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OPTIMAL OPERATING POLICIES OF AN EPQ MODEL WITH STOCK DEPENDENT PRODUCTION RATE AND TIME DEPENDENT DEMAND HAVING PARETO DECAY

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ABSTRACT

In this paper an economic production quantity model is developed and analyzed for deteriorating items. Here it is assumed that the production rate is dependent on stock on-hand having Pareto rate of decay. It is further assumed that the demand follows a power pattern with an index parameter. The model behavior is analyzed by deriving the instantaneous state of inventory at time t, stock loss due to deterioration and production quantity. By minimizing the total cost of production the optimal values of production downtime (time point at which production stops), production uptime (time point at which production resumes) and optimal production quantity are derived. The sensitivity analysis of the model revealed that the stock dependent production rate can reduce the total production cost and unnecessary inventory of goods. It is also observed that the Pareto rate of decay can well characterize the deterioration of a commodity like cement. This model also includes several of the earlier models as particular cases.

Keywords EPQ, Pareto decay, stock dependent production, power demand pattern

I. INTRODUCTION

Recently much emphasis is given for analyzing economic production quantity models which provide the basic frame work for monitoring and controlling the production processes for deteriorating items like cement, oil, food processing, ic chips, textiles etc. Goyal and Giri [6], Ruxian Li et al. [10], Pentico and Drake [9] have reviewed several types of inventory models for deteriorating items. Deterioration is characterized as loss of life or obsolete or decay or evaporate due to various facto` Since the deterioration is influenced by several random factors, the life time of the commodity is considered as random (Nahmias [8]). Several authors develop various economic production quantity models for deteriorating items with various assumptions on life time of the commodity, pattern of demand and production. (Skouri and Papachristos [11], Chen and Chen [4], Balkhi [2], Manna and Chaudhari [7], Teng et al. [17], Abad [1], Goyal and Giri [5]) have developed inventory models for deteriorating items with finite rate of production. Recently Srinivasa Rao et al. [13], Srinivasa Rao et al. [15], Begum et al. [3] have developed and analyzed EPQ models for deteriorating items with generalized Pareto decay and constant rate of production. Umamaheswara Rao et al. [18], Venkata Subbaiah et al. [19], Srinivasa Rao et al. [14] have developed and analyzed economic production quantity models with weibull rate of decay and constant rate of production. Sridevi et al. [12], Srinivasa Rao et al. [16] have developed and analyzed economic production quantity models with random production having constant decay.

In all these models, the production rate considered to be independent of stock on hand. But in reality if the product is produced without considering the on hand stock the situation may lead excess inventories or heavy shortages. Therefore it is the common practice in several of the industries dealing with perishable items cements, chemicals etc., the production rate is adjusted depending on the stock on hand i.e. if more stock is there then the production rate is reduced and if the less stock is there the production rate is increased. This type of production rate is known as stock dependent production. Very little work has been reported in literature regarding economic production quantity models with stock dependent production. Hence in this paper an economic production quantity model with stock dependent production rate and time dependent demand having Pareto decay is developed and analyzed. The Pareto decay is capable of characterizing the life time of the commodities having asymmetrically distributed variates. Its instantaneous rate of decay is inversely propositional to ageing. Assuming shortages are allowed and fully backlogged the model is developed. It is also developed to the case of without shortages.

The rest of the paper is organized as follows: section 2 deals with notations and assumptions and section 3 development of the EPQ model. In section 4, the optimal operating policies of the model are derived. In section 5, a case study dealing with cement industry is presented. Section 6, is concerned with sensitivity analysis of the model with respect to parameters and costs. Section 7, deals with the EPQ model without shortages. Section 8 is to discuss the conclusions and scope for further work.

II. NOTATIONS AND ASSUMPTIONS OF THE MODEL

2.1 Notations

A Set up cost

π Shortage cost per unit per time

D (t) Demand rate

EPQ Economic Production Quantity

h Inventory holding cost per unit time

H Total inventory holding cost in a cycle time

I (t) Inventory level at any time T

c Unit production cost of the item

Q Production Quantity

S_1 Maximum inventory level

S_2 Maximum Shortage level

R (t) Rate of production at any time t

s Selling price of the items

s_h Total Shortage cost in a cycle time

t_1 Time Point at which Production stops (Production down time)

t_2 Time Point at which shortages begins

t_3 Time Point at which Production resumes (Production up time)

T Production cycle time

TC Total production cost per unit time

TP Profit rate per unit time

TR Total revenue of the system pr unit time

b Deterioration rate parameter

(r, n, τ, a, d) Demand rate parameters

(η, k) Production rate parameters

2.2 Assumptions

i. Life time of the commodity is random and follows a Pareto distribution having probability density function of the form $f(t) = \frac{b\theta^b}{t^{b+1}}$; $t \geq \theta$, $b, \theta > 0$

The instantaneous rate of deterioration is $h(t) = \frac{f(t)}{1-F(t)} = \frac{b}{t}$; $b > 0$, $t > \theta$

ii. The demand is known and the demand rate is time dependent i.e. of the form $D(t) = \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$. This is known as power demand pattern, where r is the fixed quantity and n is the parameter of power demand pattern, the value of n may be any positive number.

iii. The rate of production is dependent on stock on hand and is of the form
$$R(t) = \begin{cases} \eta - kI(t), & 0 \leq t \leq t_1 \\ \eta, & t_3 \leq t \leq T, \\ 0, & \text{otherwise} \end{cases}$$
where η is a constant such that $\eta > 0$, k is the stock dependent production rate parameter, $0 \leq k \leq 1$. It is assumed that $R(t) \geq D(t)$ at any time where replenishment takes place. If k=0, then it includes the finite rate of production.

iv. There is no repair or replacement of deteriorated items.

v. The planning horizon is finite. Each cycle will have length T.

vi. Lead time is zero.

vii. The inventory holding cost per unit time (h), the shortage cost per unit per unit time (π), the unit production cost per unit time (c) and set up cost(A) per cycle are fixed and known.

