EFFECT OF PRESSURE ANGLE OF SPUR GEARS ON BENDING AND CONTACT STRESSES: A COMPARATIVE STUDY USING FINITE ELEMENT SOFTWARE

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ABSTRACT

The principle objective of this paper is the comparison study of the static stresses for spur gear with different pressure angles. The analyzed results of a symmetric type involute profiled spur gear pair at different pressure angles are compared. Gears are one of the most important and crucial component in a mechanical power transmission unit and also in most of the industrial rotating machineries. Generally, a spur gear pair in action undergoes two types of stresses: the bending stress and the contact stress. In this paper, both these stresses on the gear tooth pair are analyzed using the finite element analysis and are compared. The stresses on the gear tooth are first analyzed using a finite element software and then those results are validated using the conventional formulae for finding stresses in gear tooth.

Keywords*: *Bending Stress, Contact Stress, Finite Element Analysis, Involute Profiled Spur Gear, Pressure Angle.

1. INTRODUCTION

Gears are important mechanical components that are used in the power transmission unit of different mechanical and automobile systems. Therefore, it becomes a necessary and an important area of research in order to improve the efficiency of different mechanical power transmission systems by improving the workability of gear systems. The finite element static analyses of the involute profiled spur gears are carried out in order to obtain different static stresses in gears for the performance of gears. The static stresses are found out within the elastic limit of the gear, i.e., the linear static analysis of involute profiled spur gear pair. In [1] the analysis was carried out by charging the structure with a normal concentration of force along the side of the tooth. The outcomes of [1] was, the areas that undergo highest stress are those closest to the tooth root fillet. Symmetric spur gears are those whose pressure angle at both coast and drive sides are the same, i.e., the commercially used involute teeth profiled spur gears. The stress analysis, determination of the dynamics loads, the transmission errors, gear noise, and the optimal design for gear pairs are still a major concern in gear design and analysis.

In this paper, both the bending stresses and the contact stresses are analyzed. Firstly, in the bending stress analysis, the stresses that are induced at the gear tooth fillet are obtained by the finite element analysis. This bending stress that is obtained, if more than that of the allowable stress of the gear material, the gear is prone to bending tooth breakage, one of the tooth failure. Secondly, in the contact stress analysis, the stresses that are induced at the induced at the gear tooth face are obtained by the finite element method.

In this paper, to validate the obtained bending and the contact stress results by the finite element method, two theoretical formulae have been used, namely, the LEWIS BENDING stress equation for the static bending analysis, and AGMA CONTACT Stress Equation.

The 3D modeling of the gear pairs with different pressure angles are done using one of the solid modeling software. The solid model of the gear pairs with different pressure angles are converted from the basic solid model format to IGES or PARASOLID format and then import to the finite element software and then analyzed to get the different stress results.

II EQUATIONS AND CALCULATIONS

This chapter includes the discussion about the three dimensional model of the gear and pinion in mesh and also the finite element method that has been used to analyse the static conditions and calculate analytically the maximum and minimum stresses and strains when the spur gear are in mesh, for the spur gear pairs with different pressure angles (14.5°, 20°, 25° and 30°).

The 3D models of the pinion and the gear are created in a CAD software.

2.1. Gear Calculation formulae used in this paper:

1. Circular pitch, $p = \pi \cdot d'/N_P$
2. Diametral pitch, $P = N_P/ d'$
3. Module, $m = 1.26 \times \sqrt[3]{\frac{T}{y[\sigma b]\ \psi m\ N_P}}$
4. Tip circle diameter, $d_a = d'+2\cdot m$
5. Root circle diameter, $d_t = d'-(2\times 1.25\cdot m)$
6. Base circle diameter, $d_b = d' \times \cos\phi$
7. Addendum, $h_a = m$
8. Dedendum, $h_f = 1.157\cdot m$
9. Clearance, $c = 0.157\cdot m$
10. Fillet radius, $r = 0.4\times m$

