $r(u, v_0)$. The tangent vector at $(x_{0,} y_0)$ to this image curve is

$$r_u = g_u(u_0, v_0)i + h_u(u_0, v_0)j = \frac{\partial x}{\partial u} i + \frac{\partial y}{\partial u} j$$

Similarly, the tangent vector at $(x_{0,} y_0)$ to the image curve of the left side of S ( namely, $u = u_0$) is $r_v = g_v(u_0, v_0)i + h_v(u_0, v_0)j = \frac{\partial x}{\partial v} i + \frac{\partial y}{\partial v} j$

We can approximate the image region $R = T(S)$ by a parallelogram determined by the secant vectors $\quad a = r(u_0 + \Delta u, v_0) - r(u_0, v_0) \quad b = r(u_0, v_0 + \Delta v) - r(u_0, v_0)$ show in Figure 3.

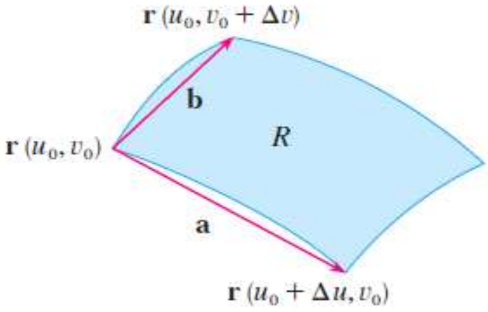


**Figure 3**

But $r_n = \lim_{\Delta v \to 0} \dfrac{r(u_0 + \Delta u, v_0) - r(u_0, v_0)}{\Delta u}$ and so $r(u_0 + \Delta u, v_0) - r(u_0, v_0) \approx \Delta u r_u$

Similarly $r(u_0, v_0 + \Delta v) - r(u_0, v_0) \approx \Delta v r_v$

This means that we can approximate R by a parallelogram determined by the vectors $\Delta u r_u$ and $\Delta v r_v$ (See Figure 4). Therefore we can approximate the area of R by the area of this parallelogram, $|(\Delta u r_u) \times (\Delta v r_v)| = |r_u \times r_v| \Delta u \Delta v$

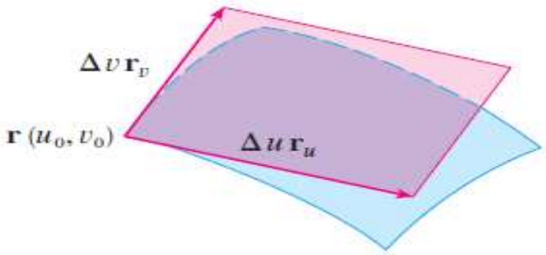


**Figure 4**

Let T be a transformation that maps a region S in uvw-space onto a region R in xyz-space by means of the equations

$$x = g(u.v,w) \qquad y = h(u,v,w) \qquad z = k(u,v,w)$$

**Definition:** Tha Jacobian of transformation T given by $x = g(u,v)$ and $y = h(u,v)$ is

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

Next we divide a region $S$ in the $uv$-plane into rectangles $S_{ij}$ and call their images in the $xy$ −plane $R_{ij}$ (See Figure 5).

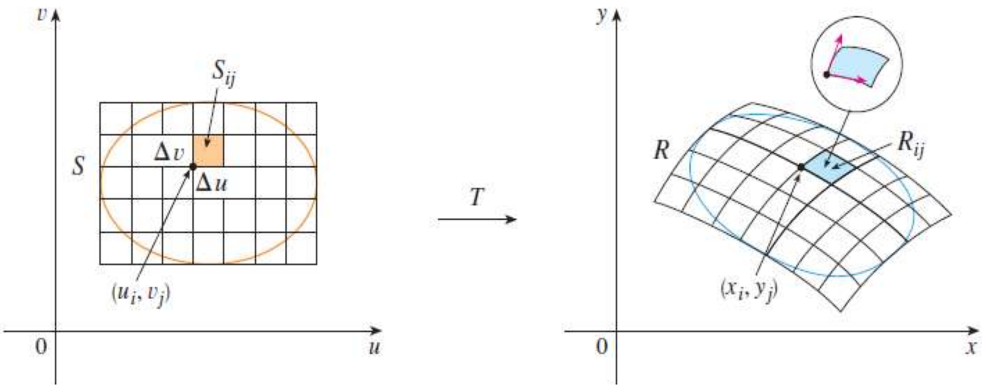


**Figure 5**

Applying the approximation $\Delta A \approx \left|\frac{\partial(x,y)}{\partial(u,v)}\right| \Delta u \Delta v$ to each $R_{ij}$, we approximate the double integral of f over R as follows:

$$\iint_R f(x,y)dA \approx \sum_{i=i}^{m}\sum_{j=1}^{n} f(x_i, y_j)\Delta A$$

$$\approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(g(u_i, v_j))\left|\frac{\partial(x,y)}{\partial(u.v)}\right|\Delta u \Delta v$$

where the Jacobian is evaluated at $(u_i, v_j)$. Notice that this double sum is Riemann sum for the integral

$$\iint_S f(g(u,v), h(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv$$

The foregoing argument suggests that the following theorem is true.

**Theorem 1 (Change of Variables in a Double Integral)**

Suppose that T is a continuous function and has continuous partial derivatives transformation whose Jacobian is nonzero and that maps a region S in the uv-plane onto a region R in the xy-plane. Suppose that f is continuous on R and that R and S are type I and type II regions. Suppose also that T is one-to-one, except perhaps on the boundary of S. Then

$$\iint_R f(x,y)dA = \iint_S f(x(u,v), y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv$$

**Example 1** Use the above theorem to drive the formula for double integration in polar coordinates.

**Solution** Here the transformation T from the $r\theta - plane$ to the $xy - plane$ is given by

$$x = g(r,\theta) = rcos\theta \qquad y = h(r,\theta) = rsin\theta$$

T maps an ordinary rectangle in the $r\theta - plane$ to a polar rectangle in the $xy - plane$. The Jacobian of T is

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} cos\theta & -rsin\theta \\ sin\theta & rcos\theta \end{vmatrix} = rcos^2\theta + rsin^2\theta = r > 0$$

Thus the above theorem gives

$$\iint_R f(x,y)dxdy = \iint_S f(r\cos\theta, r\sin\theta)\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right|drd\theta = \iint_S f(r\cos\theta, r\sin\theta)rdrd\theta$$

Example 2 Use the change of variables$x = u^2 - v^2$, $y = 2uv$ to evaluate the integral$\iint_R \, ydA$, where R is the region bounded by the x-axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x, y \geq 0$.

**Solution:** First we need to compute the Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix} = 4u^2 - 4v^2 \geq 0.$$

Therefore, by the above theorem,

$$\iint_R \, ydA = \iint_S \, 2uv\left|\frac{\partial(x,y)}{\partial(u,v)}\right|dA = \int_0^1\int_0^1(2uv)4(u^2+v^2)dudv = 8\int_0^1\int_0^1(uv)(u^2+v^2)dudv$$

$$= \int_0^1 \left[\frac{1}{4}u^4v + \frac{1}{2}u^2v^2\right]_{u=0}^{u=1} dv = \int_0^1 (2v + 4v^2)dv = [v^2 + v^4]_0^1 = 2$$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial z}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

## CONCLUSION

Now we define the triple integral over a general bounded region E in three – dimensional space (a solid) by much the same procedure that we used for double integrals and finding cylindrical coordinates and spherical coordinates and change of variables in multiple integrals .By the application triple integrals finding Volume, Center of mass of Solid Region.

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