FUZZY γ-CONTINUOUS MULTIFUNCTION

Anjana Bhattacharyya[1]

Department of Mathematics, Victoria Institution (College)

78 B, A.P.C. Road, Kolkata – 700009(India)

ABSTRACT

This paper deals with a new type of fuzzy multifunction, viz., fuzzy upper (lower) γ-continuous multifunction. Several characterizations and properties of this newly defined multifunction have been studied here. Also the mutual relationships of this fuzzy multifunction with fuzzy upper (lower) δ-precontinuous multifunctions [8] have been established here.

Keywords: Fuzzy nbd of a fuzzy set, γ-nbd of a subset A of a topological space, γ-frontier of a subset A of a topological space, fuzzy compact space.

2000 Ams Subject Classification Code Primary 54a40, Secondary 54c99

I INTRODUCTION

A fuzzy multifunction, introduced by Papageorgiou [21] in 1985 is a function from an ordinary topological space X to a fuzzy topological space (fts, for short) Y in the sense of Chang [10]. In this paper we use the definition of upper inverse of fuzzy multifunction given by Papageorgiou [21] and the lower inverse of fuzzy multifunction given by Mukherjee and Malakar [18].

Throughout this paper, (X, τ) or simply X will stand for an ordinary topological space, while by (Y, τ_Y) or simply by Y will always be denoted a fuzzy topological space. A fuzzy set A [26] in Y is a function from Y into the unit interval $I = [0, 1]$ i.e., $A \in I^Y$. The support [26] of a fuzzy set A in Y will be denoted by $suppA$ and is defined by $suppA = \{y \in Y : A(y) \neq 0\}$. The fuzzy point [22] with the singleton support $y \in Y$ and the value t $(0 < t \leq 1)$ at y will be denoted by y_t. For a set A in X (resp., a fuzzy set A in Y), clA and $intA$ will respectively stand for closure and interior of A in X (resp., fuzzy closure and fuzzy interior of A in Y [10]). For a fuzzy set A in Y, the complement of A in Y will be denoted by $1_Y \setminus A$ and is defined by $(1_Y \setminus A)(y) = 1 - A(y)$, for each $y \in Y$ [26]. 0_Y and 1_Y will respectively stand for the constant fuzzy sets taking values 0 and 1 on Y. For two fuzzy sets A, B in Y, we write

[1]The author acknowledges the financial support from UGC (Minor Research Project), New Delhi

$A \leq B$ [26] iff $A(y) \leq B(y)$, for all $y \in Y$ while AqB means A is quasi-coincident [22] (q-coincident, for short) with B if there is some $y \in Y$ such that $A(y) + B(y) > 1$. The negation of the last two statements are written as $A \nleq B$ and $A\bar{q}B$ respectively. A fuzzy set A in Y is called a fuzzy neighbourhood (fuzzy nbd, for short) [22] of a fuzzy set B in Y if there is a fuzzy open set U in Y such that $B \leq U \leq A$.

