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DYNAMIC MODELING AND VIBRATION ANALYSIS OF A BALL-SCREW DRIVE SYSTEM

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ABSTRACT

Many of the papers studying the screw-drive system dynamic model concentrated on the control methodologies that is why vibrational models are either simple lumped models or simplified flexible beam models. In this paper a screw-drive system dynamic model and its vibrations are discussed. The screw is considered as an Euler-Bernoulli type flexible beam with fixed-free boundary conditions and the table inertia is considered as a moving force on the screw. First three modes of the longitudinal and torsional vibration of the flexible screw are extracted. The table travels with a triangular velocity motion profile. Table vibrations are studied with respect to the ratio of the table mass to the screw mass and the screw length.

Keywords: Screw-Drive System, Vibration, Continuous System, Elastic Shaft, Moving Force

I. INTRODUCTION

Demands for increased productivity and higher part quality have motivated a significant amount of research dedicated to developing faster and more accurate feed drive systems. Gu and Cheng (2004) studied the dynamic response of a high-speed spindle subject to a moving mass. The system consists of a ball screw and a moving nut. The ball screw is modeled as a high-speed rotating shaft using Timoshenko beam theory and the nut is modeled as a moving concentrated mass. Varanasi and Nayfeh (2004) developed a model of lead-screw system dynamics that accounts for the distributed inertia of the screw and the compliance and damping of the thrust bearings, nut, and coupling. The distributed-parameter model of the lead-screw drive system is reduced to a low-order model using a Galerkin procedure and verified by experiments performed on a pair of ball-screw systems. Kim and Chung (2006) modeled the feedback and feed-forward controllers described in discrete-time domain which are incorporated in the motion controller. Design requirements of the servomechanism such as stability, geometric errors, resonance of the driving mechanism, deformation of the structure, actuator saturation are described. Ouyanga and Wang (2007) presented a dynamic model for the vibration of a rotating Timoshenko beam subjected to a three-directional load moving in the axial direction. The model takes into account the axial movement of the axial force component. The bending moment produced by this component acting on the surface of the beam is also included in the model. Kamalzadeh and Erkorkmaz (2007 and 2007) presented a precision control strategy for ball screw drives. Axial vibrations are modeled and actively compensated in the control law, which enables the realization of high positioning bandwidth. Lead errors, arising from imperfections of the screw, are modeled and removed from the loop by offsetting their effect from the command trajectory and position feedback signals. Zhou et al. (2007) studied dynamic behaviors of free torsional vibrations of lead-screw feed drives using the finite element method considering of changeable table position and work-piece mass, to which the effect of changing table position and work-piece mass on the first three modes of the free vibration is analyzed. Hosseinkhani and Erkorkmaz (2012) proposed to improve the track following and disturbance rejection performance of the controller at selected frequencies by the use of adaptive feedforward cancellation method. Vicente et al. (2012) presented in their work a high-frequency dynamic model of a ball screw drive. The analytical formulation follows a comprehensive approach, where the screw is modeled as a continuous subsystem, using Ritz series approximation to obtain an approximate N-degree-of-freedom model. Gordon and Erkorkmaz (2013) studied a pole-placement technique to achieve active vibration damping, as well as high bandwidth disturbance rejection and positioning in ball screw drives.

Models in the papers of Gu and Cheng (2004) and Ouyanga and Wang (2007) consider rotating Timoshenko beam having a moving mass with constant velocity. Lateral deflections of the screw-mass system are studied. Varanasi and Nayfeh (2004) models screw as a continuous beam with longitudinal and torsional vibrations but the system reduced to a low-order model to analyze the system dynamics. Kim and Chung (2006) models the system as two disks attached to each other with torsional spring and the table mass is attached to the moving disk with a linear spring. In the papers of Kamalzadeh and Erkorkmaz (2007), Kamalzadeh and Erkorkmaz (2007), Hossainkhani and Erkorkmaz (2012), and Gordon and Erkorkmaz (2013) screw-drive system is modeled as two masses separated by a linear spring. Zhou et al. (2007) models the screw torsional vibration with finite element modeling. Vicente et al. (2012) models the screw as a continuous beam with longitudinal and torsional vibration but only some fixed table positions are considered.

