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BOUNDARY LAYER FLOW OF A VISCOELASTIC LIQUID NEAR A STAGNATION POINT WITH *SLIP*

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ABSTRACT

In the present investigation, we have studied the flow of Walter's liquid B near a stagnation point with slip using boundary layer approximation. We obtained the approximate solution of the equation of motion by extermal point collocation method. We have investigated the behavior of viscoelastic liquid which impinges on a rigid wall with slip. The effect slip condition and effect of viscoelasticity of the liquid is studied. Velocity profiles with and without slip are shown in different graphs.

Keywords: Viscoelastic Liquid, Boundary Layer, Stagnation Point, Slip Condition, Extermal Point Collocation Method.

I. INTRODUCTION

A stagnation point occurs when a liquid stream impinges on a wall at right angles to it and flows away radially in all directions. The behavior of the flow of liquids near a stagnation point is a fundamental topic in fluid dynamics and has attracted the attention of many researchers in last few decades due to its wide industrial and technical applications. This principle is applied in heat exchangers placed in a low velocity environment, cooling of nuclear reactors during emergency shutdown, solar central receivers exposed to wind currents, cooling of electronic devices by fans and many hydrodynamic processes. Authors like Howarath (1935) and Froessling (1940) have obtained solutions for the flow of a viscous liquid near a stagnation point. Srivastava (1958) has obtained an approximate solution for an axially symmetric flow near a stagnation point of a non-Newtonian Reiner Rivlin fluid with constant coefficients of viscosity and cross viscosity adopting the Kerman Pohlhausen method used for the study of boundary layer equations in Newtonian fluids. Stuart (1959) studied the viscous flow near a stagnation point when the external flow has uniform verticity. Also Tamada (1979) discussed the two dimensional stagnation point flow impinging obliquely on a plane wall. Rajeswari et.al. (1962) used the same method and obtained approximate solutions for the two dimensional and axially symmetric flows near a stagnation point of Rivlin Ericksen viscoelastic liquid. Sharma (1959) has discussed the problem of axially symmetric flow of a Maxwell liquid near a stagnation point. Jain and Balram (1961), Jain (1961), Kapur and Gupta (1963) and Mishra et. al. (1972) have studied the stagnation point flow of different non-Newtonian fluids with and without suction at the impinging wall. Krechetmikov et. al (2002) studied the problem of two dimensional and three dimensional boundary layer flow for Newtonian and non-Newtonian fluids and obtained set of similar solutions in the case of steadily moving jets. Three dimensional boundary layer flow was studied by Kiril (2002) for the existence and smoothness of the Navier-Stokes equations. Rothorayer et. al. (2002) discussed the boundary layer flow through the evolution of interfacial waves on thin liquid films which are driven by a boundary layer and obtained the boundary layer thickness. Prandtl-Bathcelor free boundary layer problem was investigated by Acker (2002) who obtained an analytical solution on a convex cone of constant verticity. Three dimensional boundary layer flows was studied by Naceur et.al.(2002) for incompressible fluid through a numerical scheme. Earlier in 1964, Beard and Walter have a detailed investigation on viscoelastic boundary layer two dimensional flow near a stagnation point. They have shown that the effect of elasticity is to increase the velocity in the boundary layer and also to increase the stress on the solid boundary. Slip velocity and slip layer thickness in the flow of concentrated suspension was discussed by Soltani et.al. (1998). Derek et.al. (2002) studied the apparent fluid slip at hydrophobic micro channel wall and got interesting results. The problem of stagnation point flow with slip appears in some applications. Wang (2003) studied the stagnation point flow of viscoelastic fluids with slip. Labropulu et. al. (2004) studied the steady two dimensional stagnation point flow of non-Newtonian Walter's B liquid with slip. Bhatacharya et.al. (2011) analyzed the effect of partial slip on steady boundary layer stagnation point flow of an incompressible fluid and heat transfer towards a shrinking sheet. Rosali at.al. (2014) analyzed the effect of unsteadiness on mixed convection boundary layer stagnation point flow over a vertical flat surface embedded in a porous medium. Mabood et. al. (2015) worked on the heat and mass transfer of MHD stagnation point flow towards a permeable stretching surface. Madhu et.al. (2015) made an exhaustive study of MHD mixed convection stagnation point flow of a power law non-Newtonian Nano fluid towards a stretching surface with radiation and heat source/ sink.

