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ON THE PRESSURE DISTRIBUTION OVER THE NORMAL SHOCK INTERACTING WITH A THIN WEDGE MOVING AT SUPERSONIC SPEED

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ABSTRACT

Lighthill considered the diffraction of a normal shock passing over a small bend. Smyrl considered the impact of a plane shock of arbitrary strength which encounters a thin two dimensional wedge moving at supersonic speed. In the present work attempt has been made to obtain the pressure distribution over the normal diffracted shock.

Keyword: Pressure distribution, Diffracted shock, Moving wedge, Supersonic.

I. INTRODUCTION

Smyrl (1963) considered the impact of a plane shock of arbitrary strength which encounters a thin two dimensional wedge moving at supersonic speed. Srivastava (1998, 2000, 2005) has earlier solved several aspects of the problem. In the present investigation we have obtained the pressure distribution over the diffracted shock for the parameters involved in the problem. In fact this pressure distribution is over a Mach Stem and is a useful problem in aeronautics. Srivastava (1994) book may be used for reference. The mathematical theory is dependent on the paper of Smyrl (1963) and it is desirable to consider his paper for any clarification.

We consider a plane shock, the plane of which coincides with (Y,Z) plane moving with velocity U in the direction of x-axis into a uniform region (0) of still air. A thin wedge of infinite span, whose leading coincides at time $t = 0$ with the *z*-axis and whose plane of symmetry lies approximately in the (X,Z) plane moves in the supersonic velocity W in the direction of negative *x*-axis. When $t \leq 0$ the flow regions consists of three uniform regions (0), (1), (2); regions (0) and (1) are separated by the shock, while regions (0) and (2) are separated by weak bow shock waves attached to the leading wedge.

The solution of the problem is sought for $t > 0$ and Smyrl (1963) in his Figure-1, gives the different regions that develop when the wedge has penetrated the normal shock front. In Figure-1, the mathematical theory predicts that the disturbed region is enclosed by the arc of the unit circle, the diffracted shock AB and the wall EA. The fundamental data defining the problem is the Mach number of the shock wave $M\left({}^{U}/_{C_0}\right)$ and Mach number $M'\left({}^{W}/_{C_0}\right)$ of the wedge. U is the velocity of shock wave, W is the velocity of the wedge and C_0 is the sound velocity ahead of shock wave.

Mathematical Formulation: Following Smyrl (1963), the disturbed pressure p' over the diffracted shock has been worked out to be

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$$\frac{\partial p'}{\partial \xi} = \frac{[K_1(\xi-\xi_1)+K_2(\xi-\xi_2)+K_3(\xi-\xi_4)(\xi-\xi_2)](\xi-1)^{1/2}(r_1+r_2)}{[r_1^2+(\xi-1)]\,[r_2^2+(\xi-1)]\,[\xi^2-1]^{1/2}[(\xi-\xi_1)(\xi-\xi_2)]} \qquad -(1)$$

The relation between physical variable y and transformed variable ξ is given by

$$y/y_0 = \left(\frac{\xi-1}{\xi+1}\right)^{1/2} \qquad -(2)$$

Where $y_0 = \sqrt{1-x_0^2}$, $x_0 = \frac{U-V_1}{C_1}$, U is the velocity of shock wave, V_1 is the flow behind the shock wave and C_1 is the velocity of sound behind the shock wave.

From the relation (2) we see that $y/y_0 = 0$, when $\xi = 1$ and $y/y_0 = 1$ when $\xi \to \infty$. So on the diffracted shock, as ξ varies from 1 to ∞, y/y_0 varies from 0 to 1. $y/y_0 = 0$ is the wall shock intersection and $y/y_0 = 1$ corresponds to shock, unit circle intersection.

A word about p is equation (1). This p is

$p' = \frac{p-p_1}{\epsilon\,\rho_1 C_1^2}$, p' being the non-dimensional disturbed pressure, p is the disturbed pressure, p_1, ρ_1, C_1 are the pressure, density and sound velocity behind the shock, $\in$ is the wedge angle.