In this paper the screw is modeled as a rotating Euler-Bernoulli beam for longitudinal and torsional vibrations. Table mass is assumed traveling according to a triangular velocity time function. Table inertia is considered as a moving load on the rotating screw. Normal modes of a constrained structure method is used to find dynamic equations of the motion, first three modes of the longitudinal and torsional vibration are considered, table mass vibrations during the travel are studied.

II. SYSTEM MODELING

Fig. 1 shows the model of the ball-screw drive. The system consists of a screw of length L, electrical motor and bearings to support the screw and a table.

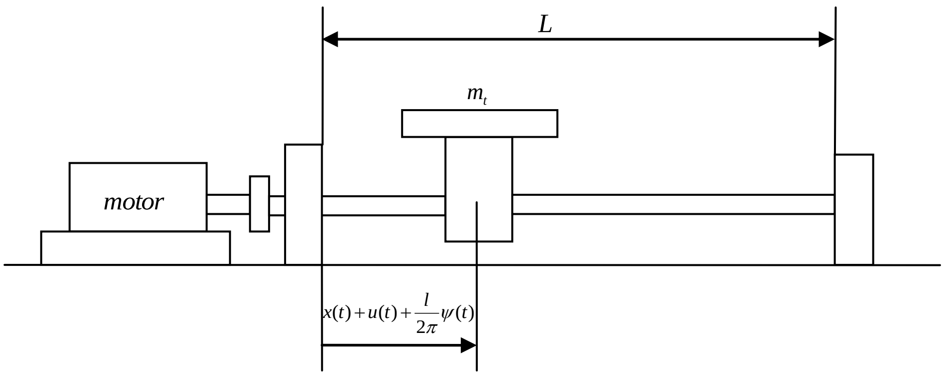

Fig. 1 Screw-Drive System

Table is guided not to have lateral motions, friction between screw and nut and table and guide are assumed negligible. It is assumed that table travels with triangular velocity function which can be formulated as,

$$x(t) = \frac{2L_t}{T^2}t^2 \qquad 0 < t < \frac{T}{2}$$

$$x(t) = -\frac{2L_t}{T^2}t^2 + \frac{4L_t}{T}t - L_t \qquad \frac{T}{2} < t < T$$

$$\ddot{x}(t) = \pm\frac{4L_t}{T^2} \tag{1}$$

Here L_t is the travel length of the table and T is the total travel time. Since the table is accelerating and decelerating during its motion, rotating screw will experience a force because of the table inertia. The screw is assumed a continuous shaft and it will have longitudinal and torsional vibrations under the table inertial force and motor torque. Longitudinal and torsional vibration of the screw is assumed as,

$$u(x,t) = \sum_{i=1}^{n} \phi_{li}(x) q_{li}(t)$$

$$\psi(x,t) = \sum_{i=1}^{n} \phi_{ti}(x) q_{ti}(t) \tag{2}$$

Since there is a table with a mass moving on the screw, the screw can be considered as a constrained structure. The generalized coordinates of the screw for longitudinal vibration can be calculated from; Thomson (1981)

$$\ddot{q}_{li} + 2\zeta_{li}\omega_{li}\dot{q}_{li} + \omega_{li}^2 q_{li} = \frac{1}{M_{li}}[f_l(x,t)\phi_{li}] \tag{3}$$

ζ_{li} is the viscous damping ratio, ω_{li} is the ith longitudinal natural frequency of the screw, M_{li} is the generalized mass which is,

$$M_{li} = \int_0^l m_s \phi_{li}^2(x)\,dx \tag{5}$$

Here m_s is the mass per unit length of the screw. Total motion of the table is the sum of the prescribed table motion of $x(t)$ plus longitudinal vibration of the table $u(x,t)$ and plus the table motion because of the screw torsional vibration.

$$x_t(t) = x(t) + u(x,t) + \frac{\psi(x,t)\,l}{2\pi} \tag{6}$$

Here l is the lead of the screw as meter per rotation. Then the inertial force acting on the screw because of the table mass m_t will be

$$f_l(x,t) = -m_t\ddot{x}_t = -m_t\left(\ddot{x}(t) + \ddot{u}(x,t) + \frac{\ddot{\psi}(x,t)\,l}{2\pi}\right) \tag{7}$$

Table motion in terms of the generalized coordinates and normal modes is

$$x_t(t) = x(t) + \sum_{i=1}^{n} \phi_{li}(x) q_{li}(t) + \frac{l}{2\pi}\sum_{i=1}^{n} \phi_{ti}(x) q_{ti}(t) \tag{8}$$

