International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 09, September 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

BUCKLING ANALYSIS OF A CRACKED STEPPED COLUMN BY USING FINITE ELEMENT METHOD

SK. Bifathima

Dept. Of Civil, NOVA College of Engg. Vijayawada, (India)

ABSTRACT

Components with varying cross-sections are most common in buildings and bridges as well as in machine parts. The stability of such structural members subjected to compressive forces is a topic of considerable scientific and practical interest. Tapered and stepped columns are very much useful in structural engineering because of their reduced weight compared to uniform columns for the same axial load carrying capacity or buckling load. In the current study free vibration and buckling analysis of a cracked two stepped cantilever column is analyzed by finite element method for various compressive loads. Simple beam element with two degrees of freedom is considered for the analysis. Stiffness matrix of the intact beam element is found as per standard procedures. Stiffness matrix for cracked beam element is found from the total flexibility matrix of the cracked beam element by inverse method in line with crack mechanics and published papers by researchers. Eigen value problem is solved for free vibration analysis of the stepped column under different compressive load. Variation of free vibration frequencies for different crack depths and crack locations is studied for successive increase in compressive load. Buckling load of the column is estimated from the vibration analysis.

Keywords: Stepped Coloumn, Cracked Beam, Buckling Analysis

I. INTRODUCTION

Components with varying cross-sections are most common in buildings and bridges as well as in machine parts. The stability of such structural members subjected to compressive forces is a topic of considerable scientific and practical interest that has been studied extensively, and is still receiving attention in the literature because of its relevance to structural, aeronautical and mechanical engineering. Tapered and stepped columns are very much useful practically in structural engineering because of their reduced weight compared to uniform columns for the same axial load carrying capacity or buckling load.

International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 09, September 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

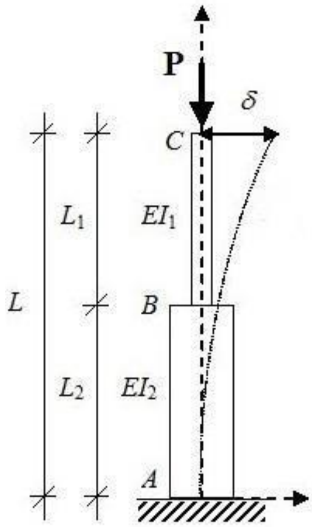

Figure1.: Stepped Column with Fixed-Free Conditions

Stepped columns are also frequently used in multistory structures where columns have to support intermediate floor loads. To attain reduction in weight and decrease costs of steel carrying structures, the engineers tend to design steel columns as multi-stepped carriers with a non-uniform cross-section. Since columns are usually compressed by applied, self-weight, etc., one of the most important aspects of using such carriers is their elastic stability.

II. CRACKS IN COLUMNS

Columns are important structural members and their stability under different cases of loading is studied by many researchers to obtain critical buckling loads and critical stresses. The cracks may develop from impact, applied cyclic load, mechanical vibrations, aero-dynamic loads etc. Due to the effect of fault or weakness that occurred due to crack in a cracked section, the stability of column may be decreased. The critical buckling loads of cracked columns are affected by effect of depths, locations, and number of cracks. Cracked section is modeled as massless rotational spring.Since axial load and stiffness are not constant along the length of the column the analysis of a stepped column is usually much more complicated than uniform column. The change in the cross-sectional areas and distribution of loads generates discontinuity in deriving the deflection equation of a stepped beam.Connection between foundation of a structure and super structure is most vulnerable and damage locations during and after earthquakes. So, a stepped column is used to palliate or retrofit such disadvantage. Stepped column is used to substitute rigid connections between foundation and upper structure.The study of cracked structures and members are topic of study for decades and many researches are still going on the topic. A fault in the structure causes serious damage to the structure if left unchecked. When a structure is damaged due to crack, modal parameters assigned with the structure are greatly affected because due to damage to a structure the stiffness of the structure is decreased. Many researches are currently are concerned on damage location and damage size. The main study is concerned on extent of damage and location of damage. Many damage detecting methods use sensitivity method which uses natural frequencies. These frequency based method require a lot of computations especially for large and complex structures. Frequency changes alone are not sufficient to damage position. Similar frequency changes may occur for different damage positions. Vibration mode shapes can be heavily influenced by local damage. The greatest change occurs around the defect, thus offering the possibility of locating the damage. The crack section is modeled as modified beam

IJARSE
ISSN 2319 - 8354

element due to presence of crack. In several studies importance is given to detection of crack that is affected by the change in natural frequencies and mode shapes of the beam.