11. Face width, B = $9.5\times m$
12. Working depth, $h_k = 2\cdot m$
13. Whole depth, $h_t = 2.157\cdot m$
14. Tooth thickness on pitch circle, $s = (\pi\times m)/2$
15. Addendum circle radius, r_A=Pitch circle radius× Addendum
16. Angular velocity of pinion, $\omega_1 = (2\pi n_p)/60$
17. Pitch line velocity, $v = \omega_1\times$ Pitch circle radius

From the Appendix of the Handbook of Gear Design [5], the standard dimensions for the symmetric spur gears are taken, as follows:

For Pinion, shaft diameter = 10mm.

For Gear/Wheel, shaft diameter = 10mm

2.2 Material Selection for the gear and the pinion

The materials for the pinion and the gear are chosen as ASTM Class 35 Cast Iron and ASTM 1045 or well known as C45 steel respectively. The reason behind choosing of these materials are, firstly, that these two are standard materials used for the gear pair and secondly, their availability. Also, the machining easiness of these materials lead to choosing of the materials. The mechanical properties of these materials are listed.

2.2.1. ASTM Class 35 Cast Iron

1. Young's Modulus=1.14×10^5MPa
2. Density= 7150 kg/m^3
3. UTS= 252 MPa
4. Poisson's ratio= 0.29
5. Yield Strength= 165 MPa
6. HB= 200 to 260

2.2.2. ASTM 1045 or C45 steel

1. Young's Modulus=2×10^5MPa
2. Density= 7870kg/m^3
3. UTS= 565 MPa
4. Poisson's ratio= 0.29
5. Yield Strength= 310 MPa
6. HB= 175-215

2.3 The calculations for the other gear pairs are the same except the base circle diameters.

Table 2.1. Calculated base circle diameters for 20°, 25°, and 30° involute profile gear pair

Profile		20°	25°	30°
Base Circle Diameters	Pinion	52.63 mm	50.75 mm	48.49 mm
	Gear	63.89 mm	61.63 mm	58.89 m

2.4 Meshing of Gears

Meshing is a very important criteria in solving Finite Element Analysis (FEA) problems. The basic theme of Finite Element Method is to make calculations at only finite number of points and interpolating the results of entire surface or volume of a body or an object.

The gear pairs are meshed with hexa-hedron meshing, which is commonly known as Hex-Dominant mesh type or Brick type meshing in FEA. This type of meshing has the advantage that they can be highly controlled and also can be generated automatically for optimal solution efficiency and accuracy [4].

2.5 Contact criteria for the contact stress in gears

For finding the contact stress in gears, Multi-point constraints (MPC) linear contact formulation is being used. Bonded (pure penalty) contact is a linear form of contact based connection. This contact connection between two bodies or surfaces must have contact elements on one body and target elements on other.

III REULTS AND DISCUSSIONS

The 3D models of different symmetric spur gear assemblies were modeled in CAD software with the dimensions mentioned in the precious chapter.