III. EPQ MODEL WITH SHORTAGES

3.1 Model formulation

Consider an inventory system for deteriorating items in which the life time of the commodity is random and follows a Pareto distribution. Here, it is assumed that shortages are allowed and fully backlogged. In this model the stock level for the item is initially zero. Production starts at time t=0 and continues adding items to stock until the on hand inventory reaches its maximum level S_1. At time t = 0 deterioration of the item starts and stock is depleted by consumption and deterioration while production is continuously adding to it. At time $t = t_1$, the production is stopped and stock will be depleted by deterioration and demand until it reaches zero at time $t = t_2$. As demand is assumed to occur continuously, at this point shortage begin to accumulate until it reaches its maximum level of S_2 at $t = t_3$. At this point production will resume meeting the current demand and clearing the backlog. Finally shortages will be cleared at time t = T. Then the cycle will be repeated indefinitely. These types of production systems are common in cement industries where production rate is stock dependent.

The schematic diagram representing the inventory system is shown in fig. 1

Inventory level I (t)

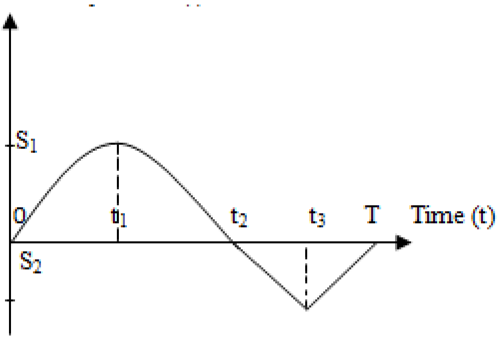


Fig.1 Schematic diagram representing the inventory level of the system with- shortages.

The differential equations governing the system in the cycle time (0, T) are;

$$\frac{dI(t)}{d\eta} = \eta - kI(t) - h(t)I(t) - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, 0 \le t \le t_1 (1)$$

$$\frac{dI(t)}{dt} = -h(t)I(t) - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \qquad t_1 \le t \le t_2 (2)$$

$$\frac{dI(t)}{dt} = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \qquad t_2 \le t \le t_3 (3)$$

$$\frac{dI(t)}{dt} = \eta - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, \qquad t_3 \le t \le T\ (4)$$

With the initial conditions, I (0) = 0, I (t_2) = 0 and I (T) = 0,

Solving the differential equations (1) to (4) the on hand inventory levels at time t are respectively.

$$I(t) = e^{-kt}t^{-b}\left\{\eta g(t,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t,b,k)\right\},$$

$$0 \le t \le t_1 (5)$$

$$\text{where}, g(t,b,k) = \int_0^t e^{ku} u^b du, f(t,b,k) = \int_0^t e^{ku} u^{b+\frac{1}{n}-1} du \qquad (6)$$

$$I(t) = \frac{r}{(1+bn)T^{\frac{1}{n}}}\left[t_2^{b+\frac{1}{n}} t^{-b} - t^{\frac{1}{n}}\right], \ t_1 \le t \le t_2 (7)$$

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}} - t^{\frac{1}{n}}\right], \qquad t_2 \le t \le t_3\ (8)$$

$$I(t) = \eta(t - T) + \frac{r}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} - t^{\frac{1}{n}}\right], \quad t_3 \le t \le T \quad (9)$$

The total inventory in the time period $0 \leq t \leq t_1$ is

$$\int_0^{t_1} I(t)\,dt = \int_0^{t_1} e^{-kt} t^{-b}\left\{\eta g(t,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t,b,k)\right\}dt \qquad (10)$$

where, g (t, b, k) and f(t, b, k) are defined as in equation (6)

The total inventory in the period (t_1, t_2) is

$$\int_{t_1}^{t_2} I(t)\,dt = \int_{t_1}^{t_2} \frac{r}{(1+bn)T^{\frac{1}{n}}}\left[t_2^{b+\frac{1}{n}} t^{-b} - t^{\frac{1}{n}}\right]dt \quad (11)$$

Since I (t) is continuous at t_1, equating (5) and (7) to establish the relationship between t_1 and t_2.

$$t_2 = \left[\left(\frac{1+bn}{r}\right)e^{-kt_1} T^{\frac{1}{n}}\left\{\eta g(t_1,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t_1,b,k)\right\} + t_1^{b+\frac{1}{n}}\right]^{\frac{n}{1+bn}} \qquad (12)$$

$$\text{where, } g(t_1,b,k) = \int_0^{t_1} e^{ku} u^b\,du,\ f(t_1,b,k) = \int_0^{t_1} e^{ku} u^{b+\frac{1}{n}-1}\,du \qquad (13)$$

The maximum inventory level $S_1 = I(t_1)$ is

$$S_1 = e^{-kt_1} t_1^{-b}\left\{\eta g(t_1,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t_1,b,k)\right\} \qquad (14)$$

where, $g(t_1, b, k)$ and $f(t_1, b, k)$ are defined as in equation (13)

Similarly, since I (t) is continuous at t_3 equating equations (8) and (9) to establish the relationship between t_2 and t_3, therefore

$$t_2 = [1 + \frac{\eta}{r}(t_3 - T)]^n T \qquad (15)$$

The maximum shortage level $S_2 = I(t_3)$ is

$$S_2 = \frac{r}{T^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}} - t_3^{\frac{1}{n}}\right] \qquad (16)$$

The backlogged demand is

$$B(t) = \int_{t_2}^{t_3} \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\,dt \qquad (17)$$

Stock loss due to deterioration at time t is

$$L(t) = \int_0^t R(t)\,dt - \int_0^t D(t)\,dt - I(t)$$

$$= \eta(t_1 + T - t_3) - k\eta\int_0^{t_1} t^{-b} e^{-kt} g(t,b,k)\,dt$$

$$+ \frac{kr}{nT^{\frac{1}{n}}}\int_0^{t_1} t^{-b} e^{-kt} f(t,b,k)\,dt - \int_0^{T} \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\,dt - I(t) \qquad (18)$$

where, g (t, b, k) and f (t, b, k) are as defined in equation (6)

