



COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

In this paper, we prove a common fixed point theorem for six self mapping in intuitionistic fuzzy metric spaces under various conditions. Our results generalizes the recent result of Ranadive and Chauhan [6].

Keywords: Absorbing maps, ϵ –chainable fuzzy metric space, Intuitionistic fuzzy metric space, Reciprocal continuous maps, Semi-compatible.

I.INTRODUCTION

The foundation of fuzzy mathematics is laid by Zadeh [7] with the introduction of fuzzy sets in 1965, as a way to represent vagueness in everyday life. Attanassov [1] introduced and studied the concept of intuitionistic fuzzy metric set as generalizations of fuzzy sets. Intuitionistic fuzzy metric set deals with both the degree of nearness and non-nearness. Park [4] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as generalizations of fuzzy metric space due to George and Veeramani[3].

Ranadive, Gopal and Mishra [5] introduced the concept of absorbing maps in metric space and proved common fixed point theorem in this spaces with observing that the new notion of absorbing maps is neither a sub-class of compatible maps nor a sub-class of non compatible maps. Cho and Jung [2] introduced ϵ –chainable fuzzy metric space and proved common fixed point theorems for weakly compatible in ϵ –chainable fuzzy metric space. Ranadive and Chauhan [6] proved a common fixed point theorem for six self mappings using absorbing maps with ϵ –chainable fuzzy metric space.

II. PRELIMINARIES

Definition 2.1 A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions:

- (i) $*$ is commutative and associative
- (ii) $*$ is continuous;
- (iii) $a * 1 = a$ for all $a \in [0, 1]$;
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2 A binary operation $\diamond : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:

- (i) $\diamond$ is commutative and associative;
- (ii) $\diamond$ is continuous;
- (iii) $a \diamond 0 = a$ for all $a \in [0, 1]$;
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3 A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying following conditions:

- (i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
- (ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
- (iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
- (iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
- (v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
- (vi) for all $x, y \in X$, $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous;
- (vii) $\lim_{n \rightarrow \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
- (viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
- (ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
- (x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
- (xi) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
- (xii) for all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is right continuous;
- (xiii) $\lim_{n \rightarrow \infty} N(x, y, t) = 0$ for all $x, y \in X$.

The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t respectively.

Definition 2.4 Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be

- (i) Convergent to a point $x \in X$ if

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_n, x, t) = 0 \text{ for all } t > 0,$$

- (ii) Cauchy sequence if

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0 \text{ for all } t > 0 \text{ and } p > 0.$$

Definition 2.5 An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.6 Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\epsilon > 0$. A finite sequence

$x=x_0, x_1, x_2, \dots, x_n=y$ is called ϵ - chain from x to y if

$$M(x_i, x_{i-1}, t) > 1 - \epsilon \text{ and } N(x_i, x_{i-1}, t) < \epsilon \text{ for all } t > 0 \text{ and } i=1, 2, \dots, n.$$

A intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is called ϵ - chainable if for any $x, y \in X$ there exists an ϵ - chain from x to y .

Definition 2.7 Let A and B be two self-maps on intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. then A is called B -absorbing if there exists a positive $R > 0$ such that

$$M(Bx, BAx, t) \geq M(Bx, Ax, \frac{t}{R}) \text{ and } N(Bx, BAx, t) \leq M(Bx, Ax, \frac{t}{R}) \text{ for all } x \in X.$$

Similarly, B is called A -absorbing if there exists a positive $R > 0$ such that

$$M(Ax, ABx, t) \geq M(Ax, Bx, \frac{t}{R}) \text{ and } N(Ax, ABx, t) \leq M(Ax, Bx, \frac{t}{R}) \text{ for all } x \in X.$$

Definition 2.8 Let A and S be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. Then a pair (A, S) is said to be commuting if $M(ASx, SAx, t) = 1$ and $N(ASx, SAx, t) = 0$.

Definition 2.9 Let A and S be mappings from a fuzzy metric space $(X, M, *)$ into itself. Then,

the mappings are said to be compatible if $\lim_{n \rightarrow \infty} M_*(ASx_n, SAx_n, t) = 1, \forall t > 0$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x \in X$.

Definition 2.10 Self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be semi-compatible if and only if $M(ASx_n, Sp, t) \rightarrow 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \rightarrow p$ for some p in X as $n \rightarrow \infty$.