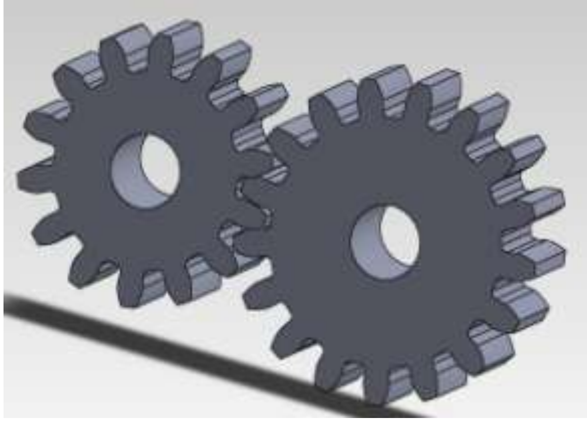


Fig.1. 14.5° Involute Teeth Profile

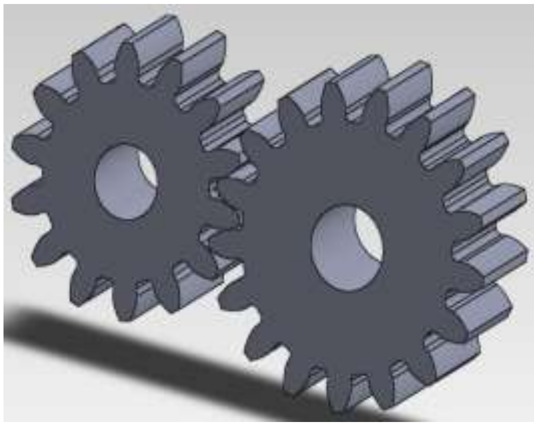


Fig.2. 20° Involute Teeth Profile

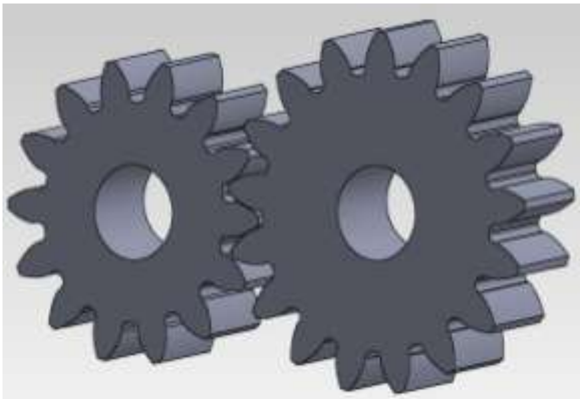


Fig. 3. 25° Involute Teeth Profile

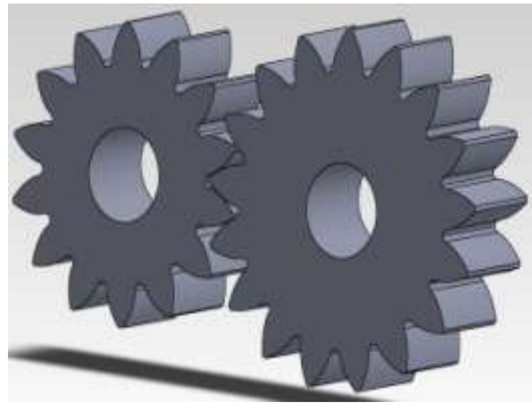


Fig.4. 30° Involute Teeth Profile

The four types of the spur gear assembly with different pressure angles (14.5°, 20°, 25° and 30°) were then analyzed using the finite element method based finite element software, in order to get the bending stress on the pinion tooth fillet and contact stress on the area where the two teeth of the gear pair are in contact with eachother.

3.1 Bending Stress analyses

The bending stress and maximum deformation of 14.5° involute profiled spur gear is shown.