Total production in the cycle time (0, T) is

$$Q = \int_0^{t_1} R(t)\,dt + \int_{t_3}^{T} R(t)\,dt$$

$$= \int_0^{t_1} \eta\,dt + \int_{t_3}^{T} \eta\,dt - k\int_0^{t_1} I(t)\,dt$$

$$= \eta(t_1 + T - t_3) - k\eta\int_0^{t_1} t^{-b}\,e^{-kt} g(t,b,k)\,dt + \frac{kr}{nT^{\frac{1}{n}}}\int_0^{t_1} t^{-b} e^{-kt} f(t,b,k)\,dt \quad (19)$$

where, g (t, b, k) and f(t, b, k) are defined as in equation (6)

Let TC (t_1, t_3, T) is the total cost per unit time. Then, TC (t_1,t_3, T) is the sum of the setup cost per unit time, the production cost per unit time, inventory holding per unit time and the shortage cost per unit time i.e.,

$$TC(t_1, t_3, T) = \frac{A}{T} + \frac{c}{T}Q + \frac{H}{T} + \frac{S_h}{T} \qquad (20)$$

Holding cost in a cycle time is

$$H = h\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_2} I(t)\,dt\right] \qquad (21)$$

Shortage cost in a cycle time is

$$S_h = \pi\left[\int_{t_2}^{t_3} -I(t)\,dt + \int_{t_3}^{T} -I(t)\,dt\right] \qquad (22)$$

Therefore, the total cost per unit time is

$$TC(t_1, t_3, T) = \frac{A}{T} + \frac{c}{T}Q + \frac{h}{T}\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_2} I(t)\,dt\right] \qquad (23)$$

This implies,

$$\begin{aligned} TC(t_1, t_3, T) = {} & \frac{A}{T} + \frac{c\eta}{T}(t_1 + T - t_3) + \frac{(h - ck)\eta}{T}\int_0^{t_1} t^{-b}\,e^{-kt} g(t,b,k)\,dt \\ & + \left[\frac{(ck - h)r}{nT^{\frac{1}{n}+1}}\right]\int_0^{t_1} t^{-b}\,e^{-kt} f(t,b,k) \\ & + \frac{hr}{(1+bn)(1-b)(1+n)T^{\frac{1}{n}+1}}\Bigg[(1+bn)\left\{1 + \frac{\eta}{r}(t_3 - T)\right\}^{1+n} T^{\frac{1}{n}+1} \\ & - (1+n)t_1^{1-b}\left\{1 + \frac{\eta}{r}(t_3 - T)\right\}^{(1+bn)} T^{b+\frac{1}{n}} + n(1-b)t_1^{\frac{1}{n}+1}\Bigg] \\ & - \frac{\pi r}{(1+n)T^{\frac{1}{n}+1}}\left[(1+n)\frac{\eta}{r}(t_3 - T)T^{\frac{1}{n}}t_3 + T^{\frac{1}{n}+1}\left(1 - \left\{1 + \frac{\eta}{r}(t_3 - T)\right\}^{1+n}\right)\right] \\ & + \frac{\pi\eta}{2T}(T - t_3)^2 \qquad (24) \end{aligned}$$

where, g (t, b, k) and f(t, b, k) are as defined in equation (6)

3.2 Optimal Operating Policies of the Model

In this section, the optimal policies of the inventory system developed in section (3.1) are derived. To find the optimal values of production down time t_1 and production up time t_3 one has to minimize the total cost TC (t_1, t_3,T) in equation (24) with respect to t_1 and t_3 and equate the resulting equations to zero. The condition for the solutions to be optimal (minimum) is that the determinant of the Hessian matrix is positive definite i.e.

$$D = \begin{vmatrix} \frac{\partial^2 TC(t_1,t_3,T)}{\partial t_1^2} & \frac{\partial^2 TC(t_1,t_3,T)}{\partial t_1 \partial t_3} \\ \frac{\partial^2 TC(t_1,t_3,T)}{\partial t_1 \partial t_3} & \frac{\partial^2 TC(t_1,t_3,T)}{\partial t_3^2} \end{vmatrix} > 0 \qquad (25)$$

Differentiating TC (t_1, t_3, T) with respect to t_1 and equating it to zero implies

$$\frac{\partial TC(t_1,t_3,T)}{\partial t_1} = \frac{c}{T}\eta + \frac{(h-ck)\eta}{T} t_1^{-b} e^{-kt_1} g(t_1,b,k) + \frac{(ck-h)r}{nT^{\frac{1}{n}+1}} t_1^{-b} e^{-kt_1} f(t_1,b,k) + \frac{hr}{(1+bn)T^{\frac{1}{n}+1}}\left[t_1^{\frac{1}{n}} - t_1^{-b}\left\{1+\frac{\eta}{r}(t_3-T)\right\}^{1+bn} T^{b+\frac{1}{n}}\right] = 0 \qquad (26)$$

where, g (t_1, b, k) and f (t_1, b, k) are as defined in equation (13)

Differentiating TC (t_1, t_3, T) with respect to t_3 and equating it to zero implies

$$\frac{\partial TC(t_1,t_3,T)}{\partial t_3} = -\frac{c}{T} + \left[\frac{h}{(1-b)} + \pi\right]\left\{1+\frac{\eta}{r}(t_3-T)\right\}^{1+bn} - \frac{h}{(1-b)}\left\{1+\frac{\eta}{r}(t_3-T)\right\}^{bn} t_1^{1-b} T^{b-1} - \frac{\pi}{T} t_3 = 0 \qquad (27)$$

Solving the equations (26) and (27) simultaneously using numerical methods one can obtain the optimal values of t_1 and t_3. Substituting these optimal values of t_1 and t_3 in the equation (15), (19) and (24) one can get the optimal values of t_2, production quantity Q and total cost TC (t_1, t_3,T) respectively. For each set of optimal values, the determinant of the Hessian matrix is computed and verified for positive semi definiteness.

3.3 Numerical Illustration

To expound the model developed, consider the case of deriving and economic production quantity, production down time and production up time for a cement industry. Here the product is of a deteriorating type and has a random life time which is assumed to follow Pareto distribution. Form the records and discussions held with the production and marketing personnel the values of various parameters are considered. For different values of the parameters and costs, the optimal values of production down time, production up time, optimal production quantity and total cost are computed and presented in Table 1.