A subset A of X (resp., a fuzzy set A in Y) is called semiopen[15], α-open [20], β-open [1], regular open [24], preopen [16], γ-open [11] (resp., fuzzy semiopen [4], fuzzy α-open [9], fuzzy β-open [12], fuzzy regular open [4], fuzzy preopen [19], fuzzy γ-open) respectively if $A \subseteq ci(intA)$, $A \subseteq int(cl(intA))$, $A \subseteq cl(int(clA))$, $A = int(clA)$, $A \subseteq int(clA)$, $A \subseteq \left(cl(intA)\right) \cup (int(clA))$ (resp., $A \leq cl(intA)$, $A \leq int(cl(intA))$, $A \leq cl(int(clA))$, $A = int(clA)$, $A \leq int(clA)$, $A \leq \left(cl(intA)\right) \cup (int(clA))$) respectively. The complements of above mentioned sets in X (resp., in Y) are called semiclosed [15], α-closed [20], β-closed [1], preclosed [16], regular closed [24], γ-closed [11] (resp., fuzzy semiclosed [4], fuzzy α-closed [9], fuzzy β-closed [12], fuzzy regular closed [4], fuzzy preclosed [19], fuzzy γ-closed) respectively. The intersection of all semiclosed, α-closed, β-closed, preclosed, γ-closed sets in X (resp., fuzzy sets in Y) containing A are called semiclosure [15], α-closure [20], β-closure [1], preclosure [16], γ-closure [11] (resp., fuzzy semiclosure [4], fuzzy α-closure [9], fuzzy β-closure [12], fuzzy preclosure [19], fuzzy γ-closure) respectively and are denoted by $sclA$, αclA, βclA, $pclA$, γclA respectively. The δ-interior [25] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $\delta intA$. A subset A of X is called δ-open if $A = \delta intA$ [25]. The complement of a δ-open set in X is called δ-closed. For a set A in X, $\delta clA = \{x \in X : A \cap \left(int(clU)\right) \neq \emptyset, U \in \tau, x \in U\}$. A subset A of X is called δ-preopen [2] if $A \leq int(\delta clA)$. A fuzzy set B is called a quasi-neighbourhood (q-nbd, for short) [21] of a fuzzy set A if there is a fuzzy open set U in Y such that $AqU \leq B$. If, in addition, B is fuzzy regular open, then B is called fuzzy regular open q-nbd of A. A fuzzy point x_α is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts Y if every fuzzy regular open q-nbd U of x_α is q-coincident with A [13]. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A and is denoted by δclA [13]. A fuzzy set A in an fts Y is called fuzzy δ-preopen [5] if $A \leq int(\delta clA)$. The complement of a fuzzy δ-preopen set is called fuzzy δ-preclosed [5]. The intersection of all fuzzy δ-preclosed sets containing a fuzzy set A in an fts Y is called fuzzy δ-preclosure of A and is denoted by $\delta pclA$ [5]. A fuzzy set A in Y is δ-preclosed iff $A = \delta pclA$ [5].

The set of all semiopen, α-open, β-open, preopen, γ-open, δ-preopen sets in X are denoted by $SO(X)$, $\alpha O(X)$, $\beta O(X)$, $PO(X)$, $\gamma O(X)$, $\delta PO(X)$ respectively. A semiopen (resp., α-open, β-open, preopen, γ-open, δ-preopen) set U in X containing a point $x \in X$ will be denoted by $U \in SO(X,x)$(resp., $U \in \alpha O(X,x)$, $U \in \beta O(X,x)$, $U \in PO(X,x)$, $U \in \gamma O(X,x)$, $U \in \delta PO(X,x)$). The union of all γ-open sets contained in A is called γ-interior of A [11], to be denoted by $\gamma int A$. A subset U of a topological space X is called a γ-nbd of a point $x \in X$ [11] if there exists $V \in \gamma O(X, x)$ such that $V \subseteq U$.

II SOME WELL KNOWN DEFINITIONS, THEOREM AND LEMMAS

In this section we recall, some definitions, lemmas and theorem for ready references.

Definition 2.1 [21]. Let (X, τ) be an ordinary topological space and (Y, τ_Y) be an fts. We say that $F : X \to Y$ is a fuzzy multifunction if corresponding to each $x \in X$, $F(x)$ is a unique fuzzy set in Y.

Henceforth by $F : X \to Y$ we shall mean a fuzzy multifunction in the above sense.

Definition 2.2 [21, 18]. For a fuzzy multifunction $F : X \to Y$, the upper inverse F^+ and the lower inverse F^- are defined as follows :

for any fuzzy set A in Y, $F^+(A) = \{x \in X : F(x) \leq A\}$, $F^-(A) = \{x \in X : F(x)\, q A\}$.

Theorem 2.3 [18]. For a fuzzy multifunction $F : X \to Y$, we have $F^-(1_Y \setminus A) = X \setminus F^+(A)$, for any fuzzy set A in Y.

Definition 2.4 [10]. Let A be a fuzzy set in an fts Y. A collection $\mathscr{U}$ of fuzzy sets in Y is called a fuzzy cover of A if $\sup\{U(x) : U \in \mathscr{U}\} = 1$ for each $x \in supp A$. If, in addition, the members of $\mathscr{U}$ are fuzzy open, then $\mathscr{U}$ is called a fuzzy open cover of A. In particular, if $A = 1_Y$, we get the definition of fuzzy cover (resp., fuzzy open cover) of the fts Y.