Our aim in the present paper is to study the flow of Walter's liquid B near a stagnation point with slip using boundary layer approximation by extremal point collocation method. This problem appears in some applications where a thin film of oil is attached to the plate or when the wall is coated with special coating such as a thick monolayer of a type of lubricant. Flows in the slip flow region is taken as

$$u_t = A_p \frac{\partial u_t}{\partial n} \qquad [1]$$

Where u_t is the tangential velocity component, n is normal to the plate and A_p is the slip coefficient. In the present analysis, we investigate the behavior of viscoelastic liquid which impinges on a rigid wall with slip. The liquid impinges on the wall orthogonally. The effect of slip condition and effect of viscoelasticity of the liquid is studied. In a way the present paper is an extension of the work done by Wang (2003).

II. BASIC EQUATIONS

The constitutive equations for flow of the viscoelastic liquid model considered here are given by

$$p'^{ij} = 2\eta_0 e^{ij} - 2k_0 e^{-ij} \qquad [2]$$

Where p^{ij} is the stress tensor, η_0 is the coefficient of viscosity, k_0 is the elastic parameter of the liquid and e^{ij} is the rate of strain tensor.

The equation of motion and continuity are given by

$$\rho\left(\frac{\partial v^i}{\partial t}+v^j v^i,j\right)=-p_{,j}+p^{ij}{}_{,j} \qquad [4]$$

And $v^{\alpha}{}_{,\alpha}=0$ [5]

Where ρ s the density of the medium and p is an arbitrary isotropic pressure.

III. BOUNDARY LAYER EQUATIONS

We consider the steady state two dimensional flow parallel to y-axis at infinite impinging on a flat wall placed along y=0. The flow divides it into two streams on the wall and proceeds in two opposite directions. The velocity components in the x and y directions are considered as

$u=u(x,y)$, $v=v(x,y)$, $w=0$ [6] so as to satisfy the equation of continuity.

The equations of motion in a steady flow with the stress strain rate relation given by [2] are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$-k_0^*[u\frac{\partial^3 u}{\partial x^3}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y\partial x^2}+v\frac{\partial^3 u}{\partial y^3} \qquad [7]$$
$$-3\frac{\partial u}{\partial x}.\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial x}.\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-2\left\{\frac{\partial v}{\partial y}.\frac{\partial^2 u}{\partial y^2}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 u}{\partial x\partial y}\right\}]$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$-k_0^*[u\frac{\partial^3 v}{\partial x^3}+u\frac{\partial^3 v}{\partial x\partial y^2}+v\frac{\partial^3 v}{\partial y\partial x^2}+v\frac{\partial^3 v}{\partial y^3} \qquad [8]$$
$$-3\frac{\partial v}{\partial y}.\frac{\partial^2 v}{\partial y^2}-\frac{\partial v}{\partial y}.\frac{\partial^2 v}{\partial x^2}-\frac{\partial v}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-2\left\{\frac{\partial u}{\partial x}.\frac{\partial^2 v}{\partial x^2}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 v}{\partial x\partial y}\right\}]$$

Where

$$\nu=\frac{\eta_0}{\rho},k_0^*=\frac{k_0}{\rho}$$

The equation of continuity reduces to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad [9]$$

Now within the boundary layer u, $\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial p}{\partial x}$ are assumed to be $o(1)$ and y to be in the order δ, where δ is the thickness of the boundary layer near a solid boundary at $y=0$. From the equation of continuity [9], we have

$$v = o(\delta)$$

In order that viscous, viscoelastic and inertial terms in the equation of motion shall be the same order of magnitude, it is necessary that

$$v = o(\delta^2) \text{ and } k_0^* = o(\delta^2)$$

Also the change of pressure across the boundary layer is $o(\delta^2)$ and the pressure gradient term in [6] can therefore be obtained from the flow just outside the boundary layer. In this region, under the steady flow condition, we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = U\frac{\partial U}{\partial x} \quad [10]$$

Where U is the mainstream velocity.