From Table 1, it is observed that when increase in deterioration parameter b from 1.1 to 1.5 units results a decrease in production down time t_1^*, production quantity Q* and an increase total cost TC* i.e. t_1^* from 2.393 to 2.015 months, Q* from 189.24 to 187.172 units and total cost TC* from

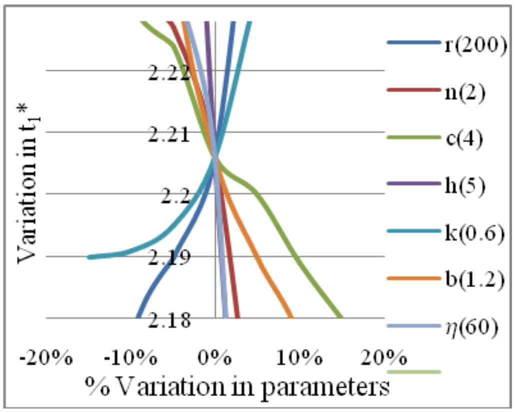
335.323 to 352.088. There is a slight decrease in production up time t_3^*, from 10.951 to 10.722 months. When the demand rate τ increases then the optimal production down time t_1^*, production up time t_3^*, production quantity Q* and total cost TC* are increasing. The increase in holding cost h results a decrease in production down time t_1^* from 2.319 to 2.017 months, production up time, t_3^* from 11.001 to 10.701 months and increase in production quantity Q* from 183.417 to 187.781 units and increase in total cost TC* from ` 319.54 to `358.885. The increase in shortage cost π has significant effect on all optimal policies viz. production down time t_1^* from 1.988 to 2.940 months, production up time t_3^* from 10.76 to 11.322 months, production quantity Q* from 182.897 to 202.281 units and total cost TC* from ` 304.398 to `357.851. The increase in production rate parameter k results an increase in production down time, decrease in production up time, production quantity and increase in total cost i.e. Production quantity Q* from 222.058 to 186.715 and total cost from ` 341.124 to `342.588.

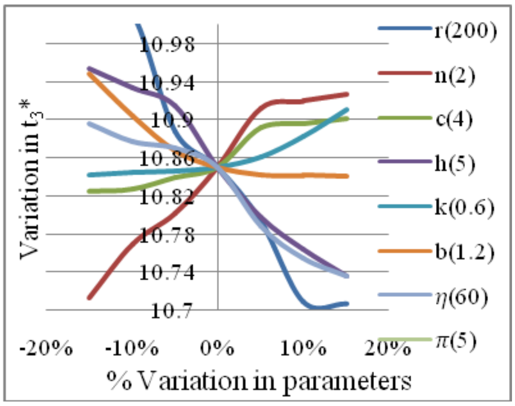
3.4 Sensitivity analysis

To study the effect of changes in the parameters and costs on the optimal values of production down time, production up time and production quantity sensitivity analysis is performed taking the values A = `100, c = ` 4, h = ` 5, T = 12 months, π = ` 5, r = 200 units, b = 1.2, k = 0.4, η = 60.

Sensitivity analysis is performed by changing the parameters by -15%, -10%, -5%, 0%, 5%, 10% and 15%. First changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values of t_1, t_3, Q and TC are computed and the results are presented in Table 2. The relationship between parameters, costs and the optimal values are shown in fig 2.

From Table 2, it is observed that the demand rate τ and deteriorating parameter b have significant effect on production down time, production up time, production quantity and total cost. Decrease in unit cost c results increase in Q* from 183.221 to 193.049 units and decrease in TC* from ` 364.810 to ` 314.671. The increase in production rate parameter η increases the production quantity Q*and total cost TC* from 162.866 units to 212.126 units and ` 323.045 to ` 387.294. The increase in shortage cost results in decrease in production quantity Q* from 224.753 units to 170.239 units and increase in total cost from ` 335.244 to ` 359.132.





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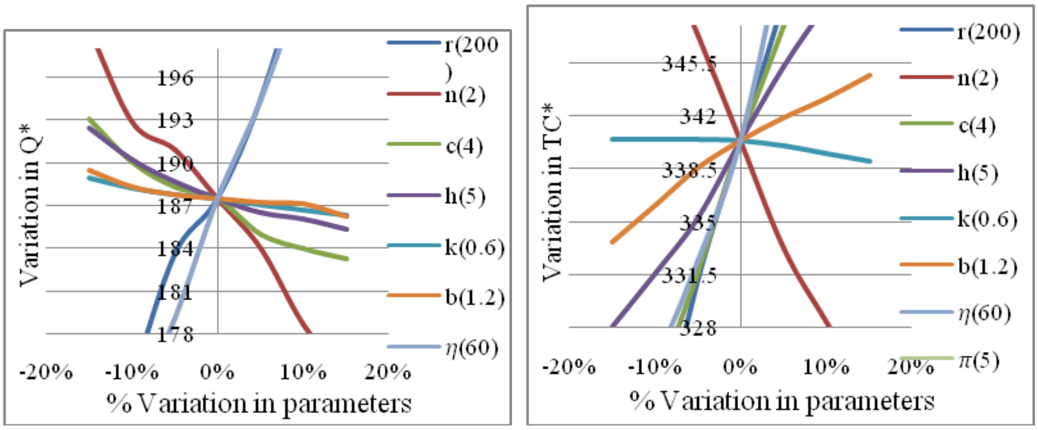

Fig. 2 Relationship between optimal values and parameters

IV. EPQ MODEL WITHOUT SHORTAGES

4.1 Model formulation

Consider a production system in which the production starts at time t = 0 and inventory level gradually increases with the passage of time due to production and demand during the time interval (0, t_1). At time t_1 the production is stopped and let S_1 be the inventory level at that time. During the time interval (t_1, T) the inventory decreases partly due to demand and partly due to deterioration of items. The cycle continues when inventory reaches zero at time t = T. The schematic diagram representing the model is shown in fig 3.