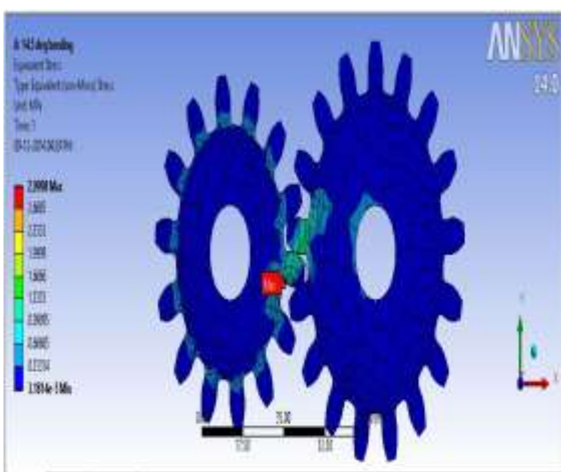


Fig. 5. Maximum Bending stress

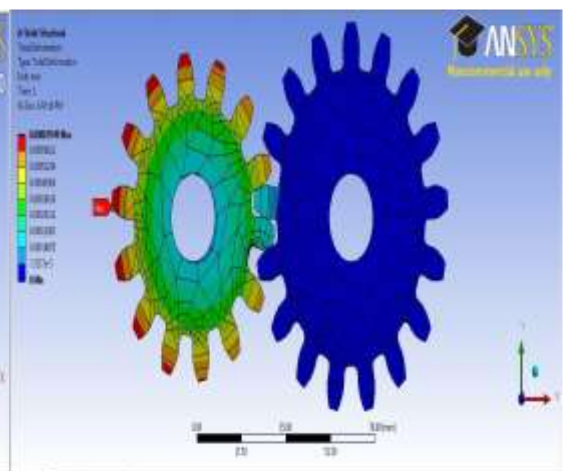


Fig. 6. Maximum Deformation

The bending stress and maximum deformation of 20° involute profiled spur gear is shown

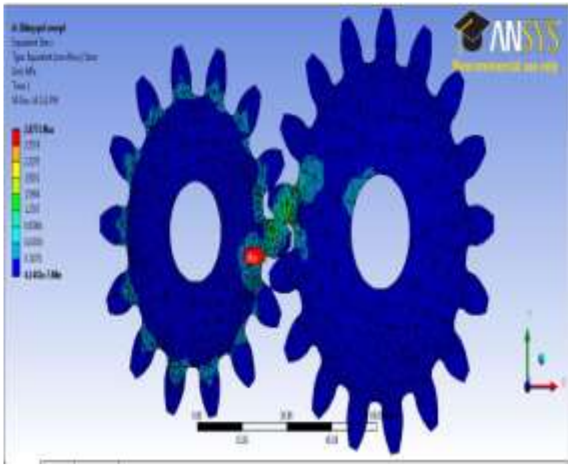


Fig. 7. Maximum Bending stress

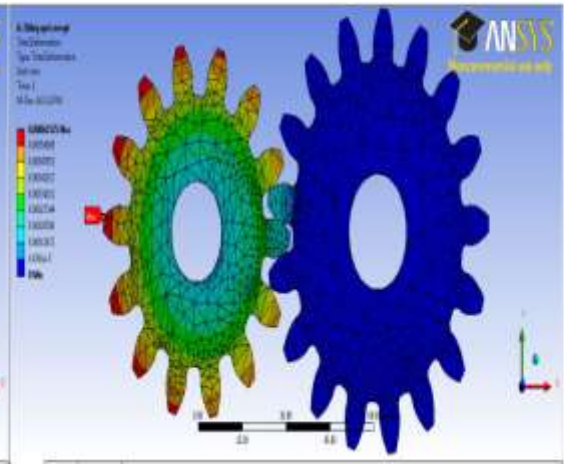


Fig. 8. Maximum Deformation

The bending stress and maximum deformation of 25° involute profiled spur gear is shown