Thus the reduced boundary equation is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{du}{dx} + v\frac{\partial^2 u}{\partial y^2} - k_0^*\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x \partial y}\right)$$

[11]

From the equation of continuity [9], it is possible to define a stream function ψ as follows.

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad [12]$$

The approximate form of ψ for the present problem is

$$\psi = \sqrt{vU_1}\, x f(\eta) \quad [13]$$

Where $\eta = \sqrt{(U_1/v)}\, y$ [14]

From the above we see that

$$u = U_1 x f'\left(\sqrt{(U_1/v)}\, y\right) \quad [15]$$

The boundary conditions to be satisfied on the velocity component are

$$\left.\begin{array}{l} y = 0 : u = 0, \dfrac{\partial \psi}{\partial x} = 0 \\ y \to \infty : u = U, \psi(x, y) \to y \end{array}\right\} \quad [16]$$

The slip condition in equation-[1] is

$$\frac{\partial \psi}{\partial y} = \gamma \frac{\partial^2 \psi}{\partial y^2} \text{ Where } \gamma = A_p\sqrt{\beta \upsilon}$$

Here β has the unit of inverse time.

A search for solution of [11] in which the velocity profiles are similar in different sections reveals that the only possible form of U under these conditions is

$$U = U_1 x \quad [17]$$

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Where U_1 is a constant.

Substituting ψ from [12] and U from [17], equation-[11] reduces to

$$\left(f''' + ff'' + 1 - f'^2\right) + k_0^* \frac{U_1}{\nu}\left(ff^{iv} - 2f'f''' + f''^2\right) = 0$$

Or $\left(f''' + ff'' + 1 - f'^2\right) + k\left(ff^{iv} - 2f'f''' + f''^2\right) = 0$ [18]

Where the non dimensional k is positive and given by

$$k = k_0^* \frac{U_1}{\nu}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &f(0) = 0,\ f'(0) = \gamma f''(0) \\ &f'(\infty) = 0 \end{aligned}\right\} \qquad [19]$$

IV. SOLUTION OF THE PROBLEM

In this section we shall obtain an approximate solution for the equation [18] by using extremal point collocation method. We use for $f(\eta)$ an approximate function $g(\eta)$ satisfying the boundary condition [19]. Thus the approximate function $g(\eta)$ can be written as

$$f(\eta) \approx g(\eta) = \left(-1 + \eta + e^{-\eta}\right) + a_1\left(1 - 2e^{-\eta} + e^{-2\eta}\right) + a_2\left(2 - 3e^{-\eta} + e^{-3\eta}\right) \qquad [20]$$

The boundary conditions are

$$\left.\begin{aligned} &g(0) = 0, g'(0) = \gamma g''(0) \\ &g'(\infty) = 1 \end{aligned}\right\}$$

Substituting [20] in equation-[18], we get

$$\in\left(\eta, a_p\right) = g''' + gg'' + 1 - g'^2 + k\left(gg^{iv} - 2g'g''' + g'^2\right) \qquad [21]$$

Where $\in\left(\eta, a_p\right)$ represents the defect in the differential equations and is a function of two arbitrary parameters a_1 and a_2.

Let us take the approximate extremal points at $\eta_0 = 0.0, \eta_1 = 0.5, \eta_2 = 1.3$. After obtaining values of $\in\left(0, a_p\right), \in\left(0.5, a_p\right) \& \in\left(1.3, a_p\right)$, we form a set of equations for different values of k from

$$\left.\begin{aligned} &\in\left(0, a_p\right) + \in\left(0.5, a_p\right) = 0 \\ &\in\left(0.5, a_p\right) + \in\left(1.3, a_p\right) = 0 \end{aligned}\right\} \qquad [22]$$

Hence we get a set of values of a_1 and a_2 corresponding to the above set of equations and then we substitute these values in the defect function [21]. Then we equate the differential coefficient with respect to η to zero, i.e.