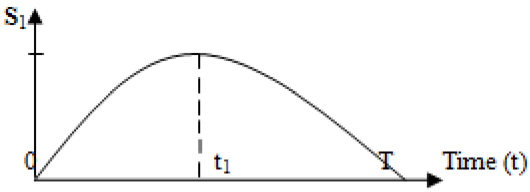

Fig 3 The schematic diagram representing the inventory level of the system without shortages

The differential equations governing the system in the cycle time (0, T) are;

$$\frac{dI(t)}{dt} = \eta - kI(t) - h(t)I(t) - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, 0 \le t \le t_1 \quad (28)$$

$$\frac{dI(t)}{dt} = -h(t)I(t) - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}, t_1 \le t \le T \quad (29)$$

with the boundary conditions I (0) = 0 and I (T) = 0, Solving the differential equations (28) and (29)

The instantaneous state of inventory at any time t during the interval (0, t_1) is obtained as

$$I(t) = e^{-kt}t^{-b}\left\{\eta g(t,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t,b,k)\right\},$$

$$0 \le t \le t_1 \quad (30)$$

where, g (t, b, k) and f (t, b, k) are as defined in equation (6)

The instantaneous state of inventory at any time t during the interval (t_1, T) is obtained as

$$I(t) = \frac{r}{(1+bn)T^{\frac{1}{n}}}\left[T^{b+\frac{1}{n}}t^{-b} - t^{\frac{1}{n}}\right],$$

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$t_1 \le t \le T$ (31)

The total inventory in the time period $0 \le t \le t_1$ is

$$\int_0^{t_1} I(t)\,dt =$$

$$\int_0^{t_1} e^{-kt} t^{-b} \left\{ \eta g(t,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t,b,k) \right\} dt\,,$$

$$0 \le t \le t_1 \quad (32)$$

where, g (t, b, k) and f (t, b, k) are as defined as in equation (6)

and

the total inventory in the time period $t_1 \le t \le T$ is

$$\int_{t_1}^{T} I(t)\,dt = \int_{t_1}^{T} \frac{r}{(1+bn)T^{\frac{1}{n}}} \left[T^{b+\frac{1}{n}} t^{-b} - t^{\frac{1}{n}} \right] dt \quad (33)$$

The maximum inventory level $S_1 = I(t_1)$ is

$$S_1 = e^{-kt_1} t_1^{-b} \left\{ \eta g(t_1,b,k) - \frac{r}{nT^{\frac{1}{n}}} f(t_1,b,k) \right\} \quad (34)$$

where, g (t_1, b, k) and f (t_1, b, k) are as defined in equation (13)

Stock loss due to deterioration at time t is

$$L(t) = \int_0^t R(t)\,dt - \int_0^t D(t)\,dt - I(t) \quad (35)$$

$$L(t) = \eta t_1 - k\eta \int_0^{t_1} t^{-b} e^{-kt} g(t,b,k)dt + \frac{kr}{nT^{\frac{1}{n}}} \int_0^{t_1} t^{-b} e^{-kt} f(t,b,k)dt \quad - \int_0^{t_1} \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} dt \int_{t_1}^{T} \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\, dt - It$$

$$0 \le t \le T \quad (36)$$

where, g (t, b, k) and f (t, b, k) are as defined in equation (6)

Total production quantity in the cycle time (0, T) is

$$Q = \int_0^{t_1} R(t)\,dt$$

$$= \eta t_1 - k\eta \int_0^{t_1} t^{-b} e^{-kt} g(t,b,k)dt +$$

$$\frac{kr}{nT^{\frac{1}{n}}} \int_0^{t_1} t^{-b} e^{-kt} f(t,b,k)dt \quad (37)$$

where, g (t, b, k) and f (t, b, k) are as defined in equation (6)

Let TC (t_1, T) is the total cost per unit time. Then, TC (t_1, T) is the sum of the setup cost per unit time, the production cost per unit time and inventory holding cost per unit time i.e.,

$$TC(t_1,T) = \frac{A}{T} + \frac{c}{T} Q + \frac{H}{T} \quad (38)$$

Holding cost in the cycle time T is

$$H = h\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{T} I(t)\,dt\right] \qquad (39)$$

Therefore the total cost per unit time is

$$TC(t_1, T) = \frac{A}{T} + \frac{c}{T}Q + \frac{h}{T}\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{T} I(t)\,dt\right] \qquad (40)$$

This implies,

$$TC(t_1, T) = \frac{A}{T} + \frac{C}{T}\eta t_1 + \frac{(h-ck)\eta}{T}\int_0^{t_1} e^{-kt} t^{-b} g(t,b,k)dt + \frac{(ck-h)r}{nT^{\frac{1}{n}+1}}\int_0^{t_1} e^{-kt} t^{-b} f(t,b,k)dt + \left[\frac{hr}{(1+bn)T^{\frac{1}{n}+1}}\right]\left\{\frac{(1+bn)T^{\frac{1}{n}+1} - (n+1)T^{b+\frac{1}{n}} t_1^{-b+1} + n(1-b)t_1^{\frac{1}{n}+1}}{(1-b)(1+n)}\right\} \qquad (41)$$

where, g (t, b, k) and f (t, b, k) are as defined as in equation (6)

4.2 Optimal Operating Policies of the Model

In this section, we obtain the optimal policies of the inventory system developed in section (4.1). The problem is to find the optimal values of production down time t_1 that minimize the total cost TC (t_1, T) over (0, T). To obtain the optimal values we differentiate TC (t_1, T) in equation (41) with respect to t_1 and equate the resulting equation to zero. The condition for the solutions to be optimal (minimum) is that

$$\frac{\partial^2 TC(t_1, T)}{\partial t_1^2} > 0 \qquad (42)$$

Differentiating TC (t_1, T) with respect to t_1 and equating to zero one can get

$$\frac{\partial TC(t_1, T)}{\partial t_1} = 0 \qquad (43)$$

$$\frac{c}{T}\eta + \frac{\eta(h-ck)}{T} e^{-kt_1} t_1^{-b} g(t_1, b, k) + \frac{(c-hk)r}{nT^{\frac{1}{n}+1}} e^{-kt_1} t_1^{-b} f(t_1, b, k) + \frac{hr}{(1+bn)T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}} - t_1^{-b} T^{b+\frac{1}{n}}\right] = 0 \qquad (44)$$

where, g (t_1, b, k) and f (t_1, b, k) are as defined as in equation (13)

Solving the equation (44) using numerical methods one can get the optimal value of t_1. Substituting the optimal value of t_1 in the equation (37) and (41) the optimal values of production quantity Q and total cost TC can be obtained respectively.

