



CHAOS SYNCHRONIZATION OF TWO IDENTICAL SYSTEMS

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ABSTRACT

This paper studied the chaos synchronization of two identical systems. Lyapunov stability theorem is used for the stability of two identical systems. For the synchronization of two systems nonlinear control method is used. Numerical results show the agreement with analytical results.

Keywords: *Chaos, Lyapunov Stability, Numerical, Simulation, Synchronization*

I. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect [1]. Since chaotic attractors were found by Lorentz in 1963, many chaotic systems have been constructed, such as the Lorentz system, Chen system, and Lü system [13-19]. Now these days, the study of chaotic systems has attracted more and more attention. Since the pioneering work of Ott, Grebogi, and Yorke and the seminal work of Pecora and Carroll, which are simultaneously reported in 1990, Chaos control and synchronization of chaotic systems have been interesting research fields. In many fields, such as secure communication, neural networks, optimization of nonlinear system performance, ecological systems, modeling brain activity, the synchronization such as complete synchronization, phase synchronization, partial synchronization, generalized synchronization, projective synchronization, Chaos synchronization problem was first described by Fujisaka and Yemada [2] in 1983. This problem did not receive great attention until Pecora and Carroll [3-4] published their results on chaos synchronization in early 1990s. From then on, chaos synchronization has been extensively and intensively studied in the last three decades [3-11]. Chaos theory has been explored in a variety of fields including physical systems [5], chemical systems [6] and ecological systems [7], secure communications [8-10] etc. Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem. In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. In this paper we studied the Chaos synchronization of two identical systems.



II. CHAOS SYNCHRONIZATION OF TWO IDENTICAL SYSTEMS

Consider the dynamical system

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= (Y - \alpha)x_1 - x_1x_3 + Yx_2 \\ \dot{x}_3 &= -\beta x_3 - \delta x_2 + x_1x_2\end{aligned}\quad (1.1)$$

The system (1.1) is considered as master system that is an input system

Now consider the slave as an output system

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (Y - \alpha)y_1 - y_1y_3 + Yy_2 + u_2 \\ \dot{y}_3 &= -\beta y_3 - \delta y_2 + y_1y_2 + u_3\end{aligned}\quad (1.2)$$

The system (1.2) is the slave system where u_1, u_2, u_3 are the controllers

The error dynamics of the given system is

$$\begin{aligned}e_i &= y_i - x_i \\ e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3\end{aligned}\quad (1.3)$$

And the system

$$\begin{aligned}\dot{e}_1 &= \dot{y}_1 - \dot{x}_1 \\ \dot{e}_2 &= \dot{y}_2 - \dot{x}_2 \\ \dot{e}_3 &= \dot{y}_3 - \dot{x}_3\end{aligned}\quad (1.4)$$

Thus

$$\begin{aligned}\dot{e}_1 &= -\alpha(x_2 - x_1) + \alpha(y_2 - y_1) \\ \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= (Y - \alpha)e_1 + x_1x_3 + Ye_2 - y_1y_3 + Yy_2 + u_2 \\ \dot{e}_3 &= -\beta e_3 + y_1y_2 - \delta e_2 - x_1x_3 + u_3\end{aligned}\quad (1.5)$$

Now choose the controller as

$$\begin{aligned}u_1 &= -\alpha e_2 \\ u_2 &= -(Y - \alpha)e_1 - x_1x_3 + y_1y_3 + Yy_2 + u_2 \\ u_3 &= -y_1y_2 + \delta e_2 + x_1x_3 + u_3\end{aligned}\quad (1.6)$$

After putting the values of (1.6) in (1.5)

The error system becomes

$$\begin{aligned}\dot{e}_1 &= -\alpha e_2 \\ \dot{e}_2 &= Ye_2 \\ \dot{e}_3 &= -\beta e_3\end{aligned}\quad (1.7)$$

Consider the Lyapunov function

$v(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2)$ which is a positive definite function of R^3

After differentiating the function (1.7)

$$v'(e) = -\alpha e_1^2 - \gamma e_2^2 - \beta e_3^2 < 0$$

Which is a negative definite function, thus the error system becomes globally asymptotically stable function.

III. NUMERICAL SIMULATION

By using the MATLAB two equations (1.1) and (1.2) are numerically solved by selecting the values of $(\alpha, \beta, \gamma, \delta)$ as (10,10,20,10) and the values of initials conditions are $y(0) = (10,10,5)$ and $x(0) = (1,1,0)$. Fig 2 shows the error dynamics with time t , shows that error system converges to zero and the two system are synchronized. Fig 1 (a) to 1(c) shows the time series of signals between x_i and y_i where $i = 1, 2, 3$ Fig 3(a) to 3(b) shows the chaotic behaviour of the system (1.5) and (1.6) for the values of $(\alpha, \beta, \gamma, \delta)$ as (10,10,20,10)