Definition 2.5 [14]. A fuzzy cover $\mathscr{U}$ of a fuzzy set A in an fts Y is said to have a finite subcover $\mathscr{U}_0$ if $\mathscr{U}_0$ is a finite subcollection of $\mathscr{U}$ such that $\cup \mathscr{U}_0 \geq A$. Clearly, if $A = 1_Y$, in particular, then the requirements on $\mathscr{U}_0$ is $\cup \mathscr{U}_0 = 1_Y$.

Definition 2.6 [10]. An fts Y is said to be fuzzy compact if every fuzzy open cover of Y has a finite subcover.

Definition 2.7 [18]. A fuzzy multifunction $F : X \to Y$ is said to be fuzzy upper (lower) semi-continuous if $F^+(V)$ (resp., $F^-(V)$) is open in X for every fuzzy open set V in Y.

Definition 2.8 [8]. A fuzzy multifunction $F : X \to Y$ is said to be fuzzy

(i) upper δ-precontinuous (resp., upper quasi continuous) if for each $x \in X$ and each fuzzy open set V in Y with $F(x) \leq V$, there exists $U \in \delta PO(X, x)$ (resp., $U \in SO(X, x)$) such that $F(U) \leq V$,

(ii) lower δ-precontinuous (resp., lower quasi continuous) if for each $x \in X$ and each fuzzy open set V in Y with $F(x)\, qV$, there exists $U \in \delta PO(X, x)$ (resp., $U \in SO(X, x)$) such that $F(u)\, qV$, for all $u \in U$,

(iii) upper (lower) δ-precontinuous (resp., upper (lower) quasi continuous) if it has this property at each point of X.

Lemma 2.9 [3, 11]. Let A and X_0 be subsets of a topological space (X,τ). If $A \in \gamma O(X)$ and $X_0 \in \alpha O(X)$, then $A \cap X_0 \in \gamma O(X_0)$.

Lemma 2.10 [11]. Let $A \subseteq X_0 \subseteq X, X_0 \in \gamma O(X)$ and $A \in \gamma O(X_0)$, then $A \in \gamma O(X)$.

III FUZZY UPPER AND LOWER γ-CONTINUOUS MULTIFUNCTIONS : SOME CHARACTERIZATIONS

Now we define fuzzy upper (lower) γ-continuous multifunction and characterize these fuzzy multifunctions in several ways.

Definition 3.1. A fuzzy multifunction $F : X \to Y$ is said to be fuzzy

(i) upper γ-continuous at a point $x \in X$ if for each fuzzy open set V in Y with $F(x) \leq V$, there exists $U \in \gamma O(X,x)$ such that $F(U) \leq V$,

(ii) lower γ-continuous at a point $x \in X$ if for each fuzzy open set V of Y with $F(x)qV$, there exists $U \in \gamma O(X,x)$ such that $F(u)\,qV$, for all $u \in U$,

(iii) upper (lower) γ-continuous if F has this property at each point of X.

Theorem 3.2. For a fuzzy multifunction $F : X \to Y$, the following are equivalent :

(a) F is fuzzy upper γ-continuous,

(b) $F^+(V) \in \gamma O(X)$ for any fuzzy open set V of Y,

(c) $F^-(V)$ is γ-closed in X for any fuzzy closed set V of Y,

(d) $\gamma cl(F^-(B)) \subseteq F^-(clB)$, for any $B \in I^Y$,

(e) for each point $x \in X$ and each fuzzy nbd V of $F(x)$, $F^+(V)$ is a γ-nbd of x,

(f) for each point $x \in X$ and each fuzzy nbd V of $F(x)$, there exists a γ-nbd U of x such that $F(U) \leq V$,

(g) $(cl(int(F^-(B)))) \cap \left(int\left(cl(F^-(B))\right)\right) \subseteq F^-(clB)$, for any $B \in I^Y$,

(h) $F^+(intB) \subseteq \left(int\left(cl(F^+(B))\right)\right) \cup (cl\left(int(F^+(B))\right))$, for any $B \in I^Y$.

Proof (a) ⇒ (b): Let V be a fuzzy open set of Y and $x \in F^+(V)$. Then by (a), there exists $U \in \gamma O(X,x)$ such that $F(U) \leq V$. Therefore, we obtain $x \in U \subseteq (cl(intU)) \cup (int(clU)) \subseteq$

$\left(cl\left(int\left(F^+(V)\right)\right)\right) \cup (int\left(cl\left(F^+(V)\right)\right))$ and so we have $F^+(V) \subseteq \left(cl\left(int\left(F^+(V)\right)\right)\right) \cup (int\left(cl\left(F^+(V)\right)\right)) \Rightarrow F^+(V) \in \gamma O(X)$.