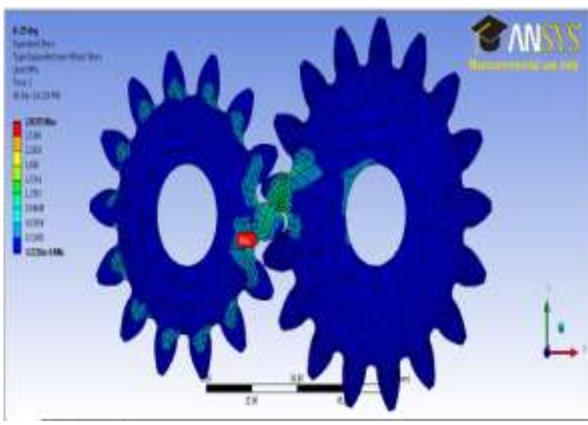


Fig. 9. Maximum Bending stress

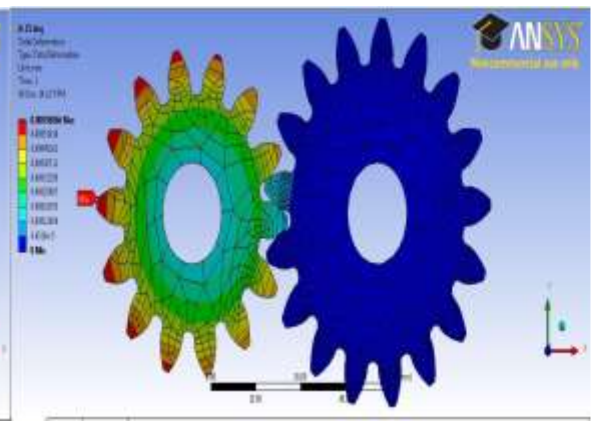


Fig. 10. Maximum Deformation

The bending stress and maximum deformation of 30° involute profiled spur gear is shown

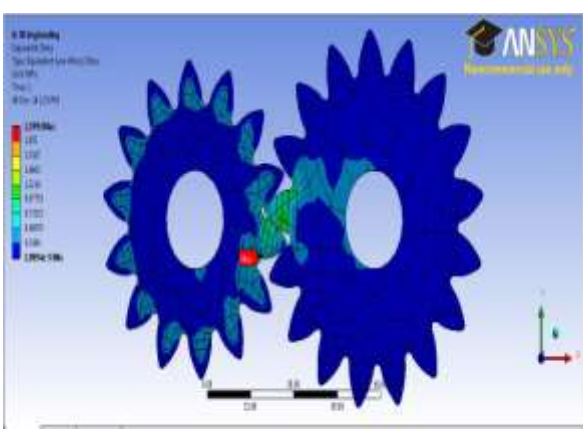


Fig. 11. Maximum Bending stress

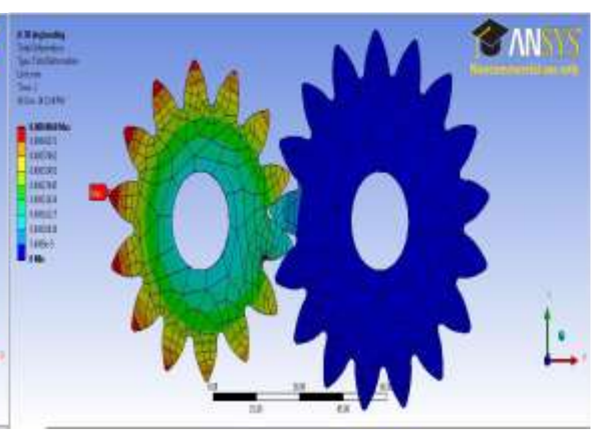


Fig. 12. Maximum Deformation

3.2 Contact Stress analyses

The Contact Stresses on the mating tooth surface of pinion and gear for the different pressure angle values of symmetric gear pair arrangement.

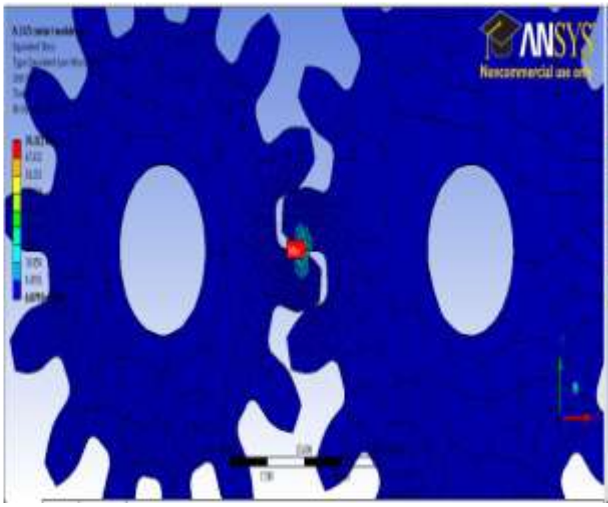
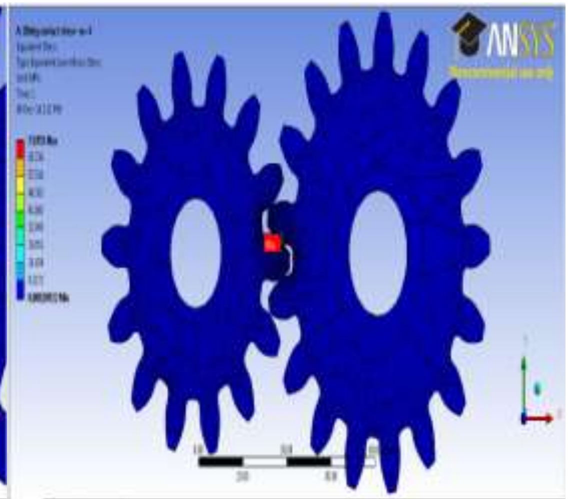