Acceleration of the table can be obtained as

$$\ddot{x}_t(t) = \ddot{x}(t) + \sum_{i=1}^{n} \phi_{li}''(x) q_{li}(t)\, \dot{x}^2 + \sum_{i=1}^{n} \phi_{li}'(x) q_{li}(t)\, \ddot{x} + 2\sum_{i=1}^{n} \phi_{li}'(x) \dot{q}_{li}(t) \dot{x} + \sum_{i=1}^{n} \phi_{li}(x) \ddot{q}_{li}(t) + \frac{l}{2\pi}\left(\sum_{i=1}^{n} \phi_{ti}''(x) q_{ti}(t)\, \dot{x}^2 + \sum_{i=1}^{n} \phi_{ti}'(x) q_{ti}(t)\, \ddot{x} + 2\sum_{i=1}^{n} \phi_{ti}'(x) \dot{q}_{ti}(t) \dot{x} + \sum_{i=1}^{n} \phi_{ti}(x) \ddot{q}_{ti}(t)\right) \tag{9}$$

When the equation (9) is put into the equation (7) and used in the equation (3), the following dynamic equation for the generalized coordinates of the longitudinal vibration will be obtained.

$$\ddot{q}_{li} + 2\zeta_{li}\omega_{li}\dot{q}_{li} + \omega_{li}^2 q_{li} = -\frac{m_t}{M_{li}}\left[\ddot{x}(t) + \sum_{j=1}^{n} \phi_{lj}''(x) q_{lj}(t)\, \dot{x}^2 + \sum_{j=1}^{n} \phi_{lj}'(x) q_{lj}(t)\, \ddot{x} + 2\sum_{j=1}^{n} \phi_{lj}'(x) \dot{q}_{lj}(t) \dot{x} + \sum_{j=1}^{n} \phi_{lj}(x) \ddot{q}_{lj}(t) + \frac{l}{2\pi}\left(\sum_{j=1}^{n} \phi_{tj}''(x) q_{tj}(t)\, \dot{x}^2 + \sum_{j=1}^{n} \phi_{tj}'(x) q_{tj}(t)\, \ddot{x} + 2\sum_{j=1}^{n} \phi_{tj}'(x) \dot{q}_{tj}(t) \dot{x} + \sum_{j=1}^{n} \phi_{tj}(x) \ddot{q}_{tj}(t)\right)\right]\phi_{li} \tag{10}$$

Similar equation will be obtained for the generalized coordinates of the torsional vibration. In this case table mass inertial force that is converted to a torsion acting on the screw will be used.

$$\ddot{q}_{ti} + 2\zeta_{ti}\omega_{ti}\dot{q}_{ti} + \omega_{ti}^2 q_{ti} = -\frac{m_t}{M_{ti}}\left[\frac{2\pi}{l}\left(\ddot{x}(t) + \sum_{j=1}^{n} \phi_{lj}''(x) q_{lj}(t)\, \dot{x}^2 + \sum_{j=1}^{n} \phi_{lj}'(x) q_{lj}(t)\, \ddot{x} + 2\sum_{j=1}^{n} \phi_{lj}'(x) \dot{q}_{lj}(t) \dot{x} + \sum_{j=1}^{n} \phi_{lj}(x) \ddot{q}_{lj}(t)\right) + \sum_{j=1}^{n} \phi_{tj}''(x) q_{tj}(t)\, \dot{x}^2 + \sum_{j=1}^{n} \phi_{tj}'(x) q_{tj}(t)\, \ddot{x} + 2\sum_{j=1}^{n} \phi_{tj}'(x) \dot{q}_{tj}(t) \dot{x} + \sum_{j=1}^{n} \phi_{tj}(x) \ddot{q}_{tj}(t)\right]\phi_{ti} \tag{11}$$

First three modes of the longitudinal and torsional vibrations are assumed. In this case equations (10) and (11) will provide six coupled equations of the motion for the generalized coordinates. When the equations are arranged in matrix form we will have

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{12}$$

The vector q is $q = [q_{l1}, q_{l2}, q_{l3}, q_{t1}, q_{t2}, q_{t3}]^t$. Elements of the mass, the damping and the stiffness matrices are given in appendix. Torsional and longitudinal vibration modes of the screw are calculated assuming fixed-free flexible beam boundary condition;

$$\phi_{ti}(x) = \phi_{li}(x) = A \sin i\frac{\pi x}{2L} \qquad i = 1, 2, 3 \tag{13}$$

Longitudinal natural frequencies of a fixed-free beam are given as

$$\omega_{li} = \frac{i\pi}{2L}\sqrt{\frac{E}{\rho}} \qquad i = 1, 3, 5 \tag{14}$$

Here E is the elasticity modulus ρ is the material density and L is the screw length. Torsional natural frequencies of the screw are

$$\omega_{ti} = \frac{i\pi}{2L}\sqrt{\frac{G}{\rho}} \qquad i = 1, 3, 5 \tag{15}$$

Here G is the shear modulus.

