COMPARISON BETWEEN ALGEBRAIC GRID AND ELLIPTIC GRID OVER AN AIRFOIL

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ABSTRACT

This project work gives a platform of how to create an efficient grid over an airfoil by comparing different grid generation techniques. Here two grid generation methods such as algebraic grid generation method and elliptic grid generation method were chosen .The algebraic equation is used to relate the grid point in the computational domain to those of the physical domain. This is achieved by using an interpolation scheme between the specified boundary grid points to generate the interior grid points. The second method highlights the solution for a system of elliptic equations in the form of laplace's equation in the physical domain with the help of point gauss seidel (PGS) method. These two methods reveals the process of generating as well as communicate benefits draw backs to both algebraic and elliptic methods.

***Keywords**: Algebraic C-Grid Generation, Elliptic C-Grid Generation.*

I INTRODUCTION OF GRID GENERATION

The arrangement of the discrete points throughout the flow field is simply called a grid. The way that a grid is determined is called grid generation. In this project a grid wraps around an airfoil is to be generated. The necessary of the grid generation is,the standard finite difference methods require a uniformly spaced rectangular grid. If a rectangular grid is used,few grid poits fall on the surface. Flow close to the surface being very important in terms of forces,a rectangular grid will give poor results in such regions. Here the rectangular computational grid will be transformed into a grid that wraps around an airfoil.

II PROBLEM STATEMENT

Consider an symmetrical air foil whose geometry is described by the equation 1. An outer C-shape domain is specified by a half circle and two parallel lines, as shown in figure 1. Grid points are clustered near leading edge and trailing edge to provide adequate resolution of the viscous boundary layer**.**

$$y = \frac{t}{0.2}\left(0.2969x^{1/2} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4\right) \rightarrow 1$$

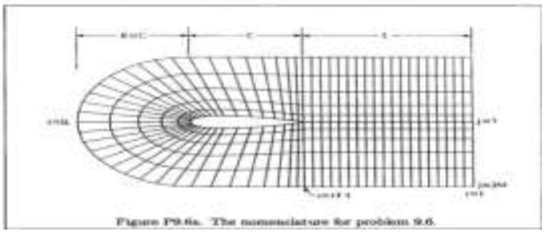


Fig 1:(Hoffmann) Figure P9-6a page 424

PHASE 1:

Generate a grid by an algebraic method with gird point clustering clustering near the airfoil surface and the wake region.

PHASE 2:

Generate a grid by the elliptic scheme with no grid clustering.

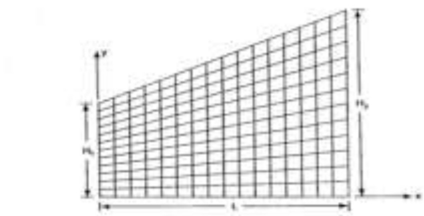
C-TYPE GRID:

C-type grid indicates a coordinate system with lines emanating from a boundary, passing around a body and returning to the boundary.the grid represents a combination of an 0-type grid in the upstream region and h-type grid in the downstream region. This type of grid provides a good treatment of all boundary and periodicity conditions.

III METHODOLOGY

3.1 Algebraic Method

The simplest grid generation technique is the algebraic method.the major advantage of this scheme is the speed with which a grid can be generated.An algebraic equation is used to relate the grid points in the computational domain to those of the physical domain.This objective is met by using an interpolation scheme between the specified boundary grid points to generate the interior grid points.clearly,many algebraic equation can beintroduced for this purpose.To illustrate the procedure,consider the simple physical domain depicted in fig(2).



Fig(2)

(Hoffmann) Figure 9-4 Page 365

The physical space which must be transformed to a uniform rectangular computational space.