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$$\frac{d}{d\eta}\in(\eta)=0 \quad [23]$$

Solving equation-[23], we get the corrected values of η_0, η_1 and η_2. After putting the corrected values of η and then adopting the same method as we adopted in obtaining equations involving a_1 and a_2. Then solving these equations, we get the corrected values of a_1 and a_2.

k	a_1	a_2
0	0.5246647	-0.1365591
0.05	0.6079025	-0.1542050
0.1	0.7045973	-0.1743611
0.15	0.8205665	-0.1981024
0.20	0.9626898	-0.2263059

Putting these values of a_1 and a_2 in equation-[20], we get the solutions for equation-[20].

It is also interesting to determine the effect of elasticity of the stress on the solid boundary. We know

$$P_{xy}=\eta_0\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)-k_0\left(u\frac{\partial^2 v}{\partial x^2}+u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 v}{\partial x\partial y}-2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}-2\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right) \quad [24]$$

Within the boundary layer, retaining only the terms of highest order, we have

$$P_{xy}=\eta_0\left(\frac{\partial u}{\partial y}\right)-k_0\left(u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right) \quad [25]$$

On the solid boundary $y=0$, we have

$$\left.P_{xy}\right]_{y=0}=\rho v^{1/2}U_1^{3/2}\times(1.2325+1.1372k) \quad [26]$$

The stress on the solid boundary is therefore increased by the presence of elasticity of the liquid.

If ψ is the stream function, then

$$u=\frac{\partial\psi}{\partial y}=U_1 x f'(\eta) \quad [27]$$

$$v=-\frac{\partial\psi}{\partial x}=-\sqrt{vU_1}\,f(\eta) \quad [28]$$

So we take

$$\psi(x,y)=\sqrt{vU_1}\,xf(\eta) \quad [29]$$

The stream function $\overline{\psi}$ is given by

$$\overline{\psi}=\frac{\psi}{\sqrt{vU_1}}=xf(\eta) \quad [30]$$

V. DISCUSSION OF RESULTS

Figure-1 shows the effect of elasticity of the liquid on the velocity component. An examination of this figure shows that the ratio of velocity in the boundary layer to the mainstream velocity u/U gradually increases as η goes on increasing. At about $\eta = 2.6$, the value of u/U approaches to unity. The presence of elasticity in the liquid increases the velocity in the boundary layer.

Details of the function f are given in table-1. The values of $f, f' \,\&\, f''$ are in the agreement with those given by Howarth. The stream function is given by $\overline{\psi} = xf(\eta)$. Figure-2 furnishes the flow pattern near a stagnation point for $k_0^* = 0 \,\&\, k_0^* = 0.05$. We find the effect of k_0^* is to flatten the velocity profile. Figure-3 shows the profiles of f' for $\gamma = 0$ and for various values of k_0^* and figure-4 shows the profiles of f' for $\gamma = 1$ and for different values of k_0^*. Here we see that as the elasticity of the fluid increases, the velocity near the wall increases and as the slip parameter increases, the velocity near the wall increases as well.

Table-1

η	f	f'	f''	f'''	k_0^*
0	0.0000	0.0000	1.2325	-1.0000	0
	0.0000	0.0000	1.2895	-1.0759	0.05
0.2	0.0234	0.2266	1.0346	-0.9728	0
	0.0243	0.2346	1.0764	-1.0462	0.05
0.4	0.0883	0.4146	0.8463	-0.9028	0
	0.0918	0.4313	0.8741	-0.9696	0.05
0.6	0.1868	0.5665	0.6753	-0.8050	0
	0.1942	0.5874	0.6905	-0.8628	0.05
0.8	0.3126	0.6860	0.5251	-0.6936	0
	0.3244	0.7090	0.5305	-0.7401	0.05
1.00	0.4593	0.7780	0.3982	-0.5778	0
	0.4760	0.7912	0.3938	-0.6128	0.05
1.20	0.6223	0.8468	0.2939	-0.4637	0
	0.6553	0.8673	0.2815	-0.4953	0.05
1.40	0.7968	0.8968	0.2112	-0.3638	0
	0.8221	0.9165	0.1985	-0.3774	0.05