(b) ⇒ (c) : It follows from the fact that $F^+(1_Y \backslash B) = X \backslash F^-(B)$, for any $B \in I^Y$.

(c) ⇒ (d) : Let $B \in I^Y$. Then clB is fuzzy closed in Y and so by (c), $F^-(clB)$ is γ-closed in X and so

$\gamma cl(F^-(clB)) \subseteq F^-(clB) \Rightarrow \gamma cl\left(F^-(B)\right) \subseteq \gamma cl\left(F^-(clB)\right) \subseteq F^-(clB)$.

(d) ⇒ (c) : Let V be a fuzzy closed set of Y. Then $clV = V$ and so by (d),
$\gamma cl\left(F^-(V)\right) \subseteq F^-(clV) = F^-(V) \Rightarrow F^-(V)$ is γ-closed in X.

(b) ⇒ (e) : Let $x \in X$ and V be a fuzzy nbd of $F(x)$. Then there exists a fuzzy open set G of Y such that $F(x) \leq G \leq V \Rightarrow x \in F^+(G) \subseteq F^+(V)$. Since $F^+(G) \in \gamma O(X)$ (by (b)), $F^+(V)$ is a γ-nbd of x.

(e) ⇒ (f) : Let $x \in X$ and V be a fuzzy nbd of $F(x)$. Put $U = F^+(V)$. By (e), U is a γ-nbd of x and $F(U) \leq V$.

(f) ⇒ (a) : Let $x \in X$ and V be a fuzzy open set such that $F(x) \leq V$. Then V is a fuzzy nbd of $F(x)$. By (f), there exists a γ-nbd U of x such that $F(U) \leq V$. Therefore, there exists $W \in \gamma O(X)$ such that $x \in W \subseteq U$ and so $F(W) \leq F(U) \leq V \Rightarrow F(W) \leq V$.

(c) ⇒ (g) : Let $B \in I^Y$. Then clB is fuzzy closed in Y and so by (c), $F^-(clB)$ is γ-closed in
$X \Rightarrow F^-(clB) \supseteq (int\left(cl\left(F^-(clB)\right)\right)) \cap (cl\left(int\left(F^-(clB)\right)\right)) \supseteq (int\left(cl\left(F^-(B)\right)\right)) \cap (cl\left(int\left(F^-(B)\right)\right))$.

(g) ⇒ (h) : Let $B \in I^Y$. Then $1_Y \backslash B \in I^Y$. By (g),
$F^-(cl(1_Y \backslash B)) \supseteq (cl\left(int\left(F^-(1_Y \backslash B)\right)\right)) \cap (int\left(cl\left(F^-(1_Y \backslash B)\right)\right)) \Rightarrow F^-(1_Y \backslash intB) \supseteq (cl\left(int\left(X \backslash F^+(B)\right)\right)) \cap (int\left(cl\left(X \backslash F^+(B)\right)\right))$

$\Rightarrow X \backslash F^+(intB) \supseteq \left(X \backslash int\left(cl\left(F^+(B)\right)\right)\right) \cap \left(X \backslash cl\left(int\left(F^+(B)\right)\right)\right) = X \backslash ((int\left(cl\left(F^+(B)\right)\right)) \cup (cl\left(int\left(F^+(B)\right)\right)))$

$\Rightarrow F^+(intB) \subseteq (int\left(cl\left(F^+(B)\right)\right)) \cup (cl\left(int\left(F^+(B)\right)\right))$.

(h) ⇒ (b) : Let V be a fuzzy open set of Y. By (h), $F^+(intV) = F^+(V) \subseteq (int\left(cl\left(F^+(V)\right)\right)) \cup \left(cl\left(int\left(F^+(V)\right)\right)\right)$

$\Rightarrow F^+(V) \in \gamma O(X)$.