Definition 2.11 Let A and S be mappings from a fuzzy metric space $(X, M, *)$ into itself. Then, the mappings are said to be reciprocally continuous if $\lim_{n \rightarrow \infty} ASx_n = Ax, \lim_{n \rightarrow \infty} SAx_n = Sx$ whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x \in X.$$

Lemma 2.1 Let $\{u_n\}$ is a sequence in an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. If there exists a constant $k \in (0, 1)$ such that $M(u_n, u_{n+1}, kt) \geq M(u_{n-1}, u_n, t)$ and $N(u_n, u_{n+1}, kt) \leq N(u_{n-1}, u_n, t)$ for $n = 1, 2, 3, \dots$, then $\{u_n\}$ is a Cauchy sequence in X .

Lemma 2.2 Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. If there exists a constant $k \in (0, 1)$ such that
that
 $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$
for all $x, y \in X$ and $t > 0$, then $x = y$.

III.MAIN RESULTS



Theorem 3.1 Let $(X, M, N, *, \diamond)$ be a complete ϵ -chainable intuitionistic fuzzy metric space with continuous t -norm and continuous t -conorm $\diamond$ defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0, 1]$. Let A, B, S, T, P and Q be mappings from X into itself such that $P(X) \subseteq ST(X)$ and $Q(X) \subseteq AB(X)$.

There $\exists$ a constant $k \in (0, 1)$ such that

$$M(Px, Qy, kt) \geq \min\{M(STx, ABy, t) * M(STx, Qy, t) * M(Qy, ABy, t) * [M(Px, ABy, t) + M(Px, STx, t)]/2\}$$

and

$$N(M(Px, Qy, kt) \leq \max\{M(STx, ABy, t) * M(STx, Qy, t) * M(Qy, ABy, t) * [M(Px, ABy, t) + M(Px, STx, t)]/2\}$$

for all $x, y \in X$.

(3) If one of $P(X), ST(X), AB(X)$ and $Q(X)$ is a complete subspace of X then

(a) The pair (A, BP) has a coincidence point.

(b) The pair (Q, ST) has a coincidence point.

(4) $AB=BA, ST=TS, PB=BP, QT=TQ$.

(5) The pairs $\{P, AB\}$ and $\{Q, ST\}$ are semi-compatible and reciprocally continuous, then A, B, S, T, P and Q have a unique common fixed point in X .

Proof: Let x_0 be an arbitrary point of X . By (1) there $\exists x_1, x_2 \in X$:

$$Px_0 = STx_1 = y_0 \text{ and } Qx_1 = ABx_2 = y_1$$

In general, we can find a sequence $\{x_n\}$ and $\{y_n\}$ in X :

$$Px_{2n} = STx_{2n+1} = y_{2n} \text{ and}$$

$$Qx_{2n+1} = ABx_{2n+2} = y_{2n+1}, \text{ for } n=1, 2, \dots$$

By taking $x = x_{2n}$ and $y = x_{2n+1}$ in (2), we have

$$M(Px_{2n}, Qx_{2n+1}, kt) \geq \min\{M(STx_{2n}, ABx_{2n+1}, t) * M(STx_{2n}, Qx_{2n+1}, t) * M(Qx_{2n+1}, ABx_{2n+1}, t) * [M(Px_{2n}, ABx_{2n+1}, t) + M(Px_{2n}, STx_{2n}, t)]/2\}$$

and

$$N(Px_{2n}, Qx_{2n+1}, kt) \leq \max\{N(STx_{2n}, ABx_{2n+1}, t) \diamond N(STx_{2n}, Qx_{2n+1}, t) \diamond N(Qx_{2n+1}, ABx_{2n+1}, t) \diamond [N(Px_{2n}, ABx_{2n+1}, t) + N(Px_{2n}, STx_{2n}, t)]/2\}$$

$$M(y_{2n}, y_{2n+1}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t) * M(y_{2n-1}, y_{2n}, t) * M(y_{2n+1}, y_{2n}, t) * [M(y_{2n}, y_{2n-1}, t) + M(y_{2n}, y_{2n-1}, t)]/2\}$$

and

$$N(y_{2n}, y_{2n+1}, kt) \leq \max\{N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n+1}, y_{2n}, t) \diamond [N(y_{2n}, y_{2n-1}, t) + N(y_{2n}, y_{2n-1}, t)]/2\}$$



$$M(y_{2n}, y_{2n+1}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n}, y_{2n-1}, t)\} \\ \geq M(y_{2n-1}, y_{2n}, t)$$

and

$$N(y_{2n}, y_{2n+1}, kt) \leq \max\{N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n+1}, y_{2n}, t) \diamond N(y_{2n}, y_{2n-1}, t)\} \\ \leq N(y_{2n-1}, y_{2n}, t)$$