Fig. 13. Contact stress (14.5° gear assembly) **Fig. 14. Contact stress (20° gear assembly)**

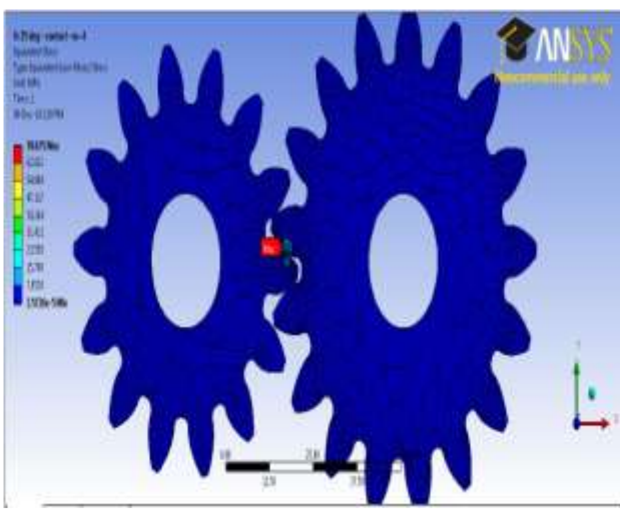
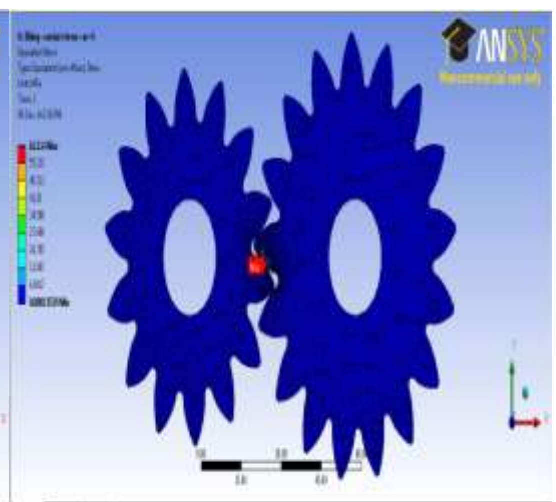



Fig. 15. Contact stress (25° gear assembly) **Fig. 16. Contact stress (30° gear assembly)**

Table 3.1.Comparison of stresses on pressure angles

Pressure angles (in degrees)	Bending Stress (in MPa)	Contact Stress (in MPa)
14.5	2.9998	76.311
20	2.8771	73.953

25	2.8335	70.675
30	2.1994	62.114

3.3 Theoretical formulae for the stresses

1. LEWI'S BENDING STRESS EQUATION:

$$\sigma_t = \frac{F_t \times P_d}{B \times y}$$

where,

σ_t – Max. Bending stress

F_t - Tangential load on gear tooth

P_d - Diametral pitch

B- Face width

y- Lewis form factor

Using the equations, **σ_t= 2.7224 N/mm^2**

2. AGMA CONTACT STRESS EQUATION:

$$\sigma_H = C_p \sqrt{\frac{F_t}{B \times d_1 \times I} k_v . k_o . k_m}$$

where,

σ_H – Contact stress

C_p – Elastic coefficient = $0.564 \times \sqrt{\frac{1}{\frac{1-v_1^2}{E_1} \times \frac{1-v_2^2}{E_2}}}$