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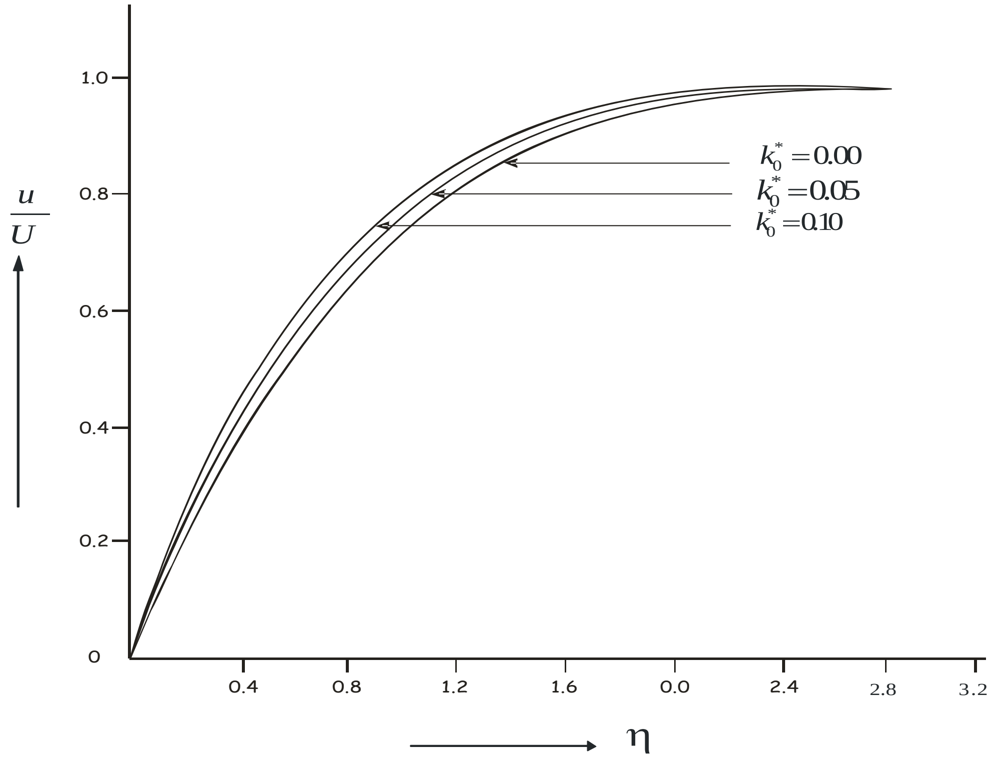

Figure-1: Effects of k_0^* On velocity profile.