III. RESULTS AND DISCUSSIONS

Parameters used for the numerical calculations are given in Table 1. The screw is 1.5 m long and has 20 mm diameter. Its mass is 3.6 kg. First three longitudinal natural frequencies of the screw are 849.4 Hz, 2548.2 Hz and 4247.1 Hz. First three torsional natural frequencies of the screw are 537.2 Hz, 1611.6 Hz and 2686.1 Hz. It is assumed that screw table travels with the triangular velocity function which is described in the equation (1)

Table 1 List of Parameters

Description	Screw length	Lead	Screw diameter	Screw mass	Material density	Elasticity modulus	Shear modulus	Damping ratio	Travel length
Symbol	L	l	d	m_s	ρ	E	G	ζ	L_t
Value	1.5 m	0.005 m/rev	0.02 m	3.6 kg	7700 kg/m^3	200 GPa	80 GPa	0.02	1.0 m

Screw table has a length of 50 cm that is why the travel length is 50 cm less than the screw length. Travel time is assumed as 2 seconds. Fig. 2, Fig. 3 and Fig. 4 are showing the first, second and third mode longitudinal vibrations separately.

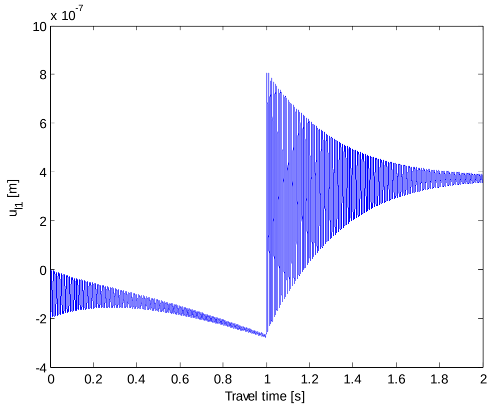

Fig. 2 First Mode Longitudinal Vibration.

L=2 m, T=2 s, m_t=20 kg.

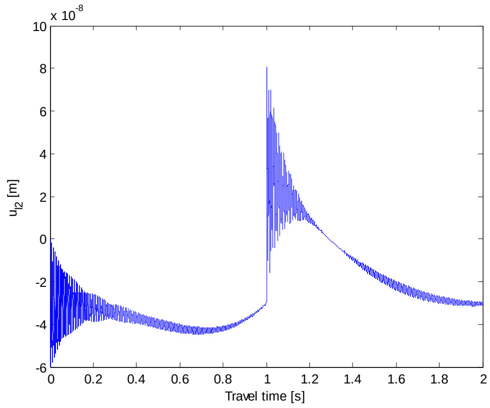

Fig. 3 Second Mode Longitudinal Vibration.

L=2 m, T=2 s, m_t=20 kg.

In the first half of the travel, screw table has positive, in the second half of the motion has negative constant valued acceleration. Table inertia acts to the screw like a pushing force in the first half of the motion in the second half like a pulling force.

Fig. 2, Fig. 3 and Fig. 4 show this effect. In the beginning of the motion, the screw has vibration in the negative direction and this vibration is damping out before the second half motion starts. Since the vibrating length of the screw is much longer in the second half of the motion, magnitudes of the vibrations are also much higher than the ones of the first half.

Fig. 5 shows the vibration amplitudes of the table when the three longitudinal modes considered together. Fig. 6, Fig. 7 and Fig. 8 show the first three torsional mode vibrations of the screw separately.