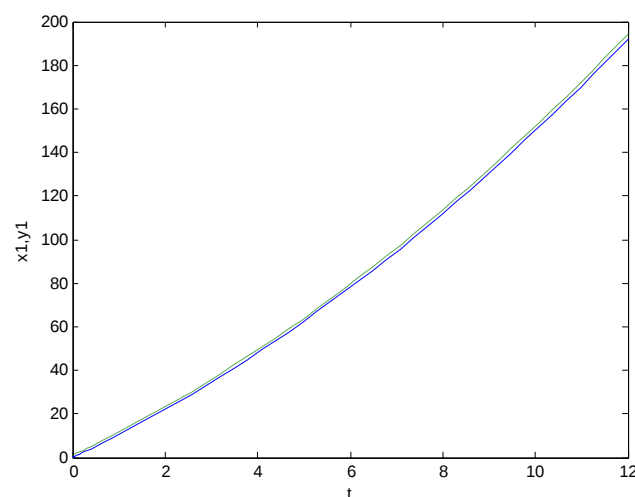


Fig 1(a). Synchronization of x_1, y_1 with Time t

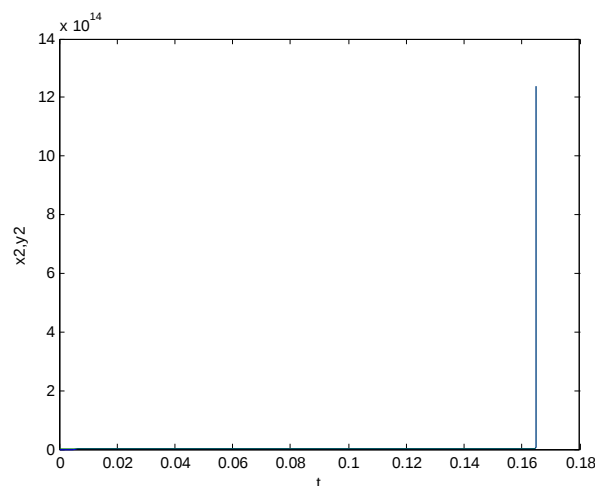


Fig 1(b). Synchronization of x_2, y_2 with Time t

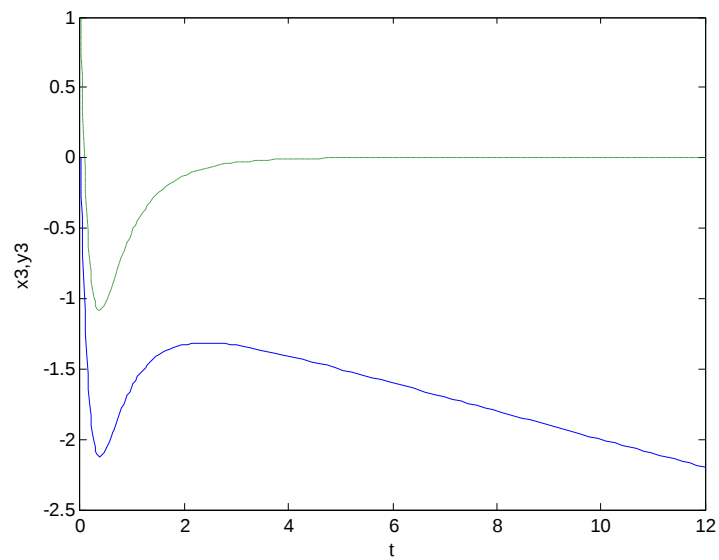


Fig 1(c). Synchronization of x_3, y_3 with time t

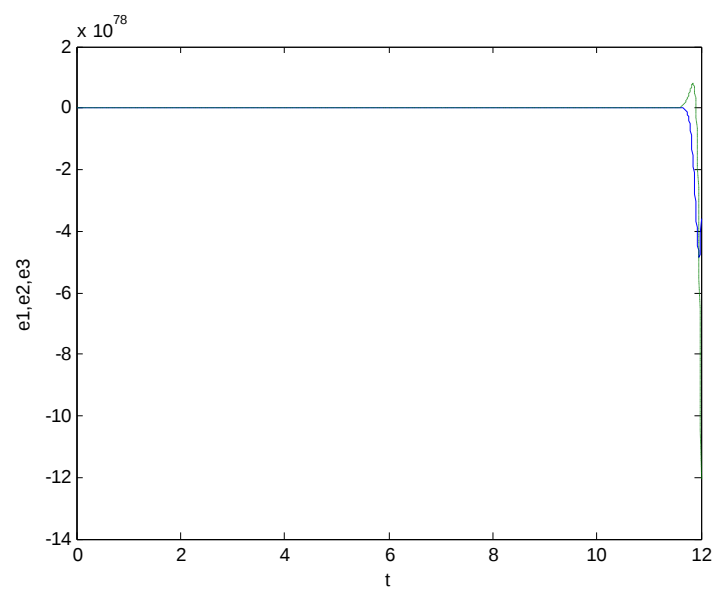


Fig 2. Synchronization of e_1, e_2, e_3 with time t

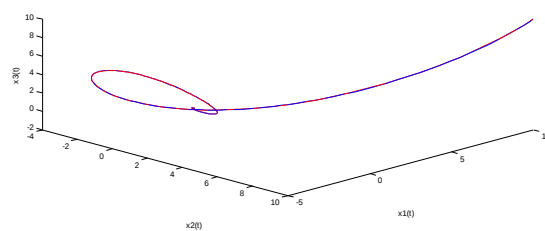


Fig 3(a). Chaotic attractor of master system between x_1, x_2, x_3

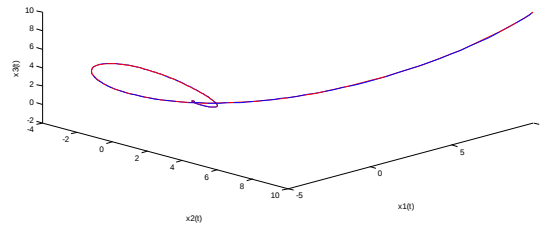


Fig 3(b). Chaotic Attractor of Master System Between y_1, y_2, y_3

IV. CONCLUSION

Nonlinear control method and Lyapunov stability theory is used in this paper to achieve global chaos synchronization for the two identical chaotic systems. The nonlinear control method is very effective and convenient to achieve global chaos synchronization for the cases of chaotic systems studied in this paper. Numerical simulations have been shown to illustrate the effectiveness of the synchronization schemes derived in this paper.

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