4.3 Numerical Illustration

To expound the model developed, consider the case of deriving an economic production quantity and production down time for a cement manufacturing unit. Here, the product is deteriorating type and has random life time and assumed to follow a Pareto distribution. Based on the discussions held with the personnel connected with the production and marketing and the records it is observed that the deterioration parameter b is estimated to vary from 1.2 to 1.6 months and demand parameter r to vary from 200 to 300 units respectively. The other parameters are considered as T = 12 months, c = ` 3 to ` 5, h = ` 4 to ` 6, k = 0.4 to 0.8 and η = 50 to 70. Substituting the values of the parameters and costs in the equation (44) and solving numerically, the optimal values of production down time t_1^*, production quantity Q* and total cost TC* are obtained and are presented in Table 3.From Table 3, It is observed that the increase in deterioration parameter b from 1.2 to 1.6 has shown an increasing trend in production down time t_1^* from 7.264 to7.616 months, production quantity Q* from 299.569 to 320.422 units and total cost TC* from `267.290 to `267.436. Whereas an increase in demand parameter r from 200 to 300 results as a increase in optimal values of t_1^* from 6.574 to 7.821 months, Q* from 265.933 to 331.535 units and TC* from ` 257.734 to Rs 272.31.

The increase in unit cost c from ` 3 to ` 5 has a decreasing effect on t_1^* and Q* an increasing effect on TC* viz. Production down time t_1^* from 7.454 to 7.078 months, production quantity Q* from 304.117 to 294.610 units, total cost TC*, from ` 242.117 to ` 292.047. The increase in holding cost h from `4 to ` 6 results an increase in optimal values t_1^*, Q* and TC* i.e. production down time t_1^* from 7.078 to 7.39 months production quantity Q* from 294.610 to 302.883 units and total cost TC* from `235.304 to `298.998

The increase in production rate parameter k from 0.4 to 0.8 results an increase in optimal values t_1^* and decrease in production quantity Q* and total cost TC^* i.e. production down time t_1^* from 6.669 to 7.763 months, production quantity Q* from 307.428 to 292.277 units and total cost TC^* from ` 292.999 to 245.862. Whereas the increase in production rate parameter $\eta$ from 50 to 70 results a decrease in production down time $t_1^*$ from 7.821 to 6.787 months, increase in production quantity Q*276.279 to 321.717 units and total cost $TC^*$ from ` 228.314 to ` 303.164

4.4 Sensitivity Analysis

To study the effects of changes in the parameters on the optimal values of production down time and production quantity a sensitivity analysis is performed taking the values of the parameters as b = 1.2, n = 2, r = 250, c = `4/-, h = `5/-, k=0.6, A = 100 and η = 60.Sensitivity analysis is performed by changing the parameter values by -15%, -10%, -5%, 5%, 10% and 15%. First changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optional values of production down time, production quantity and total cost are computed. The results are presented in Table 4. The relationship between parameters, costs and the optimal values are shown in Fig 4.

From Table 4, It is observed that variation in the deterioration parameters b has considerable effect on production down time, optimal production quantity and total cost i.e. production down time t_1^* from 7.079 to 7.432 months, production quantity Q* from 289.375 to 309.27 units and total cost TC* from ` 267.037 to Rs 267.411 Similarly variation in demand parameter r has slight effect on production down time $t_1^*$ from 6.761 to 7.692 months and moderate effect on production quantity Q* from 274.537 to 323.658 units and total cost TC* from ` 260.66 to ` 271.389. The decrease in unit cost c results an increase in production down time $t_1^*$ from 7.152 to 7.378 months, optimal production quantity Q*, from 296.593 to 302.569 units and decrease in total cost TC* ` 282.194 to `252.236 The increase in production rate parameters k and η results slight variation in production quantity Q*from 303.068 to 296.19 units and from 278.659 to 319.563 units and total cost TC* from ` 278.322 to `257.134 and from ` 232.366 to ` 299.702 respectively. The increase in holding cost h has significant effect on optimal values production down time $t_1^*$ from 7.132 to 7.363 months, production quantity Q* from 296.058 to 302.176 units and total cost TC* from ` 243.337 to ` 291.089. When all the parameters change at a time that highly effects on optimal values i.e. production down time $t_1^*$ from 7.252to 7.296 months, production quantity Q* from 259.597 to 338.122 units and total cost TC* from ` 209.856 to ` 330.065 respectively.A comparative study of with and without shortages revealed that allowing shortages has significant influence on optimal production schedule and total cost. This model includes some of the earlier inventory models for deteriorating items with Pareto decay as particular cases for specific values of the parameters When k = 0, this model includes EPQ model for deteriorating items with Pareto decay and constant rate of demand and finite rate of replenishment. When b = 0, this model becomes EPQ model with stock dependent production and time dependent demand. When n=1, this model includes EPQ model with stock dependent production and constant rate of demand.

V/. CONCLUSIONS

This paper introduces an EPQ model with a stock dependent production rate and time dependent demand having Pareto decay. The stock dependent production will avoid wastage in excess inventories and loss due to shortages. The Pareto rate of decay can well characterize the life time of the commodities like cement which exhibit high rate of decay in the early periods and slow rate of decay after certain period. That is the lifetimes are left skewed having long upper time distribution. The optimal production down time, production uptime and production quantity are derived using MathCAD code and Hessian matrix. A case study dealing with perishable item like cement has demonstrated that the stock dependent production rate has significant influence on total production cost.

The sensitivity analysis of the model reveals that the deterioration parameters and demand parameters have tremendous influence on optimal values of the production down time and up time. With historical data on lifetime and demand the production manger can estimate the parameters of the model can produce optimal production quantity. This model can also be extended with inflation and delay in payments which will be taken elsewhere. For specific values of the parameters this model provides spectra of models which are useful for scheduling a variety of production processes.