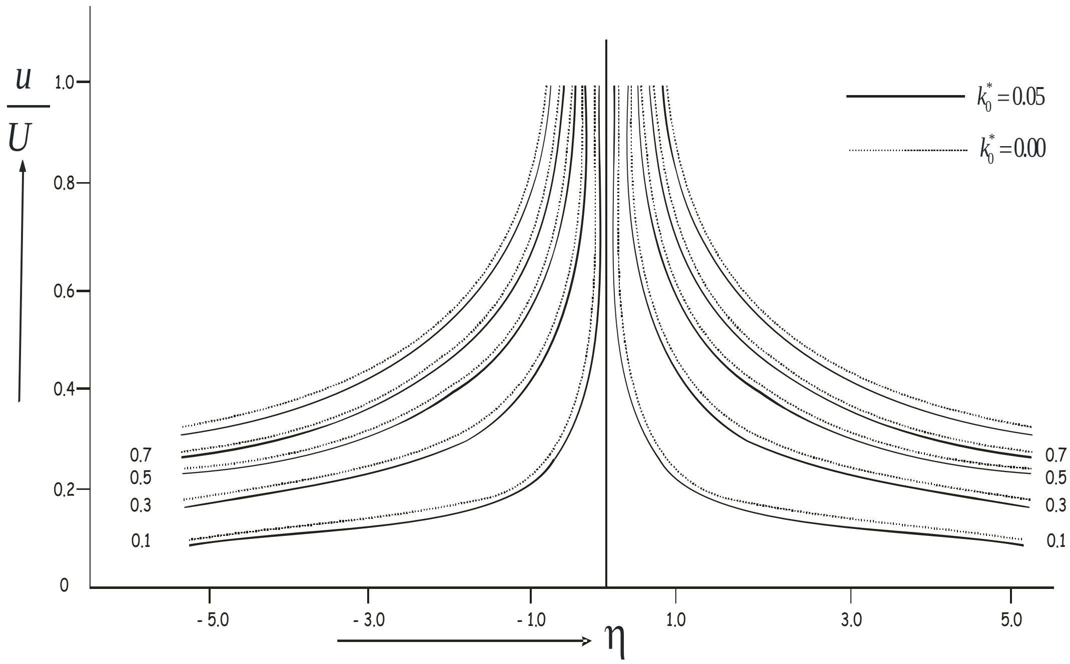

Figure-2: The Effect of k_0^*, On velocity profile.

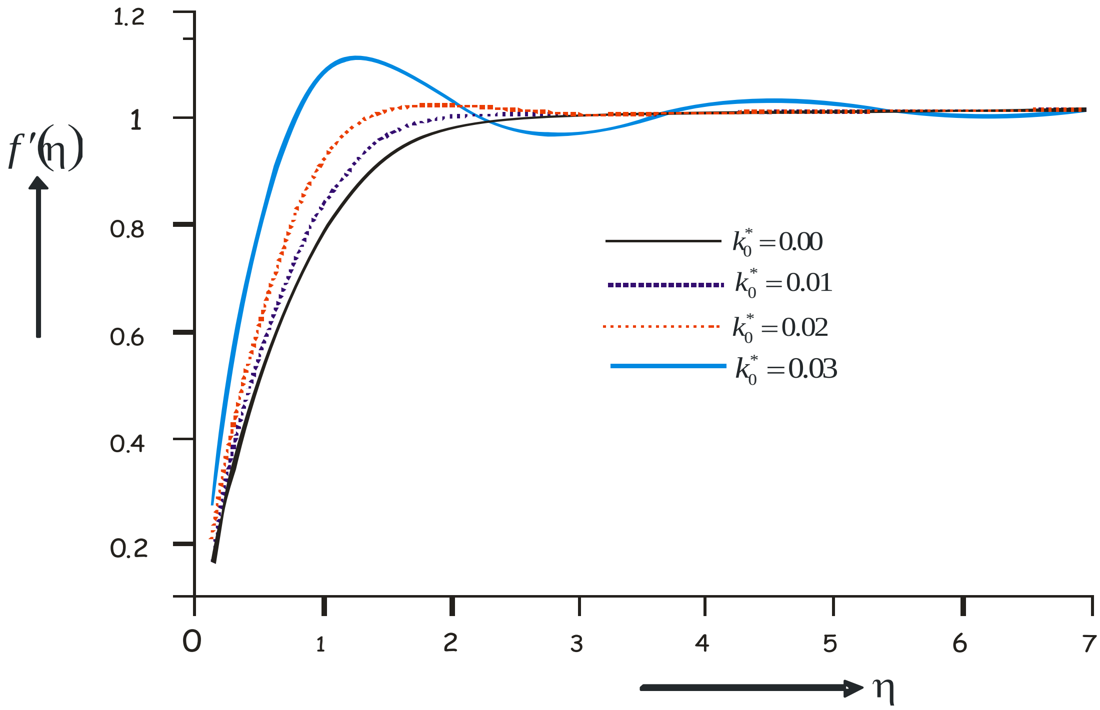

Figure-3: Variation of $f'(\eta)$ for $\gamma = 0$ and various values of k_0^*.