Similarly, we also have

$$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t)$$

$$\text{and } N(y_{2n}, y_{2n+1}, kt) \leq N(y_{2n-1}, y_{2n}, t)$$

therefore for all n , we have

$$M(y_n, y_{n+1}, kt) \geq M(y_n, y_{n-1}, \frac{t}{k}) \geq M(y_n, y_{n-1}, \frac{t}{k^2}) \geq \dots \geq M(y_n, y_{n-1}, \frac{t}{k^n}) \rightarrow 1 \text{ as } n \rightarrow \infty \text{ and}$$

$$N(y_n, y_{n+1}, kt) \leq N(y_n, y_{n-1}, \frac{t}{k}) \leq N(y_n, y_{n-1}, \frac{t}{k^2}) \leq \dots \leq N(y_n, y_{n-1}, \frac{t}{k^n}) \rightarrow 1 \text{ as } n \rightarrow \infty \text{ for any } t > 0.$$

For each $\varepsilon > 0$ and each $t > 0$, we can choose $n_0 \in \mathbb{N}$ such that

$M(y_n, y_{n+1}, t) > 1 - \varepsilon$ and $N(y_n, y_{n+1}, t) < 1 - \varepsilon$. For all $n > n_0$, $m, n \in \mathbb{N}$, we suppose that $m \geq n$. then we have that

$$M(y_n, y_m, t) \geq M(y_n, y_{n+1}, \frac{t}{m-n}) * M(y_{n+1}, y_{n+2}, \frac{t}{m-n}) * \dots * M(y_{m-1}, y_m, \frac{t}{m-n}) \\ \geq (1-\varepsilon) * (1-\varepsilon) * (1-\varepsilon) * \dots * (1-\varepsilon) \\ \geq (1-\varepsilon)$$

Similarly $N(y_n, y_m, t) \leq (1-\varepsilon)$.

Hence $\{y_n\}$ is a Cauchy sequence in X ; that is $y_n \rightarrow z$ in X ; So its subsequences, $Px_{2n}, STx_{2n+1}, ABx_{2n}, Qx_{2n+1}$ also converges to z . Since X is ε -chainable, there $\exists \varepsilon$ -chain from x_n to x_{n+1} such that

$$M(y_i, y_{i-1}, t) > 1 - \varepsilon \text{ and } N(y_i, y_{i-1}, t) < 1 - \varepsilon \text{ for all } t > 0 \text{ and } i = 1, 2, \dots, l.$$

Thus we have

$$M(x_n, x_{n+1}, t) > M(y_1, y_2, \frac{t}{l}) * M(y_2, y_3, \frac{t}{l}) * \dots * M(y_{i-1}, y_i, t) \\ > (1-\varepsilon) * (1-\varepsilon) * (1-\varepsilon) * \dots * (1-\varepsilon) \\ \geq (1-\varepsilon) \text{ and}$$

$$N(x_n, x_{n+1}, t) < N(y_1, y_2, \frac{t}{l}) \diamond N(y_2, y_3, \frac{t}{l}) \diamond \dots \diamond N(y_{i-1}, y_i, t) \\ < (1-\varepsilon) \diamond (1-\varepsilon) \diamond (1-\varepsilon) \diamond \dots \diamond (1-\varepsilon) \\ \leq (1-\varepsilon)$$

So $\{x_n\}$ is a Cauchy sequence in X and hence there $\exists z \in X$ such that $x_n \rightarrow z$.



Since the pair of (P, ST) and (Q, AB) is reciprocal continuous; we have $\lim_{n \rightarrow \infty} P(ST)x_{2n} \rightarrow Pz$ and $\lim_{n \rightarrow \infty} ST(P)x_{2n} \rightarrow STz$ and semi-compatibility of (P, ST) which gives $\lim_{n \rightarrow \infty} ST(P)x_{2n} \rightarrow STz$, therefore

$Pz = STz$, we claim $Pz = STz = z$.