There are three modes of fracture failure, they are:

Mode I: opening mode, crack faces are separated in direction normal to the plane of the crack. Normal load applied on crack plane causes open crack.

Mode II: this is due to in plane shearing load, in this two faces of crack slides against each other or shearing mode. Stresses are developed parallel to crack direction.

Mode III: it is out of pale shear or tearing mode

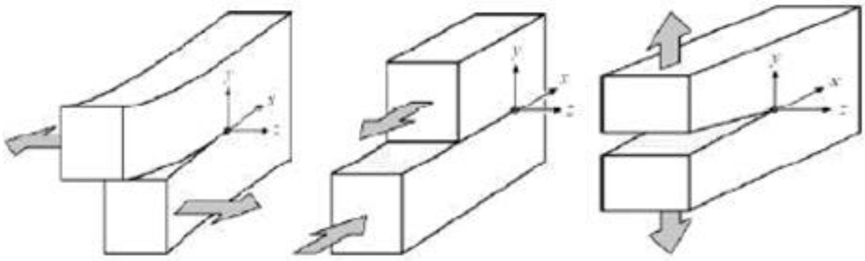

Figure2: Three Basic Modes of Fracture.

In present study various end conditions are considered such as fixed-free, hinged-hinged, fixed-hinged, and fixed-fixed. The analysis depends up on the deflection function which satisfies the end conditions. Lower bound solutions for buckling problems have been calculated by a method of successive approach done numerically by new mark method or by finite element methods. Differential equations for equilibrium of a column with small lateral displacement can be easily written in finite element equation form. Alternate methods using wavelet analysis have been done to detect damage in structures for which the effect of small cracks may not be affected much by Eigen frequencies of the structures. Mode shapes are more sensitive to local damages when compared to natural frequencies.

III. FINITE ELEMENT ANALYSIS

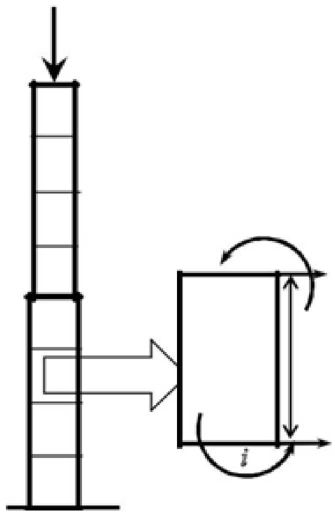

Figure 3.: Intact Stepped Column discretized into beam elements with 2 dof per node for finite element analysis

The displacement model is taken as a polynomial given by,

$q = a_1 + a_2 x + a_3 x^2 + a_4 x^3$

Shape functions as derived by Cook *et al*. (2003) are given by,

$$N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}$$

$$N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}$$

$$N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}$$

$$N_4 = -\frac{x^2}{l} + \frac{x^3}{l^2}$$

Where

$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$

$[N]$ = Shape function matrix

Similarly, the strain displacement matrix coefficients in line with Cook *et al*. (2003) is given by

$$B_1 = -\frac{6}{l^2} + \frac{12x}{l^3}$$

$$B_2 = -\frac{4}{l} + \frac{6x}{l^2}$$

$$B_3 = \frac{6}{l^2} - \frac{12x}{l^3}$$

$$B_4 = -\frac{2}{l} + \frac{6x}{l^2}$$

$$\therefore [B] = [B_1 \quad B_2 \quad B_3 \quad B_4]$$

Where $[B]$ = strain displacement matrix

IV. STIFFNESS MATRIX FOR A CRACKED BEAM ELEMENT

The key problem in using FEM is how to appropriately obtain the stiffness matrix for the cracked beam element. The most convenient method is to obtain the total flexibility matrix first and then take inverse of it.