Introducing the following algebraic relations will transform this nonrectangular physical domain into a rectangular domain:

$$\xi = x \rightarrow 2$$

$$\eta = \frac{y}{y_t} \rightarrow 3$$

In equation 3,y_t represents the upper boundary which is expressed as

$$y_t = H_1 + \frac{H_2 - H_1}{L} x$$

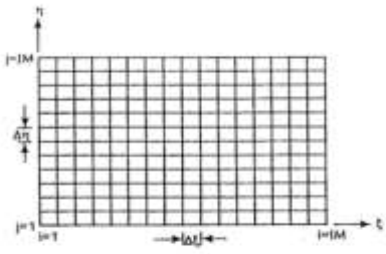
And

$$y = \left(H_1 + \frac{H_2 - H_1}{L} \xi\right) \eta \rightarrow 4$$

The equal grid spacing in the computational domain is produced as follows:

$$\Delta\xi = \frac{1}{\mathrm{IM} - 1} \rightarrow 5$$

$$\Delta\eta = \frac{1}{\mathrm{JM} - 1} \rightarrow 6$$



Fig(3)

The rectangular computational domain with uniform grid spacing.

The physical domain can be converted to the computational through the use of equations (7) and (8).

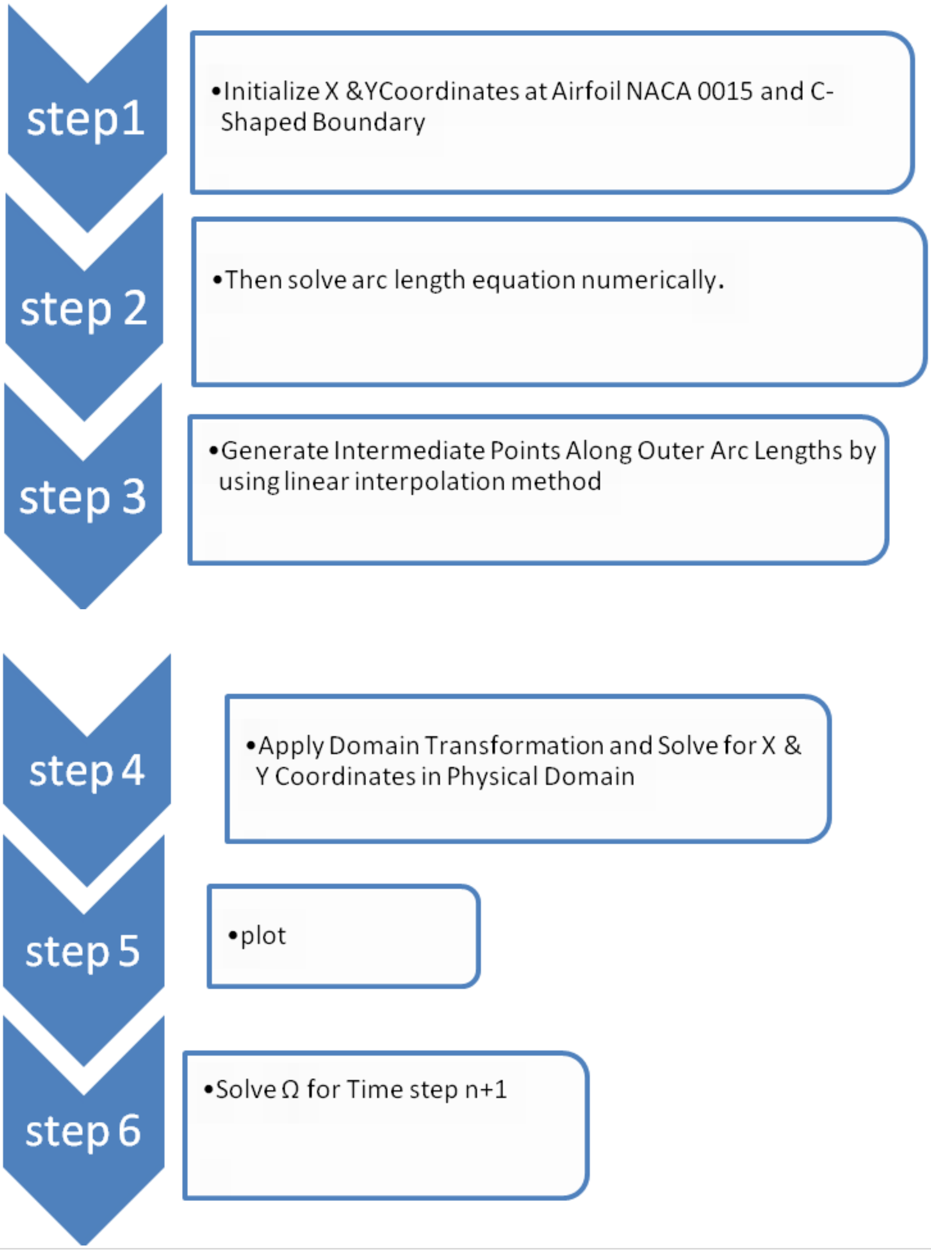
$$\xi = x \rightarrow 7$$

$$\eta = \alpha + (1 - \alpha) \frac{\left\{\{\beta + [(2\alpha + 1)y/H] - 2\alpha\}/\left\{\beta - \left[\frac{(2\alpha+1)y}{H}\right] + 2\alpha\right\}\right\}}{\mathrm{Ln}[(\beta + 1)/(\beta - 1)]} \rightarrow 8$$

Where y_t is the upper limit of y and β is the clustering term. This effectively gives η as a ratio of y to the upper limit. $\beta$is given is a clustering parameter whose range is 1 to infinity. As the value of β approaches 1,the grid points cluster near the surface, where y = 0.To be able to generate a C-type grid, a transformation from the rectangular domain to the C-type domain needs to be conducted. This can be accomplished through an algebraic transformation converting computational x and y coordinates to ξ and η coordinates of the C-type domain.The relationship that exists between the physical and computational domains can seen in Equations 2 and 3. The physical domain can be converted to the computational through the use of Equations 7 and 8.