Step 1: Putting $x = z$ and $y = x_{2n+1}$ in (2), we have

$$M(Pz, Qx_{2n+1}, kt) \geq \min\{M(STz, ABx_{2n+1}, t) * M(STz, Qx_{2n+1}, t) * M(Qx_{2n+1}, ABx_{2n+1}, t) * \\ [M(Pz, ABx_{2n+1}, t) + M(Pz, STz, t)]/2\}$$

and

$$N(Pz, Qx_{2n+1}, kt) \leq \max\{N(STz, ABx_{2n+1}, t) \diamond N(STz, Qx_{2n+1}, t) \diamond M(Qx_{2n+1}, ABx_{2n+1}, t) \diamond \\ [N(Pz, ABx_{2n+1}, t) + N(Pz, STz, t)]/2\}$$

Letting $n \rightarrow \infty$, we have

$$M(Pz, z, kt) \geq \min\{M(Pz, z, t) * M(Pz, z, t) * M(z, z, t) * [M(Pz, z, t) + M(Pz, z, t)]/2\}$$

and

$$N(Pz, z, kt) \leq \max\{N(Pz, z, t) \diamond N(Pz, z, t) \diamond N(z, z, t) \diamond [N(Pz, z, t) + N(Pz, z, t)]/2\}$$

$$M(Pz, z, kt) \geq \min\{M(Pz, z, t) * M(Pz, z, t) * 1 * M(Pz, z, t)\}$$

$$\geq 1$$

$$M(Pz, z, t) = 1$$

$$\text{and } N(Pz, z, t) = 1.$$

Hence $Pz = z = STz$.

Step 2: Putting $x = Bz$, $y = x_{2n+1}$ in (2), we have

$$M(P(Bz), Qx_{2n+1}, kt) \geq \min\{M(ST(Bz), ABx_{2n+1}, t) * M(ST(Bz), Qx_{2n+1}, t) * M(Qx_{2n+1}, ABx_{2n+1}, t) * \\ [M(P(Bz), ABx_{2n+1}, t) + M(P(Bz), ST(Bz), t)]/2\}$$

and

$$N(P(Bz), Qx_{2n+1}, kt) \leq \max\{N(ST(Bz), ABx_{2n+1}, t) \diamond N(ST(Bz), Qx_{2n+1}, t) \diamond M(Qx_{2n+1}, ABx_{2n+1}, t) \diamond \\ [N(P(Bz), ABx_{2n+1}, t) + N(P(Bz), ST(Bz), t)]/2\}$$

Letting $n \rightarrow \infty$ we have

Since $PB = BP$, $AB = BA$, so $P(Bz) = B(Pz) = Bz$ and $AB(Bz) = B(ABz) = Bz$

$$M(Bz, z, kt) \geq \min\{M(Bz, z, t) * M(Bz, z, t) * M(z, z, t) * [M(Bz, z, t) + M(Bz, z, t)]/2\}$$

and

$$N(Bz, z, kt) \leq \max\{N(Bz, z, t) \diamond N(Bz, z, t) \diamond M(z, z, t) \diamond [N(Bz, z, t) + N(Bz, z, t)]/2\}$$

$$M(Bz, z, t) = 1 \text{ and } N(Bz, z, t) = 1.$$

Therefore $Az = Bz = Pz = z$.



Step 3: $P(X) \subseteq ST(X)$, there $\exists u \in X$, such that $z = Pz = STu$.

Putting $x = x_{2n}, y = u$ in (2), we have

$$M(Px_{2n}, Qu, kt) \geq \min\{M(STx_{2n}, ABu, t) * M(STx_{2n}, Qu, t) * M(Qu, ABu, t) * \\ [M(Px_{2n}, ABu, t) + M(Px_{2n}, STu, t)]/2\}$$

and

$$N(Px_{2n}, Qu, kt) \leq \max\{N(STx_{2n}, ABu, t) \diamond N(STx_{2n}, Qu, t) \diamond M(Qu, ABu, t) \diamond \\ [N(Px_{2n}, ABu, t) + N(Px_{2n}, STu, t)]/2\}$$

Letting $n \rightarrow \infty$, we have

$$M(z, Qu, kt) \geq \min\{M(z, z, t) * M(z, Qu, t) * M(Qu, z, t) * [M(z, z, t) + M(z, z, t)]/2\}$$

and

$$N(z, Qu, kt) \leq \max\{N(z, z, t) \diamond N(z, Qu, t) \diamond M(Qu, z, t) \diamond [N(z, z, t) + N(z, z, t)]/2\}$$

$$M(z, Qu, t) = 1 \text{ and } N(z, Qu, t) = 1.$$

Therefore $z = Qu = Abu$.