Equation (1) is integrated to determine p'. The final result is Smyrl (1963)

$$p' = p_5 - \pi/2\,(C_5+C_6) - \frac{1}{2}\Bigg\{C_1\tan^{-1}\left(\frac{r_1^2-2}{\xi+1}\right) + C_2\tan^{-1}\left(\frac{r_2^2-2}{\xi+1}\right)^{1/2}$$
$$+(C_3-C_5)\tan^{-1}\left(\frac{r_3^2-2}{\xi+1}\right)^{1/2} + (C_4-C_6)\tan^{-1}\left(\frac{r_4^2-2}{\xi+1}\right)^{1/2}\Bigg\} \qquad -(3)$$

$$C_1 = \frac{4}{(r_2-r_1)(r_1^2-2)^{1/2}}\left(\frac{K_1}{r_4^2-r_1^2}+\frac{K_2}{r_3^2-r_1^2}+K_3\right) \qquad -(4)$$

$$C_2 = \frac{4}{(r_1-r_2)(r_1^2-2)^{1/2}}\left(\frac{K_1}{r_4^2-r_2^2}+\frac{K_2}{r_3^2-r_1^2}+K_3\right) \qquad -(5)$$

$$C_3 = \frac{(r_1+r_3)(r_2+r_3)}{(r_1-r_3)(r_2-r_3)}C_5 \qquad -(6)$$

$$C_4 = \frac{(r_1+r_4)(r_2+r_4)}{(r_1-r_4)(r_2-r_4)}C_6 \qquad -(7)$$

$$C_5 = \frac{2}{\pi}p_5' \qquad -(8)$$

$$C_6 = \frac{2}{\pi}p_3' \qquad -(9)$$

$$p_5' = \frac{\left\{\frac{C_0}{C_1}M'+M_1\right\}^2}{\left[\left\{\frac{C_0}{C_1}M'+M_1\right\}^2-1\right]^{1/2}} \qquad -(10)$$

M' has already been defined and M_1 is the Mach number of the uniform flow behind the shock.

$$p_3' = \frac{5\rho_0 C_0}{3\rho_1 C_1}y_3\left[M\frac{\delta}{\epsilon}+\frac{10MM'+5M^2M'^2-M^2}{10(M+M')}\right] \qquad -(11)$$

$$y_3 = \frac{[6M(M+M')]}{[(7M^2-1)(M^2+5)(M'^2-1)]^{1/2}} \quad -(12)$$

ρ_0 , C_0 is the density and sound velocity ahead of shock wave, ρ_1 , C_1 are density and sound velocity behind the shock wave.

In $\frac{\delta}{\epsilon}$, δ is the shock deflection (Smyrl 1963) and $\in$ is the wedge angle.

II. NUMERICAL RESULTS

In the following two tables we have determined p' for the combinations $M' = 1.5,\ M = 2$ and $M' = 2,\ M = 2.$

Table-1

y/y_0	0	0.2	0.4	0.6	0.8	1
p'	27.03	21.27	23.06	21.27	18.19	17.05

Table-2

y/y_0	0	0.2	0.4	0.6	0.8	1
p'	3.38	3.03	3.02	2.52	2.36	2.11

III. DISCUSSION

In the case of $M' = 1.5,\ M = 2,$ the pressure is maximum at the wall shock junction $(y/y_0 = 0)$ and continues to go down and attains the minimum value of the intersection of unit circle and shock $(y/y_0 = 1)$. In the case of $M' = 2,\ M = 2,$ the same features appear, values maintain relatively constant values in the interval.

IV. CONCLUSION

Intersection of a normal shock by a wedge moving at supersonic speed is problem of very practical interest in connection with blast effects on supersonic aircraft. Further it covers several other important effects in aeronautical engineering.

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