Theorem 3.3. For a fuzzy multifunction $F : X \to Y$, the following are equivalent :

(a) F is fuzzy lower γ-continuous,

(b) $F^-(V) \in \gamma O(X)$, for any fuzzy open set V of Y,

(c) $F^+(V)$ is γ-closed in X for any fuzzy closed set V of Y,

(d) $\gamma cl(F^+(B)) \subseteq F^+(clB)$, for any $B \in I^Y$,

(e) $F(\gamma clA) \leq clF(A)$, for any subset A of X,

(f) $(cl(int(F^+(B))) \cap \left(int\left(cl(F^+(B))\right)\right) \subseteq F^+(clB)$, for any $B \in I^Y$,

(g) $F^-(intB) \subseteq \left(int\left(cl(F^-(B))\right)\right) \cup (cl\left(int(F^-(B))\right))$, for any $B \in I^Y$,

(h) for each $x \in X$ and each fuzzy q-nbd V of $F(x)$, $F^-(V)$ is a γ-nbd of x,

(i) for each $x \in X$ and each fuzzy q-nbd V of $F(x)$, there exists a γ-nbd U of x such that

$F(u)\, qV$, for all $u \in U$.

Proof (a) ⇒ (b): Let V be a fuzzy open set of Y and $x \in F^-(V)$. Then by (a), there exists $U \in \gamma O(X, x)$ such that $F(u)\, qV$, for all $u \in U \Rightarrow U \subseteq F^-(V)$. Therefore, we obtain

$x \in U \subseteq (cl(intU)) \cup (int(clU)) \subseteq \left(cl\left(int(F^-(V))\right)\right) \cup (int\left(cl(F^-(V))\right))$ and so we have

$F^-(V) \subseteq \left(cl\left(int(F^-(V))\right)\right) \cup (int\left(cl(F^-(V))\right)) \Rightarrow F^-(V) \in \gamma O(X)$.

(b) ⇒ (c) : It follows from the fact that $F^+(1_Y \backslash B) = X \backslash F^-(B)$, for any $B \in I^Y$.

(c) ⇒ (d) : Let $B \in I^Y$. Then clB is fuzzy closed in Y. By (c), $F^+(clB)$ is γ-closed in X ⇒
$\gamma cl\left(F^+(B)\right) \subseteq \gamma cl\left(F^+(clB)\right) \subseteq F^+(clB)$.

(d) ⇒ (c) : Let V be a fuzzy closed set of Y. Then $clV = V$. By (d),
$\gamma cl\left(F^+(V)\right) = \gamma cl(F^+(clV)) \subseteq F^+(clV) = F^+(V) \Longrightarrow F^+(V)$ is γ-closed in X.

(c) ⇒ (e) : Let A be a subset of X. Then $clF(A)$ is fuzzy closed in Y. By (c), $F^+(clF(A))$ is γ-closed in X ⇒
$\gamma cl(F^+\left(clF(A)\right)) \subseteq F^+(clF(A)) \Rightarrow F(\gamma cl\left(F^+\left(clF(A)\right)\right)) \leq clF(A) \Rightarrow$
$clF(A) \geq F(\gamma cl\left(F^+\left(F(A)\right)\right)) \geq F(\gamma clA)$.

(e) $\Rightarrow$ (d) : Let $B \in I^Y$. Then $F^+(B) \subseteq X$. By (e), $F(\gamma cl(F^+(B))) \leq clF(F^+(B)) \leq clB \Rightarrow \gamma cl(F^+(B)) \subseteq F^+(clB)$.

(c) $\Rightarrow$ (f) : Let $B \in I^Y$. Then clB is fuzzy closed in Y. By (c), $F^+(clB)$ is γ-closed in $X \Rightarrow$
$F^+(clB) \supseteq \left(int\left(cl(F^+(clB))\right)\right) \cap \left(cl\left(int(F^+(clB))\right)\right) \supseteq int(cl(F^+(B))) \cap cl(int(F^+(B)))$.

(f) $\Rightarrow$ (g) : Let $B \in I^Y$. Then $1_Y \backslash B \in I^Y$. By (f),
$F^+(cl(1_Y\backslash B)) \supseteq cl\left(int(F^+(1_Y\backslash B))\right) \cap int\left(cl(F^+(1_Y\backslash B))\right) \Rightarrow F^+(1_Y\backslash intB) \supseteq cl\left(int(X\backslash F^-(B))\right) \cap$
$int\left(cl(X\backslash F^-(B))\right)$
$\Rightarrow X\backslash F^-(intB) \supseteq \left(X\backslash int\left(cl(F^-(B))\right)\right) \cap \left(X\backslash cl\left(int(F^-(B))\right)\right) =$
$X\backslash((int\left(cl(F^-(B))\right)) \cup (cl\left(int(F^-(B))\right))) \Rightarrow F^-(intB) \subseteq (int\left(cl(F^-(B))\right)) \cup (cl\left(int(F^-(B))\right))$.