International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 09, September 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

The total flexibility matrix of the cracked beam element includes two parts. The first part is original flexibility matrix of the intact beam. The second part is the additional flexibility matrix due to the existence of the crack, which leads to energy release and additional deformation of the structure.

4.1 Elements of the overall additional flexibility matrix C_{ovl}

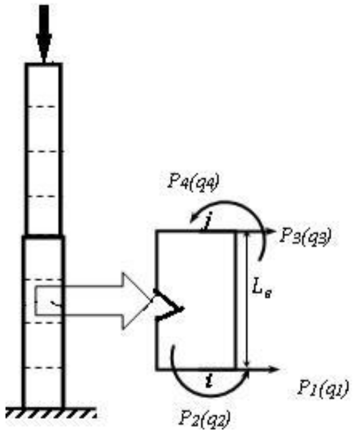

Figure 4 Cracked Beam Element with 2 Degree of Freedom

The fig. 4 shows a typical cracked beam element with a rectangular cross section. The left hand side end node ***i*** is assumed to be fixed, while the right hand side end node ***j*** is subjected to shearing force P_1 and bending moment P_2. The corresponding generalized displacements are denoted as q_1 and q_2.

B = Breath of the beam

h = Depth of the beam

a = crack depth

L_c = Distance between the right hand side end node j and the crack location

Le = Length of the beam element

A = Cross-sectional area of the beam

I = Moment of inertia

4.2 Flexibility Matrix C_{intact} of the Intact Beam Element

$$[C_{intact}] = \begin{bmatrix} \frac{L_e^3}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix}$$

4.3 Total Flexibility Matrix C_{tot} of the Cracked Beam Element

$$[C_{total}] = [C_{intact}] + [C_{ovl}]$$

International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 09, September 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

$$[C_{total}] = \begin{bmatrix} \frac{L_e^3}{3EI_2} + C_{11} & \frac{L_e^2}{2EI} + C_{12} \\ \frac{L_e^2}{2EI} + C_{21} & \frac{L_e}{EI} + C_{22} \end{bmatrix}$$

4.4 Stiffness Matrix K_C of a Cracked Beam Element

From the equilibrium conditions, the stiffness matrix K_c of a cracked beam element can be obtained as

$$[K_{crack}] = \{L\}[C_{tot}^{-1}]\{L\}^T$$

Where L is the transformation matrix for equilibrium condition

$$\{L\} = \begin{Bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{Bmatrix}$$

The results are presented for vibration of beams with cracks using the present formulation. The boundary conditions are

- Fixed end: $v = 0$, and $\theta = 0$
- Free end: no restraint

V. COMPUTATIONAL PROCEDURE

To obtain the frequencies for vibration of beam for different crack locations, crack depths and different percentages of loading by using finite element method, a finite element computational software Matlab R12b is used. Matlab R12b is user friendly, its name is derived from its functioning formula translating system, and it is used for optimization of mathematical process. Matlab R12b is used in computationally intensive areas such as numerical weather prediction, HYPERLINK "http://en.wikipedia.org/wiki/Finite_element_method" finite element analysis, computational fluid dynamics, computational physics and computational chemistry.