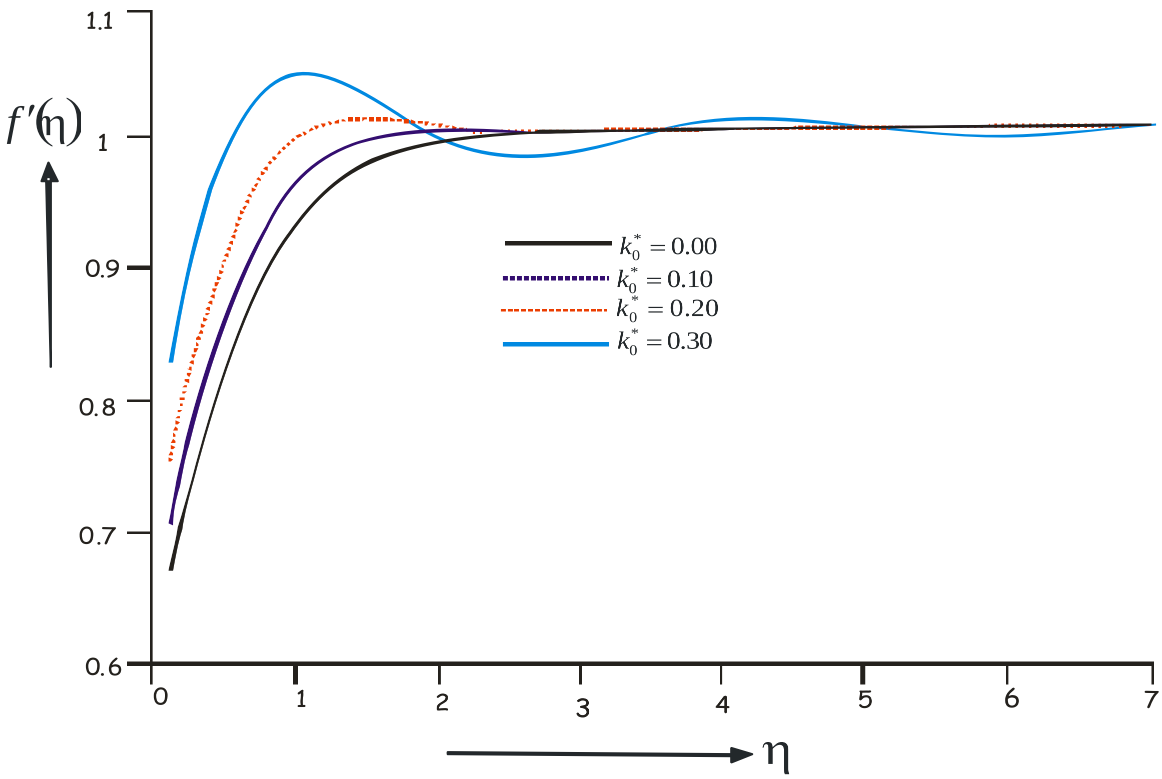

Figure-4: Variation of $f'(\eta)$ for $\gamma = 1$ and various values of k_0^*.

REFERENCES

[1]. Howarth, L. (1935) “On the calculation of steady flow in the boundary layer near the surface of a cylinder in a stream.”Arc. Rep. and Memo.-1932 .

[2]. Freossling, N. (1940)”Verdustung warmeubertragung und Geschwindigkeits vertilung bei zweidimensionalar und rotationsymmetrischer blaminarer Grenzchichtstromung ” Lnds, Univ. Arsskr. F.F, Adv.2, 35, No.4.

[3]. Srivastava, A.C. (1958), “The flow of Newtonian and non-Newtonian fluidsnear a stagnation point.”ZAMP, 9, 80.