(g) $\Rightarrow$ (b) : Let V be a fuzzy open set of Y. By (g), $F^+(intV) = F^-(V) \subseteq (int\left(cl(F^-(V))\right)) \cup \left(cl\left(int(F^-(V))\right)\right)$

$\Rightarrow F^-(V) \in \gamma O(X)$.

(b) $\Rightarrow$ (h) : Let Let $x \in X$ and V be a fuzzy q-nbd of $F(x)$. Then there exists a fuzzy open set G of Y such that $F(x)qG \leq V \Rightarrow x \in F^-(G) \subseteq F^-(V)$. Since $F^-(G) \in \gamma O(X)$ (by (b)), $F^-(V)$ is a γ-nbd of x.

(h) $\Rightarrow$ (i) : Let $x \in X$ and V be a fuzzy q-nbd of $F(x)$. Put $U = F^-(V)$. By (h), U is a γ-nbd of x and $F(u)qV$, for all $u \in U$.

(i) $\Rightarrow$ (a) : Let $x \in X$ and V be a fuzzy open set such that $F(x)qV$. Then V is a fuzzy q-nbd of $F(x)$. By (i), there exists a γ-nbd U of x such that $F(u)qV$, for all $u \in U \Rightarrow U \subseteq F^-(V)$. Therefore, there exists $W \in \gamma O(X)$ such that $x \in W \subseteq U$ and so $W \subseteq F^-(V) \Rightarrow F(w)qV$, for all $w \in W$.

Definition 3.4 [18]. For a fuzzy multifunction $F : X \to Y$, the fuzzy graph multifunction $G_F : X \to X \times Y$ of F is defined as $G_F(x)$ = the fuzzy set $x_1 \times F(x)$ of $X \times Y$, where x_1 is the fuzzy set in X, whose value is 1 at $x \in X$ and 0 other points of X. We shall write $\{x\} \times F(x)$ for $x_1 \times F(x)$.

Lemma 3.5 [6]. The following hold for a fuzzy multifunction $F : X \to Y$:

(a) $(G_F)^+(A \times B) = A \cap F^+(B)$ and
(b) $(G_F)^-(A \times B) = A \cap F^-(B)$ for every subset A of X and every fuzzy set B of Y.

Theorem 3.6. A fuzzy multifunction $F : X \to Y$ is fuzzy lower γ-continuous iff $G_F : X \to X \times Y$ is so.

Proof. Let F be fuzzy lower γ-continuous. Let $x \in X$ and W be any fuzzy open set of $X \times Y$ such that $G_F(x)qW$. Then there exists $(x,y) \in X \times Y$ such that $[\{x\} \times F(x)](x,y) + W(x,y) > 1$. Then $[F(x)](y) + W(x,y) > 1$. Let $[F(x)](y) = \alpha$. Then $W(x,y) > 1 - \alpha \Rightarrow (x,y)_\alpha qW$ so that $(x,y)_\alpha q(U \times V) \leq W$ for some open set U in X and some fuzzy open set V in Y with $y_\alpha qV$. Now $V(y) + \alpha > 1 \Rightarrow V(y) + [F(x)](y) > 1 \Rightarrow VqF(x)$. Since F is fuzzy lower γ-continuous, there exists $G \in \gamma O(X,x)$ such that $F(g)qV$, for all $g \in G \Rightarrow G \subseteq F^-(V)$. Then by Lemma 2.5(b), $U \cap G \subseteq U \cap F^-(V) = (G_F)^-(U \times V) \subseteq (G_F)^-(W)$. Moreover, $x \in U \cap G \in \gamma O(X)$ and hence the proof.