3.1.1 Block Diagram of Algebraic Coding

The coding algorithm will be illustrate with a block diagram. The code can be seen directly following the block diagram. The block diagram illustrates a short synopsis of how the code is employed to solve the vorticity equation. The code is also commented to ensure understanding.



3.2 Elliptic Method

The elliptic method system starts with a elliptic partial differential equations (PDE). The two elliptic equations that the system is derived from are Equations 9(9-61) and 10 (9-62).

$$\xi_{xx} + \xi_{yy} = 0 \rightarrow 9$$

$$\eta_{xx} + \eta_{yy} = 0 \rightarrow 10$$

With the Dirichiet boundary conditions

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \xi_1(x, y) \\ \eta_1 \end{bmatrix}; [x, y] \epsilon G_1 \rightarrow 11$$

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \xi_2(x, y) \\ \eta_2 \end{bmatrix}; [x, y] \epsilon G_2 \rightarrow 12$$

Where ,η_1 *and* η_2 Are constants and ξ_1(x, y) and ξ_2(X, y) are specified monotonic functions on G1 and G2 respectively. That is, let ξ(x, y) and η(x, y) be harmonic in D. This generation system guarantees one-to-one mapping for boundary-conforming curvilinear coordinate systems on general closed boundaries.

Equations 9 and 10 were transformed into Equations13 and 14.

$$a. x_{\xi\xi} - 2. b. x_{\xi\eta} + c. x_{\eta\eta} = 0 \rightarrow 13$$

$$a. y_{\xi\xi} - 2. b. y_{\xi\eta} + c. y_{\eta\eta} = 0 \rightarrow 14$$

Where

$$a = x_\eta{}^2 + y_\eta{}^2$$

$$b = x_\xi . x_\eta + y_\xi . y_\eta$$

$$c = x_\xi{}^2 + y_\xi{}^2$$

Equation 13 is a PDE that needs to be discretized and turned into a finite difference equation (FDE)which can be seen in Equation 15. Similarly Equation 14 is converted into equation 16.