[4]. Stuart (1959) “The viscous flow near a stagnation point when the external flow has uniform verticity”J. Aero. Sci. 26 pp. 124-125.

[5]. Sharma, S.K. (1959),” Viscoelastic steady flow” ZAMP, 39,313.

[6]. Jain, M.K., Balram, M. (1961), J. Sci. Eng. Res. 5, 259.

[7]. Jain, M.K. (1961),” The Motion of an Elastico- Viscous Liquid Contained Between Coaxial Cylinders” Appl. Phys. Quart, 7, 100.

[8]. Rajeswari, G.K., Ratna, “Flow of a particular class of non-Newtonian viscoelastic and visco-inelastic fluids near a stagnation point”S.L. (1962) ZAMM. 39, 313.

[9]. Walter, K. (1962) Journal de Mecanique. 1, 479.

[10]. Kapur, J.N., Gupta, R.C. “Power law fluid flow in the inlet length of a circular pipe”(1963) Arch. Mech. Stoso 15, 711.

[11]. Beard, D.W., Walter, K. (1964,)”Two dimensional flow near a stagnation point” Proc. Camb.Phil.Soc. , 60, pp. 667-674.

[12]. Mishra, S.P, Dash, G.C. (1972) Bull. Cal. Math. 64 pp 159-164.

[13]. Tamada, K.J. (1979) “Two dimensional stagnation point flow impinging obliquely on a plane wall” J. Phys. Soc. Japan 46 pp. 310-311.

[14]. Soltani, F., Yilmazer, V. (1998). “Slip velocity and slip layer thickness in the flow of concentrated suspension”, J. Appl. Polym. Sci. 70, No.3 pp. 515-522.

[15]. Krechetmikov, R., Lipatov, I. “Hidden Invariance in Problems of Two dimensional and Three dimensional Wall Jets for Newtonian and non-Newtonian Fluids.”(2002) SIAM. J. Appl. Math. 62, No.6 pp. 1837-1855.

[16]. Kiril, Z.S. (2002) Int. J. Diff. Equations Appl. 5 No.3 pp. 215-230.

[17]. Rothorayar, A.P., Matheis, B.D. (2002) J. Engrg. Math. 42 No.3-4 pp. 341-357.

[18]. Acker, A. (2002)” On the existence of convex classical solutions to a generalized Prandtl-Batchelor free boundary problem” Z. Angew. Math. Phy. 53, No.3, pp. 438-485.

[19]. Naceur, H.J. Buisne, D. (2002) Int. J. 38, No.8 PP. 711-727.

[20]. Wang, C.Y. (2003) “Stagnation point flow of viscoelastic fluids with slip: exact solution of Navier-Stokes Equations”. Z. Angew. Math. Phys. 54, No.1 pp-184-189.

[21]. Labropulu, F., Husain, I. and Chinichian, M. (2004) “Stagnation point flow of non-Newtonian Walter’s B liquid with slip”, IJMMS, 61, PP. 3249-3258.

[22]. Bhatacharya, K., Mukhopadhyaya, S. Layek, G.C. (2011) “Slip effect on boundary layer stagnation point flow and heat transfer towards a shrinking sheet”. Int. J. Heat and Mass Transfer 54 pp. 308-313.

[23]. Rosali, h., Nazar, A. R.(2014) “The effect of unsteadiness on mixed convection boundary layer stagnation point flow over a vertical flat surface embedded in a porous medium”. Int. J. Heat and Mass Transfer. 77, pp. 147-156.

[24]. Mabood,F.K., Ismil, W.A., Mohamad, A.I. (2015) “MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration”, J. Chem. Eng. 273 pp. 430-437.

[25]. Madhu , M. Naikoti, K. (2015) “Magneto hydrodynamics mixed convection stagnation point flow of a power law non-Newtonian Nano fluid towards a stretching surface with radiation and heat source/ sink.” Journal of Fluids vol.2015.