Since Q is AB -absorbing ; then

$$M(Abu, ABQu, kt) \geq M(Abu, Qu, \frac{t}{R}) = 1$$

$$\text{i.e. } Abu = ABQu \Rightarrow z = ABz.$$

Step 4: Putting $x = x_{2n}, y = z$ in (2), we have

$$M(Px_{2n}, Qz, kt) \geq \min\{M(STx_{2n}, ABz, t) * M(STx_{2n}, Qz, t) * M(Qz, ABz, t) * \\ [M(Px_{2n}, ABz, t) + M(Px_{2n}, STx_{2n}, t)]/2\}$$

and

$$N(Px_{2n}, Qz, kt) \leq \max\{N(STx_{2n}, ABz, t) \diamond N(STx_{2n}, Qz, t) \diamond M(Qz, ABz, t) \diamond \\ [N(Px_{2n}, ABz, t) + N(Px_{2n}, STx_{2n}, t)]/2\}$$

Letting $n \rightarrow \infty$, we have

$$M(z, Qz, kt) \geq \min\{M(z, z, t) * M(z, Qz, t) * M(Qz, z, t) * [M(z, z, t) + M(z, z, t)]/2\}$$

and

$$N(z, Qz, kt) \leq \max\{N(z, z, t) \diamond N(z, Qz, t) \diamond M(Qz, z, t) \diamond [N(z, z, t) + N(z, z, t)]/2\}$$

$$\text{i.e. } M(z, Qz, kt) \geq M(z, Qz, t) \text{ and}$$

$$N(z, Qz, kt) \leq N(z, Qz, t)$$

Therefore $z = Qz = ABz$.

Step 5: Put $x = x_{2n}, y = Tz$ in (2), we have



$$M(Px_{2n}, QTz, kt) \geq \min\{M(STx_{2n}, AB(Tz), t) * M(STx_{2n}, Q(Tz), t) * M(Q(Tz), AB(Tz), t) *$$

$$[M(Px_{2n}, AB(Tz), t) + M(Px_{2n}, STx_{2n}, t)]/2\}$$

and

$$N(Px_{2n}, QTz, kt) \leq \max\{N(STx_{2n}, AB(Tz), t) \diamond N(STx_{2n}, Q(Tz), t) \diamond M(Q(Tz), AB(Tz), t) \diamond$$

$$[N(Px_{2n}, AB(Tz), t) + N(Px_{2n}, STx_{2n}, t)]/2\}$$

Since $QT=TQ, ST=TS$, therefore $Q(Tz)=T(Qz)=Tz, ST(Tz)=T(STz)=Tz$,

Letting $n \rightarrow \infty$, we have

$$M(z, Tz, kt) \geq \min\{M(z, Tz, t) * M(z, Tz, t) * M(Tz, Tz, t) * [M(z, Tz, t) + M(z, Tz, t)]/2\}$$

and

$$N(z, Tz, kt) \leq \max\{N(z, Tz, t) \diamond N(z, Tz, t) \diamond M(Tz, Tz, t) \diamond [N(z, Tz, t) + N(z, Tz, t)]/2\}$$

i.e. $M(z, Tz, kt) \geq M(z, Tz, t)$ and

$$N(z, Tz, kt) \leq N(z, Tz, t)$$

Therefore $z=Tz=Sz=Qz$

Hence $z=Az=Bz=Pz=Sz=Qz=Tz$.

Uniqueness: Let w be another fixed point of A, B, P, S, Q and T . Then

Put $x=u, y=w$ in (2), we have

$$M(Pu, Qw, kt) \geq \min\{M(STu, ABw, t) * M(STu, Qw, t) * M(Qw, ABw, t) * [M(Pu, ABw, t) + M(Pu, STw, t)]/2\}$$

and

$$N(Pu, Qw, kt) \leq \max\{N(STu, ABw, t) \diamond N(STu, Qw, t) \diamond M(Qw, ABw, t) \diamond [N(Pu, ABw, t) + N(Pu, STw, t)]/2\}$$

$$M(u, w, kt) \geq \min\{M(u, w, t) * M(u, w, t) * M(w, w, t) * [M(u, w, t) + M(u, w, t)]/2\}$$

and

$$N(u, w, kt) \leq \max\{N(u, w, t) \diamond N(u, w, t) \diamond M(w, w, t) \diamond [N(u, w, t) + N(u, w, t)]/2\}$$

i.e. $M(u, w, kt) \geq M(u, w, t)$ and

$$N(u, w, kt) \leq N(u, w, t)$$

Hence $u=w \Rightarrow z=u=w$.

IV. CONCLUSION

Theorem 3.1 is a generalization of the result of Ranadive and Chauhan [6] and in the sense that employing the notion of semi-compatible and reciprocal continuity. Our result stronger than that Ranadive and Chauhan[6] we extend the above result of intuitionistic fuzzy metric space using absorbing maps.



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