Problem considered under current study is a 2 stepped column with each step 0.15m and total height of column 0.3m with depth of cross section of upper step is 10mm and depth of cross section of below is 10mm and the breadth is maintained a constant value of 10mm. the young"s modulus of the material is 68.215GPa, poisons ratio is 0.28, and the density of the material is 2569 kg/m^3. In the current study free vibration and buckling analysis of a cracked two stepped cantilever column is analyzed by finite element method for various compressive load. Simple beam element with two degrees of freedom is considered for the analysis. Stiffness matrix of the intact beam element is found, Stiffness matrix for cracked beam element is found from the total flexibility matrix of the cracked beam element by inverse method in line with crack mechanics. Eigen value problem is solved for free vibration analysis of the stepped column under different compressive load. Variation of free vibration frequencies for different crack depths and crack locations is studied for successive increase in compressive load. Buckling load of the column is estimated from the vibration analysis. First critical buckling load of the stepped column is obtained from the computational procedure by using Matlab R12b. Then frequencies of the stepped column by varying different parameters such as crack depth, crack location, loading are found.

5.1Convergence Study

To study the correct approximation of results obtained the convergence study is done, for this each element is sub-divided to some number of elements and with certain load vibration analysis is done. And the results for different number of elements are compared and seen that those values are approximately equal.

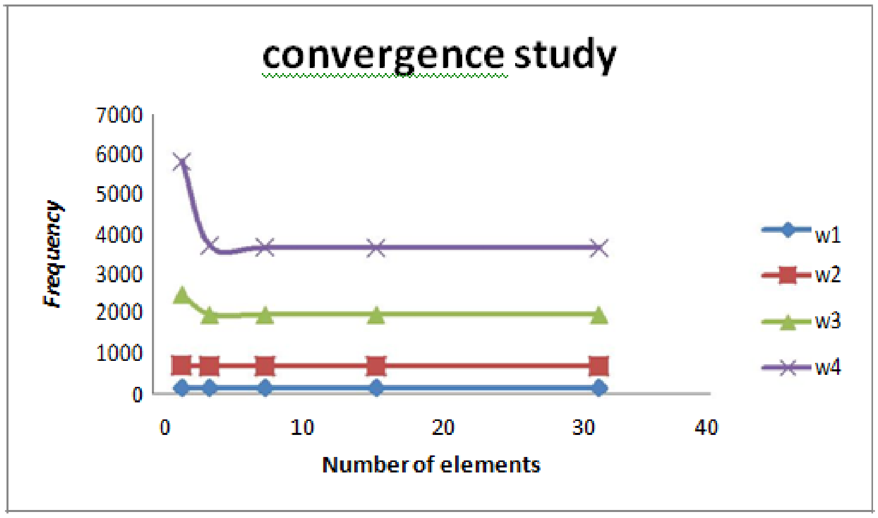

Figure 5: A Graph Between Number of Elements and Frequency for Different Frequencies

Different number of elements for each section 2, 4,8,16 are considered and for fixed-free end condition, their corresponding frequencies for free vibration are found and compared with a graph. It is observed that frequencies converge for different number of elements.

5.2 Validation of the Results of Current Study

This is result of variation of frequency ratio with crack location from Talaat H. Abdel-Lateef, Magdy Israel Salama, Buckling of slender prismatic columns with multiple edge cracks using energy approach, Alexandria Engineering Journal(2013) 52, 741-747.

A clamped- free ended column shown in with the following data is considered. h = b= 20 cm, L =3m, a =0.3 h = 6 cm and xc = 0.3 L =0.90 m