$$\mathrm{a.}\left(\frac{\mathrm{X}_{\mathrm{I+1,j}} - 2.\mathrm{X}_{\mathrm{I,j}} + \mathrm{X}_{\mathrm{I-1,j}}}{(\Delta\xi)^2}\right) - 2.\mathrm{b.}\left(\frac{\mathrm{X}_{\mathrm{I+1,j+1}} - \mathrm{X}_{\mathrm{I+1,j-1}} + \mathrm{X}_{\mathrm{I-1,j+1}} + x_{\mathrm{I-1,j-1}}}{4.\Delta\xi.\Delta\eta}\right) + \mathrm{c.}\left(\frac{\mathrm{X}_{\mathrm{I,j+1}} - 2.\mathrm{X}_{\mathrm{I,j}} + \mathrm{X}_{\mathrm{I,j-1}}}{(\Delta\eta)^2}\right) = 0 \rightarrow 15$$

$$\mathrm{a.}\left(\frac{\mathrm{Y}_{\mathrm{I+1,j}} - 2.\mathrm{Y}_{\mathrm{I,j}} + \mathrm{Y}_{\mathrm{I-1,j}}}{(\Delta\xi)^2}\right) - 2.\mathrm{b.}\left(\frac{\mathrm{Y}_{\mathrm{I+1,j+1}} - \mathrm{Y}_{\mathrm{I+1,j-1}} + \mathrm{Y}_{\mathrm{I-1,j+1}} + y_{\mathrm{I-1,j-1}}}{4.\Delta\xi.\Delta\eta}\right) + \mathrm{c.}\left(\frac{\mathrm{Y}_{\mathrm{I,j+1}} - 2.\mathrm{Y}_{\mathrm{I,j}} + \mathrm{Y}_{\mathrm{I,j-1}}}{(\Delta\eta)^2}\right) = 0 \rightarrow 16$$

Using a point gauss-seidel(PGS)iterative method the FDE equation can isolate the $x_{i,j}$ And $y_{i,j}$ Terms and then solved for.the resulting equation 17 and 18 ,after some algebraic manipulation can be seen below.

$$x_{i,j} = \left\{ \frac{\frac{a}{(\Delta\xi)^2}\left[x_{i+1,j} + x_{i-1,j}\right] - \frac{b}{2\Delta\xi\Delta\eta}\left[x_{i+1,j+1} - x_{i+1,j-1} + x_{i-1,j-1} - x_{i-1,j+1}\right] + \frac{c}{(\Delta\eta)^2}\left[x_{i,j+1} + x_{i,j-1}\right]}{2\left[\frac{a}{(\Delta\xi)^2} + \frac{c}{(\Delta\eta)^2}\right]} \right\} \rightarrow 17$$

$$y_{i,j} = \left\{ \frac{\frac{a}{(\Delta\xi)^2}\left[y_{i+1,j} + y_{i-1,j}\right] - \frac{b}{2\Delta\xi\Delta\eta}\left[y_{i+1,j+1} - y_{i+1,j-1} + y_{i-1,j-1} - y_{i-1,j+1}\right] + \frac{c}{(\Delta\eta)^2}\left[y_{i,j+1} + y_{i,j-1}\right]}{2\left[\frac{a}{(\Delta\xi)^2} + \frac{c}{(\Delta\eta)^2}\right]} \right\} \rightarrow 18$$