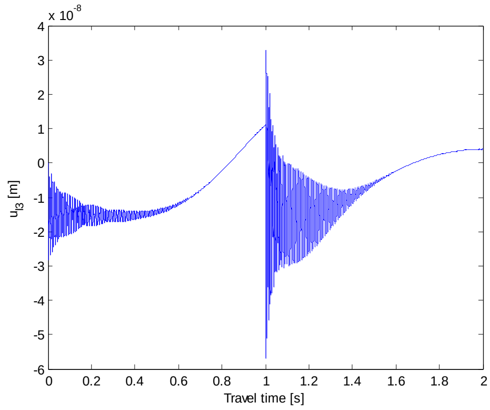

Fig. 4 Third Mode Longitudinal Vibration.

L=2 m, T=2 s, m_t=20 kg.

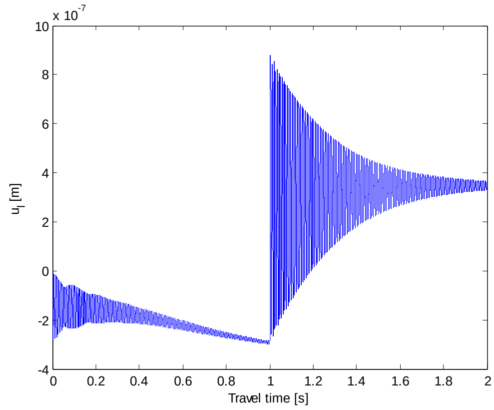

Fig. 5 Total Longitudinal Vibration.

L=2 m, T=2 s, m_t=20 kg.

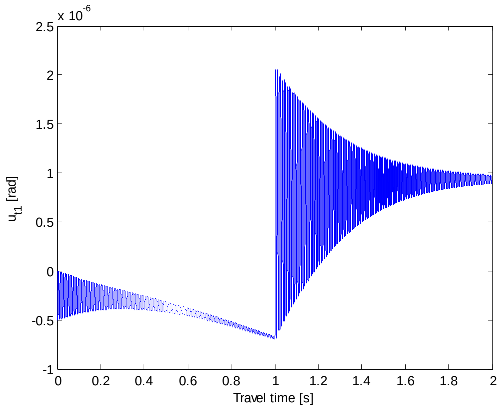

Fig. 6 First Mode Torsional Vibration.

L=2 m, T=2 s, m_t=20 kg.

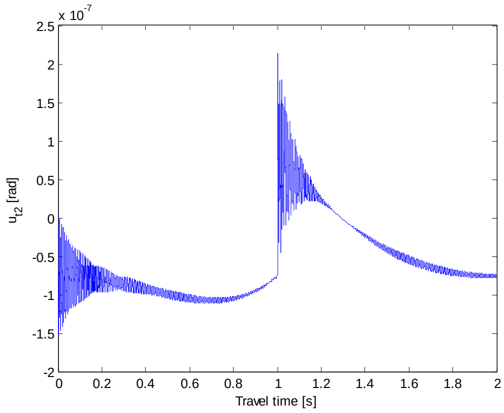

Fig. 7 Second Mode Torsional Vibration.

L=2 m, T=2 s, m_t=20 kg.

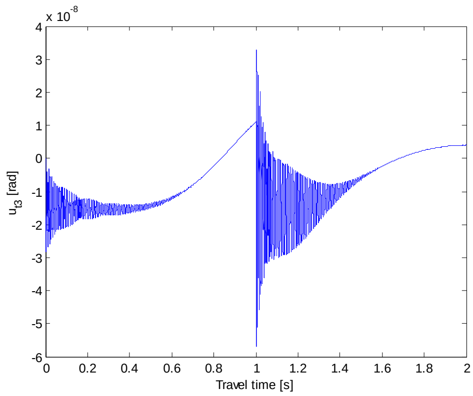

Fig. 8 Third Mode Torsional Vibration.

L=2 m, T=2 s, m_t=20 kg.

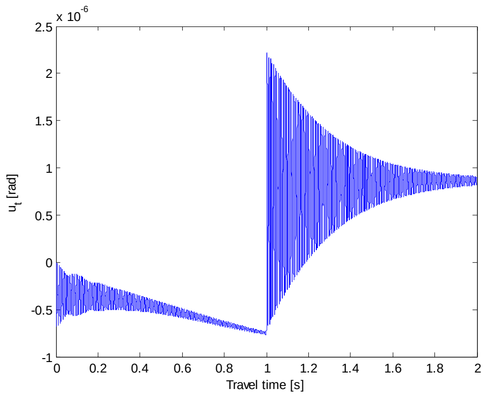

Fig. 9 Total Torsional Vibration.