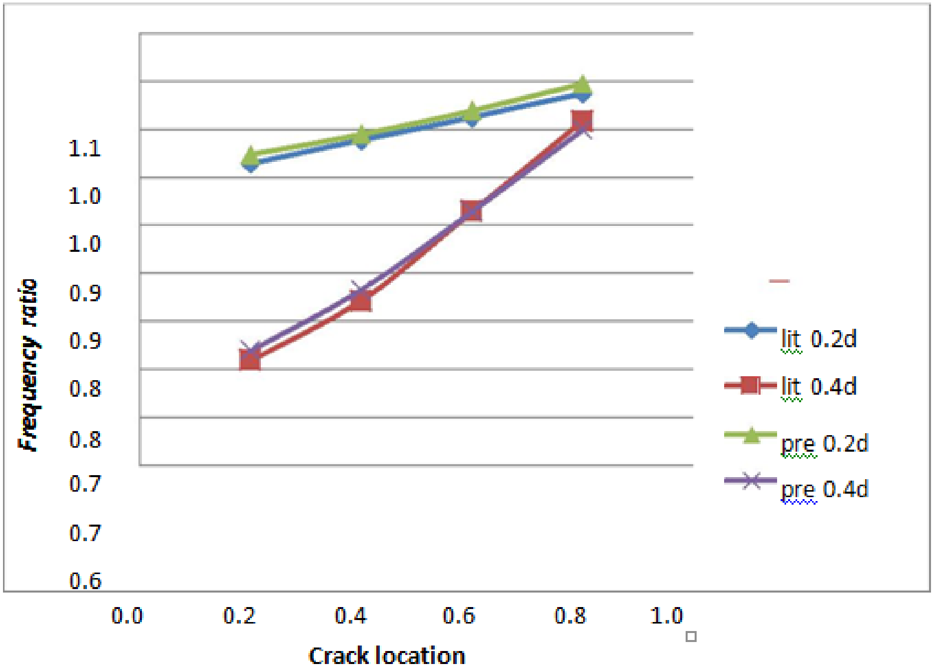

Figure 6 : A Graph Between Frequency Ratio and Crack Location for Validation of Result.

For 1st frequency and free vibration condition with no load and crack depth 0.2d the change in frequency ratio is minimal. For 0.4d the change from 0.2L to 0.4L is considerable but after that the change is negligible. For 0.6d the frequency changes almost linearly. And for 0.8d the frequency ratio increases linearly from 0.2L to 0.8L.

For 1st frequency and with free vibration under 0.2P compressive load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal. For crack depth 0.4d the change is small and linear. The change is considerable and linear for crack depth 0f 0.6d and for crack depth of 0.8d the variation of frequency ratio with crack location is it increases from 0.2L to 0.4L and the decreases at 0.6L and increases at 0.8L.

For free vibration analysis under 0.4P the frequency ratio of 1st frequency for 0.2d the change is negligible for different crack locations. For 0.4d it changes linearly. For 0.6d the change is considerable and linear. It is less at 0.2L and increase to 1 at 0.8L. For 0.8d crack depth column buckles for locations 0.2L and 0.6L.

For 1st frequency and with free vibration under 0.2P compressive load the change in frequency ratio for 0.2d the change with respect to different crack locations is minimal. For crack depth 0.4d the change is small and linear. And for crack depth of 0.6d the variation of frequency ratio with crack location is it increases from 0.2L to 0.4L and the decreases at 0.6L and increases at 0.8L.

This is result of variation of frequency ratio with crack location from Talaat H. Abdel-Lateef,
Magdy Israel Salama, Buckling of slender prismatic columns with multiple edge cracks using energy approach, Alexandria Engineering Journal(2013) 52, 741-747.

A clamped- free ended column shown in with the following data is considered. h = b= 20 cm,
=3m, a =0.3 h = 6 cm and xc = 0.3 L =0.90 m

At a crack location 0.2L and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4th frequency is 1.17% at 20% of buckling load the decrease is 1.57%, at 0.4 Pcr the decrease is

1.96% and the percentage in decrease of frequency increase to 2.38% at 0.6P and it decreases by 2.78% at 0.8P. At a crack location 0.12m and for crack depth of 0.2d the decrease in percentage of the frequencies for the increase in load compared to intact beam under free vibration is due to presence of crack even at free vibration the decrease in 4^{th} frequency is 0.122% at no load case and as load increase to 0.2P the decrease in frequency increase to 0.3% and with further increase in load to 0.4P the decrease in frequency becomes 0.7% and the decrease in frequency becomes 1.1% at a load of 0.6P and it becomes 1.5% at 0.8P. With a depth of crack 0.2d and at a crack location 0.18m the decrease in frequency of the cracked beam with respect to intact beam under free vibration for 4^{th} frequency is 0.67% for no load case and the reduction in frequency becomes 1.08% for 0.2P and with increase in load to 0.4P the decrease in frequency becomes 1.5% and the decrease in frequency is further increased to 1.91% for a load of 0.6P and it becomes 2.33% for a load of 0.8P.