3.2.1 Block Diagram of Elliptic Coding

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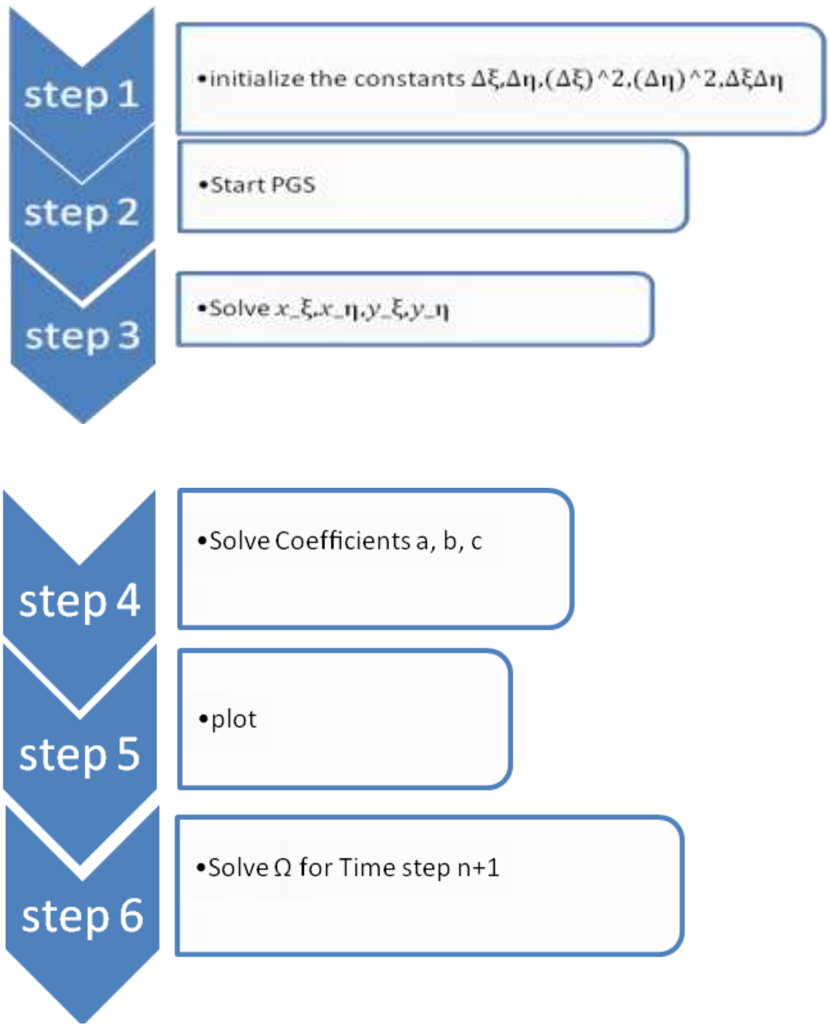


IV DISCUSSION

The numerical results will be illustrated with the plots that were generated while running the MATLAB coding . Then the results will be analyzed and discussed.

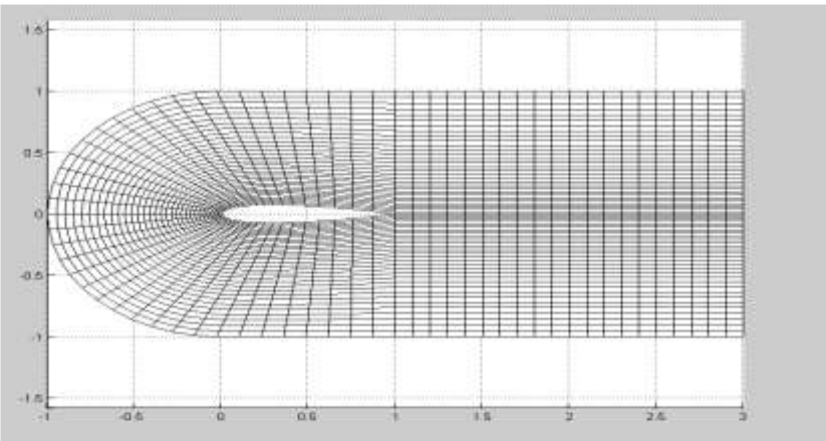
V RESULTS

The algebraic code is solved several times.

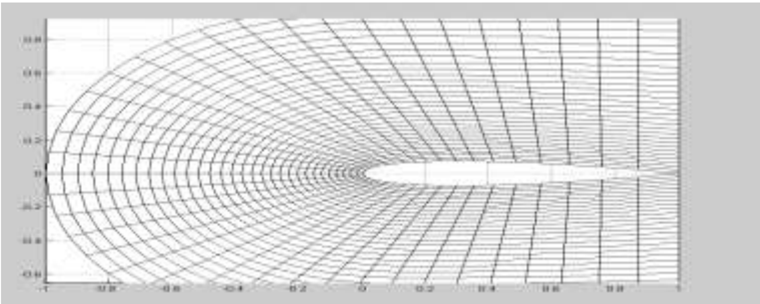
CASE 1:

```
U = 40;
V = 30;
A = 8;
%Clustering points near leading edge, inf = no clustering
B = 1;
%Clustering points near the surface, 1 = no clustering
t = .15;
c = 1;
L = 2;
```