Table 1 OPTIMAL VALUES OF t_1, t_3, Q and TC for different values of the parameters and costs for the model-without shortages

r	n	k	c	h	b	η	π	A	t_1^*	t_3^*	Q*	TC*
200	**2**	**0.6**	**10**	**8**	**1.2**	**60**	**6**	**100**	**2.206**	**10.850**	**187.45**	**340.363**
150									1.963	11.175	153.154	288.506
175									2.194	11.051	172.732	314.306
225									2.679	10.838	211.355	357.606
250									3.233	10.821	238.668	372.403
	1.5								2.256	10.537	205.49	379.722
	2.5								2.196	11.088	174.033	309.221
	3								1.975	11.186	158.517	286.593
	3.5								1.959	11.322	149.608	267.167
		0.4							2.191	10.889	222.058	341.124
		0.5							2.205	10.875	187.793	339.481
		0.7							2.216	10.846	186.715	340.922
		0.8							2.225	10.819	186.368	342.588
			8						2.805	10.807	215.707	305.714
			9						2.232	10.827	189.495	323.806
			11						2.189	10.876	185.126	356.749
			12						2.189	10.909	183.146	373.066
				6					2.319	11.001	183.417	319.540
				7					2.210	10.891	184.271	331.751
				9					2.070	10.746	187.522	351.266
				10					2.017	10.701	187.781	358.885
					1.1				2.393	10.951	189.24	335.323
					1.3				2.081	10.762	187.343	346.388
					1.4				2.028	10.729	187.22	350.023
					1.5				2.015	10.722	187.172	352.088
						50			2.904	10.814	186.755	307.674
						55			2.495	10.819	186.819	325.138
						65			1.991	10.894	188.331	353.632
						70			1.990	11.022	192.302	362.423
							4		1.998	10.76	182.897	304.398
							5		2.196	10.846	187.241	322.665
							7		2.392	10.923	191.297	353.452
							8		2.940	11.322	202.281	357.851
								90	2.206	10.850	187.45	339.529
								95	2.206	10.850	187.45	339.946
								105	2.206	10.850	187.45	340.779
								110	2.206	10.850	187.45	341.196

Table 2 Sensitivity Analysis of the model with –shortages

Parameters	Optimal values	percentage Change in parameters(Cycle Time = 12 months)						
		-15%	-10%	-5%	0%	+5%	+10%	+15%
r(200)	t_1^*	1.994	2.175	2.190	2.206	2.268	2.366	2.693
	t_3^*	11.020	11.010	10.888	10.850	10.793	10.708	10.706
	Q*	165.323	174.779	183.605	187.450	194.523	204.926	220.423
	TC*	310.368	319.551	331.15	340.363	349.787	360.156	165.973
n(2)	t_1^*	2.324	2.238	2.227	2.206	2.203	2.088	2.012
	t_3^*	10.712	10.769	10.801	10.850	10.912	10.919	10.927
	Q*	199.441	192.809	190.901	187.45	183.961	178.635	174.914
	TC*	361.232	354.074	347.303	340.363	333.22	328.404	323.691
c(4)	t_1^*	2.298	2.232	2.224	2.2060	2.206	2.189	2.180
	t_3^*	10.825	10.827	10.849	10.850	10.891	10.896	10.901
	Q*	193.049	189.995	188.317	187.45	184.99	183.926	183.221
	TC*	314.671	323.806	331.713	340.363	347.974	356.444	364.810
h(5)	t_1^*	2.460	2.380	2.320	2.206	2.114	2.069	2.007
	t_3^*	10.953	10.933	10.914	10.850	10.797	10.763	10.735
	Q*	192.446	190.173	188.681	187.45	186.472	186.456	185.279
	TC*	328.054	331.638	335.29	340.363	345.327	349.393	353.332
k(0.6)	t_1^*	2.190	2.191	2.195	2.206	2.233	2.256	2.300
	t_3^*	10.841	10.844	10.846	10.85	10.861	10.882	10.910
	Q*	188.939	188.237	187.742	187.450	187.445	186.645	186.308
	TC*	340.475	340.434	340.426	340.363	340.026	339.533	338.967
b(1.2)	t_1^*	2.401	2.307	2.239	2.206	2.190	2.177	2.153
	t_3^*	10.948	10.904	10.867	10.850	10.842	10.841	10.840
	Q*	189.420	188.250	187.698	187.450	187.406	187.066	186.210
	TC*	333.633	336.074	338.533	340.363	341.854	343.172	344.692
η(60)	t_1^*	2.289	2.288	2.237	2.206	2.090	2.045	2.034
	t_3^*	10.895	10.877	10.870	10.850	10.788	10.754	10.736
	Q*	162.866	172.326	179.125	187.450	194.416	202.931	212.126
	TC*	323.453	326.076	332.425	340.363	353.76	369.600	387.294
π (5)	t_1^*	3.022	2.864	2.548	2.206	2.066	2.032	2.023
	t_3^*	10.805	10.812	10.825	10.850	10.874	10.911	10.998
	Q*	224.753	217.812	203.897	187.450	179.658	175.874	170.239
	TC*	335.244	336.979	337.024	340.363	346.444	351.932	359.132
A(100)	TC*	339.113	339.529	339.946	340.363	340.779	341.196	341.613
All Parameters	t_1^*	2.239	2.227	2.226	2.206	2.200	2.143	1.996
	t_3^*	10.754	10.785	10.824	10.85	10.857	10.929	10.963
	Q*	165.650	172.884	180.344	187.450	196.126	202.784	208.758
	TC*	259.045	285.187	312.092	340.363	370.408	398.296	435.514