L=2 m, T=2 s, m_t=20 kg.

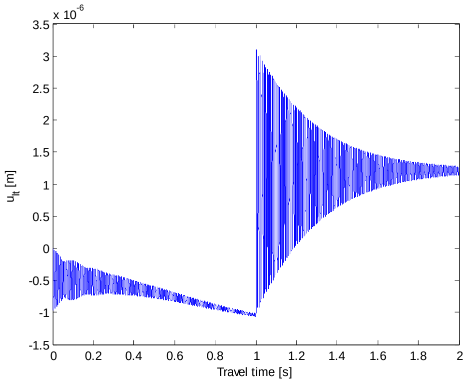

Fig. 10 Total Torsional And Longitudinal Vibration.

L=2 m, T=2 s, m_t=20 kg.

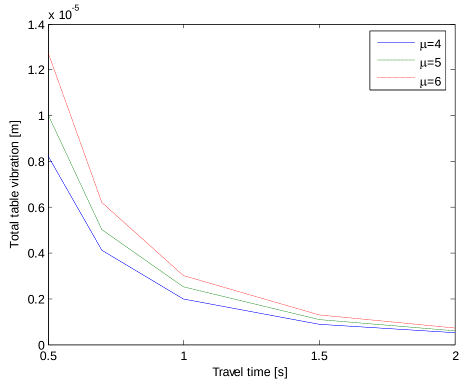

Fig. 11 Total Table Vibration Versus Travel Time

L=1 m.

Vibration trends of the torsional motion are similar to the ones of the longitudinal vibrations because the modes are same for the longitudinal and torsional vibrations for a fixed-free beam but the torsional vibration amplitudes are much higher than the longitudinal vibration amplitudes for the given example. Fig. 9 shows the torsional vibration amplitudes if the three modes are combined. Fig. 10 shows total vibration amplitudes of the table if the longitudinal and torsional vibrations of the screw are considered together. Fig. 11 shows total table vibrations for different travel times and for different table mass to screw mass ratios. When the mass ratio is increased, table inertia increases. Vibration amplitudes versus travel time curves are having exponential decay character. For longer travel times, the vibration amplitudes are also merging to each other and the effect of table mass inertia becomes negligibly small. Fig. 12 shows table vibration amplitudes for the screw lengths of 2 meters.

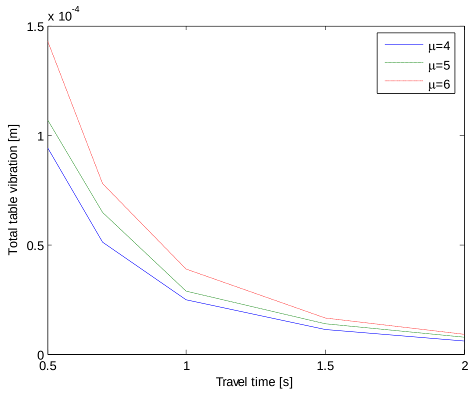

Fig. 12 Total Table Vibration Versus Travel Time. L=2 m.

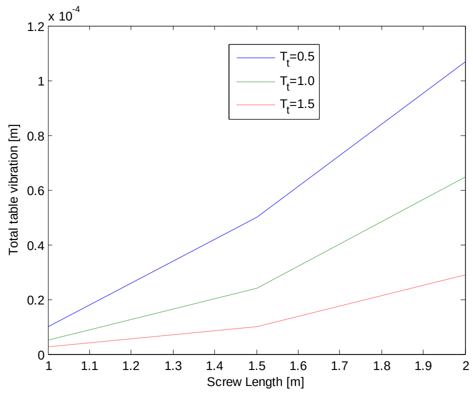

Fig. 13 Total Table Vibration Versus Screw Length. m_t=20 kg.

Fig. 13 shows the table vibration amplitude with respect to a fixed screw length but for different travel times. For longer screw and shorter travel time vibration amplitudes are exponentially increasing.

IV. CONCLUSION

In this paper a screw-drive system vibration model is studied. The screw is modeled as a Euler-Bernoulli type flexible beam with fixed-free boundary conditions. The table inertia is considered as a moving force on the screw. First three modes of longitudinal and torsional vibrations of the screw are considered. Equations of the motion are developed considering normal modes of a constrained structure method. Table is assumed travelling with a triangular velocity motion profile. Table vibrations are studied for different table mass to screw mass ratios and also for different screw lengths.

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