Algebraic method



Fig(4)



Fig(5)

CASE 2:

U = 50;

V = 20;

A = 10;

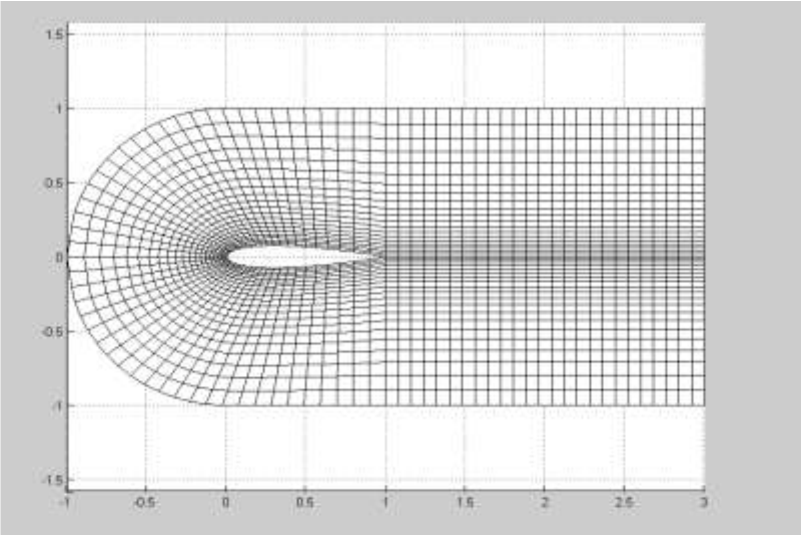
%Clustering points near leading edge, inf = no clustering

B = 2;

%Clustering points near the surface, 1 = no clustering

t = .15;

N2 = 50



Fig(6)

Elliptic Method

CASE 3:

de = 1/V;

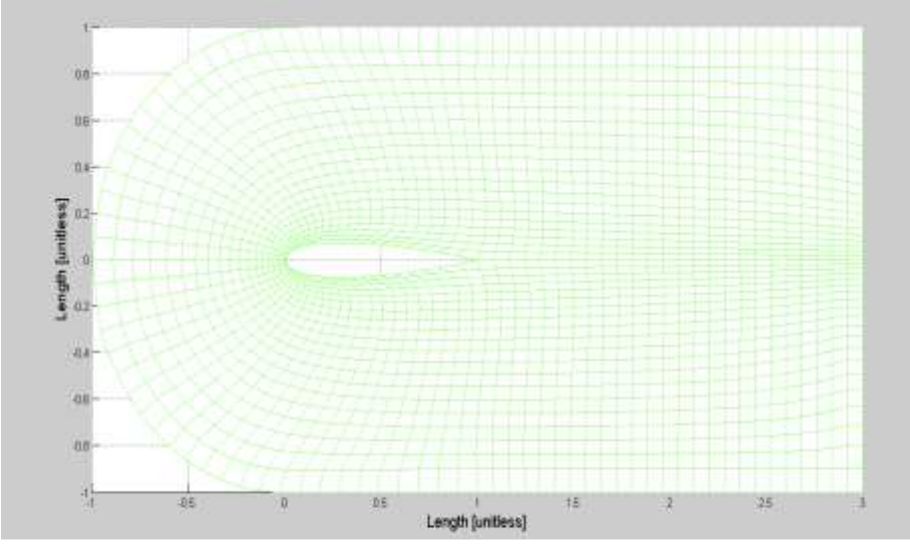
de2 = de^2;

dn = 1/U;

dn2 = dn^2;

dedn = 2/(V*U);

P = 100;



Elliptic method grid without clustering

Fig(7)

VI CONCLUSION

This project showed the algebraic the algebraic and elliptic grid generation method over an airfoil.Algebraic method is very fast and also less numerical errors.it has the ability to cluster grid points in different regions.Eventhough it has many advantages, control of grid smoothness and skewness is different taskThis drawback was overcome by implementing techniques in elliptic method.

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