The decrease in % of 4^{th} frequency with respect to frequency of intact beam at free vibration at a crack location 0.8L and crack depth 0.2d is 1.49% at no load case and it is decreased by 1.9% at 0.2P and the decrease in 2.3% of frequency at 0.4P and it increases to 2.71% at 0.6P and at 0.8P the decrease in frequency becomes 3.12%.

VI. CONCLUSION

From the analysis from the results and discussions following remarks and conclusions are observed.

1. The free vibration frequencies of a cracked stepped column decrease than the intact column for a crack of any depth and location in the column. However this decrease in free vibration frequencies is marginal for cracks of small depths and significant for larger crack depths. Thus the percentage of decrease in free vibration frequency increases with increase in crack depth.
2. The free vibration frequencies are more affected by the cracks present near the fixed end than free end. With the crack moving away from fixed end it loses its effect on free vibration frequencies and finally when the crack is near free end its effect is negligible.
3. When a crack in a stepped column coincides with the step of the column there is a severe drop in free vibration frequency. This is due to the fact that the section where the depth of the column decreases abruptly, there is a severe loss of stiffness of the cracked section.
4. The free-vibration frequencies under compression load decreases with increase in load than the free vibration frequencies under no load condition.
5. The load under which free vibration frequency vanishes or approaches zero can be assumed to be buckling load of the column.

REFERENCES

[1]. Adams RD, Cawley P. “Location of defects in structures from measurement of natural frequencies”. Journal of Strain Analysis 1997; 14:49–55.

[2]. Ameneh Maghsoodi, Amin Ghadami, Hamid Reza MIrdamadi, “Multiple crack damage detection in multi-step beams by novel local flexibility based damage index”, Journal of Soundand Vibration 332(2013)294–305.

[3]. Binici, B. „„Vibration of beams with multiple open cracks subjected to axial force“‟,

a. Journal of Sound and Vibration, 287, pp. 277–295 (2005).

[4]. Caddemi, S. and Calio, I. „„Exact solution of the multi-cracked Euler– Bernoulli column"", International Journal of Solids and Structures, 45,pp. 1332–1351 (2008).

[5]. Chondros, T. G., Dimarogonas, A. D., Yao, J., 1998. "A continuous Cracked Beam Vibration Theory", Journal of Sound and Vibration 215(1), pp. 17-34.

[6]. Wang. C. M, Wang. C. Y, Tun Myint Aung, "Buckling of a weakened column", J. Eng. Mech. ASCE 130 (2004).

[7]. Dong C, Zhang PQ, Feng WQ, Huang TC. "The sensitivity studies of the modal parameters of a cracks beam". 12th International Modal Analysis Conference 1994:98–104.

[8]. Shifrin. E. I, Ruotolo. R, "Natural frequencies of a beam with an arbitrary number of cracks", J. Sound Vib. 222 (3) (1999) 409–423.

[9]. G. Falsone, "The use of generalized functions in the discontinuous beam bending differential equations," *International Journal of Engineering Education*, vol. 18, no. 3, pp. 337–343, 2002.

[10]. Nahvi. H, Jabbari. M, "Crack detection in beams using experimental modal data and finite element model", International Journal of Mechanical Sciences 47 (2005) 1477– 1497

[11]. Jeong-Tae Kim, Yeon-Sun Ryu, Hyun-Man Cho, Norris Stubbs, "Damage identification in beam type structures: frequency based method vs mode shape based method", Engineering Structures 25 (2003) 57–67.

[12]. Krawczuk M, Ostachowiz W. "Natural vibration of a clamped– clamped arch with an open transverse crack". Journal of Vibrational Acoustics 1997;119:145–51.

[13]. Li, Q. S., "Buckling of Multi-Step Cracked Columns with Shear Deformation",

[14]. Engineering Structures, Vol. 23, (2001), 356-36