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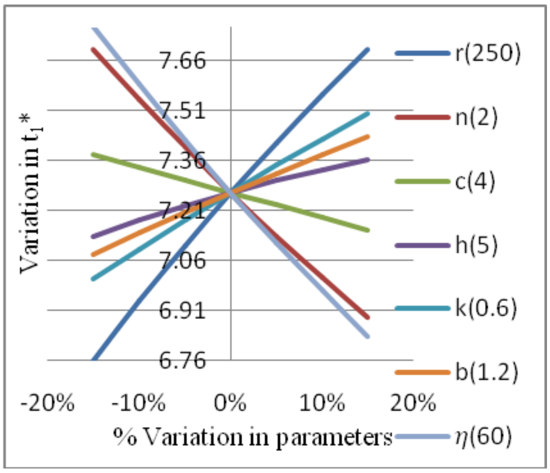
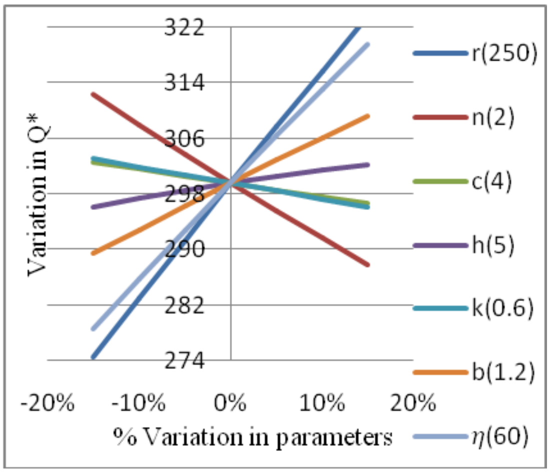
Table 3 OPTIMAL VALUES OF t_1, Q and TC for different values of the parameters and costs for the model-without shortages

Parameters (Cycle Time T = 12 months)								Optimal Values		
r	n	k	c	h	b	η	A	t_1^*	Q^*	TC^*
250	**2**	**0.6**	**4**	**5**	**1.2**	**60**	**100**	**7.264**	**299.569**	**267.29**
200								6.574	265.933	257.734
225								6.939	283.031	263.202
275								7.577	316.261	270.262
300								7.821	331.535	272.31
	0.8							9.512	355.859	312.441
	1.5							8.02	321.473	282.744
	3							6.182	264.373	243.548
	4							5.432	237.941	225.449
		0.4						6.669	307.428	292.999
		0.5						6.978	303.482	279.603
		0.7						7.526	295.822	256.059
		0.8						7.763	292.277	245.862
			3					7.454	304.553	242.117
			3.5					7.359	302.071	254.755
			4.5					7.171	297.100	279.72
			5					7.078	294.610	292.047
				4				7.078	294.610	235.304
				4.5				7.181	297.366	251.343
				5.5				7.333	297.366	283.318
				6				7.39	302.883	298.998
					1.3			7.359	305.012	267.371
					1.4			7.449	310.297	267.419
					1.5			7.535	315.445	267.439
					1.6			7.616	320.422	267.436
						50		7.821	276.279	228.314
						55		7.532	288.09	248.222
						65		7.017	310.791	285.584
						70		6.787	321.717	303.164
							90	7.264	299.569	266.456
							95	7.264	299.569	266.873
							105	7.264	299.569	267.706
							110	7.264	299.569	268.123

Table 4 Sensitivity Analysis of the model-without shortages

Parameters	Optimal Values	Percentage Change in parameters(Cycle Time = 12 months)						
		-15%	-10%	-5%	0%	+5%	+10%	+15%
r(250)	t_1^*	6.761	6.939	7.106	7.264	7.414	7.557	7.692
	Q*	274.537	283.031	291.359	299.569	307.683	315.724	323.658
	TC*	260.66	263.202	265.4	267.29	268.901	270.26	271.389
n(2)	t_1^*	7.694	7.544	7.401	7.264	7.134	7.01	6.891
	Q*	312.34	307.973	303.72	299.569	295.558	291.671	287.89
	TC*	276.164	273.086	270.131	267.29	264.552	261.911	259.36
c(4)	t_1^*	7.378	7.34	7.302	7.264	7.227	7.189	7.152
	Q*	302.569	301.572	300.572	299.569	298.589	297.579	296.593
	TC*	252.236	257.271	262.288	267.29	272.274	277.242	282.194
h(5)	t_1^*	7.132	7.181	7.225	7.264	7.3	7.333	7.363
	Q*	296.058	297.366	298.536	299.569	300.52	301.388	302.176
	TC*	243.337	251.343	259.326	267.29	275.237	283.169	291.089
k(0.6)	t_1^*	7.007	7.095	7.181	7.264	7.346	7.424	7.501
	Q*	303.068	301.897	300.738	299.569	298.446	297.294	296.19
	TC*	278.322	274.546	270.869	267.29	263.808	260.423	257.134
b(1.2)	t_1^*	7.079	7.143	7.205	7.264	7.322	7.378	7.432
	Q*	289.375	292.834	296.243	299.569	302.864	306.098	309.27
	TC*	267.037	267.139	267.222	267.29	267.343	267.383	267.411
η(60)	t_1^*	7.761	7.588	7.422	7.264	7.114	6.97	6.832
	Q*	278.659	285.754	292.711	299.569	306.348	313.008	319.563
	TC*	232.366	244.31	255.946	267.29	278.355	289.155	299.702
A(100)	TC*	266.04	266.456	266.873	267.29	267.706	268.123	268.54
All Parameters	t_1^*	7.252	7.253	7.258	7.264	7.273	7.284	7.296
	Q*	259.597	273.094	286.436	299.569	312.565	325.419	338.122
	TC*	209.886	228.396	247.536	267.29	287.639	308.569	330.065





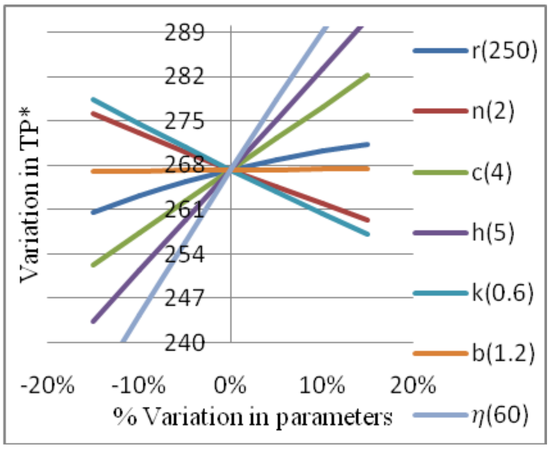


Fig 4; Relationship between optimal values and parameters

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