I – Geometry factor = $\frac{\sin\Phi \cos\Phi}{2} \times \frac{i}{i+1}$

k_v, k_o, k_m – Velocity factor, Overload factor, and Load distribution factor respectively

$d_1 = \frac{d_1'}{\sin\Phi}$;

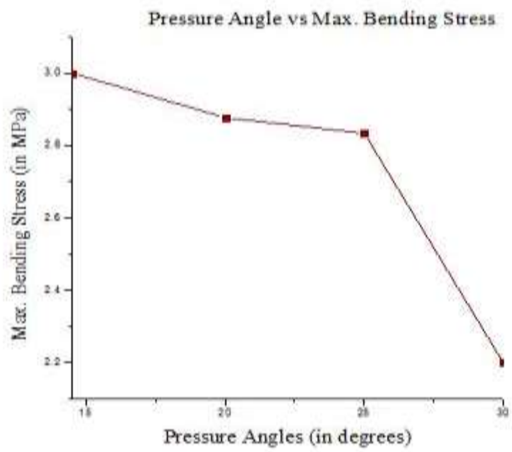
k_v=1,

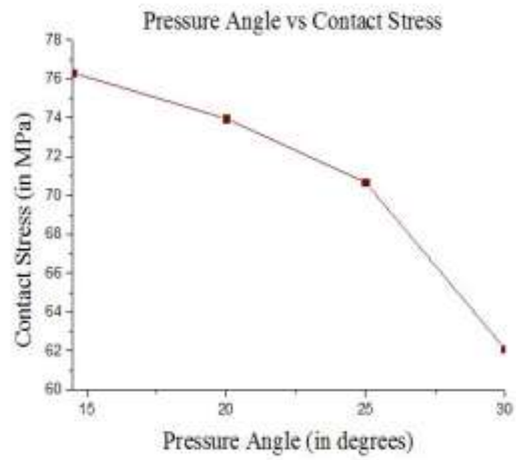
k_o=1(for uniform load),

k_m= 1.3

Using the equations, $\boldsymbol{\sigma_H}$**= 76.11 N/mm^2**

3.4 Graphs plotter between the bending stress and pressure angle and also between contact stress and pressure angle.





Both the graphs above shows that, with the increase in the pressure angles, making the profile different, decreases or reduces the bending stress and also the contact stress.

IV CONCLUSION

In this paper, the equivalent stresses and strains of the symmetric involute spur gears were studied through finite element meshing simulation for finding out the gear pair with least stress when the pressure angles are increased from 14.5° to 30°.This paper finds a comprehensive study on the variation of static stresses with four different pressure angles that might be developed in spur gear teeth (while they are in contact) using a commonly used finite element based software package. As a case study, four different pressure angles of 14.5°, 20°, 25° and 30° were considered. Available fundamental equations along with classical standard (AGMA) were also consulted while carrying out this preliminary analysis.

GEAR NOMENCLATURE

N_P and N_G	Number of teeth on Pinion and Gear
m	Module
p	Circular pitch
P	Diametral pitch
d′	Pitch circle diameter
d_a	Tip circle diameter

d_b	Base circle diameter
d_t	Root circle diameter
ϕ	Pressure Angle
h_a	Addendum
h_f	Dedendum
c	Clearance
h_t	Whole depth
h_k	Working depth
s	Tooth thickness on pitch circle
r_A	Addendum circle radius
ω_1	Angular velocity of pinion
v	Pitch line velocity
α	Half angle of tooth
σ	Bending Stress on tooth fillet
T	Torque transmitted
B	Face width
y	Lewis form factor

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