Conversely, let G_F be fuzzy lower γ-continuous. Let $x \in X$ and V be any fuzzy open set in Y such that $F(x)qV$. Then there exists $y \in Y$ such that $[F(x)](y) + V(y) > 1$. Now $1_X \times V$ is open in $X \times Y$ such that $\{x \times F(x)\}(x,y) + (1_X \times V)(x,y) > 1$ and so $G_F(x)q(1_X \times V)$. Since G_F is fuzzy lower γ-continuous, there exists $U \in \gamma O(X,x)$ such that $G_F(u)q(1_X \times V)$, for all $u \in U$. By Lemma 2.5(b), we obtain $U \subseteq (G_F)^-(1_X \times V) = 1_X \cap F^-(V) = F^-(V)$. Hence $F(u)qV$, for all $u \in U \Rightarrow F$ is fuzzy lower γ-continuous.

Theorem 3.7. Let $\{U_\alpha : \alpha \in \Lambda\}$ be an α-open cover of a topological space X. A fuzzy multifunction $F : X \to Y$ is fuzzy upper γ-continuous iff the restriction $F/_{U_\alpha} : U_\alpha \to Y$ is fuzzy upper γ-continuous.

Proof. Let $x \in X$. Then there exists $\alpha \in \Lambda$ such that $x \in U_\alpha$. Let V be a fuzzy open set of Y such that $\left(F/_{U_\alpha}\right)(x) = F(x) \leq V$. Since F is fuzzy upper γ-continuous, there exists $G \in \gamma O(X,x)$ such that $F(G) \leq V$. Put $U = G \cap U_\alpha$. Then by Lemma 1.9, $U \in \gamma(U_\alpha, x)$ and $\left(F/_{U_\alpha}\right)(U) = F(U) \leq V$. Therefore, $F/_{U_\alpha}$ is fuzzy upper γ-continuous.

Conversely, let $x \in X$ and V, a fuzzy open set of Y such that $F(x) \leq V$. Since $\{U_\alpha : \alpha \in \Lambda\}$ is an α-open cover of X, there exists $\beta \in \Lambda$ such that $x \in U_\beta$. Since $F/_{U_\beta}$ is fuzzy upper γ-continuous and $\left(F/_{U_\beta}\right)(x) = F(x) \leq V$, there exists $U \in \gamma O(U_\beta, x)$ such that $\left(F/_{U_\beta}\right)(U) = F(U) \leq V$. Then by Lemma 1.10 (as α-open sets are γ-open), $U \in \gamma O(X,x)$ and $F(U) = (F/_{U_\beta})(U) \leq V \Rightarrow F$ is fuzzy upper γ-continuous.

Theorem 3.8. Let $\{U_\alpha : \alpha \in \Lambda\}$ be an α-open cover of a topological space X. A fuzzy multifunction $F : X \to Y$ is fuzzy lower γ-continuous iff the restriction $F/_{U_\alpha} : U_\alpha \to Y$ is fuzzy lower γ-continuous.

Proof. Let $x \in X$. Then there exists $\alpha \in \Lambda$ such that $x \in U_\alpha$. Let V be a fuzzy open set of Y such that $\left(F/_{U_\alpha}\right)(x) = F(x)qV$. Since F is fuzzy lower γ-continuous, there exists $G \in \gamma O(X,x)$ such that $F(g)qV$, for all

$g \in G$. Put $U = G \cap U_\alpha$. Then by Lemma 1.9, $U \in \gamma O(U_\alpha, x)$ and $\left(F/_{U_\alpha}\right)(u) = F(u)qV$, for all $u \in U$. Therefore, $F/_{U_\alpha}$ is fuzzy lower γ-continuous.

Conversely, let $x \in X$ and V, a fuzzy open set of Y such that $F(x)qV$. Since $\{U_\alpha : \alpha \in \Lambda\}$ is an α-open cover of X, there exists $\beta \in \Lambda$ such that $x \in U_\beta$. Since $F/_{U_\beta}$ is fuzzy lower γ-continuous and $\left(F/_{U_\beta}\right)(x) = F(x)qV$, there exists $U \in \gamma O(U_\beta, x)$ such that $\left(F/_{U_\beta}\right)(u) = F(u)qV$, for all $u \in U$. Then by Lemma 1.10 (as α-open sets are γ-open), $U \in \gamma O(X, x)$ and $F(u) = (F/_{U_\beta})(u)qV$, for all $u \in U \Rightarrow F$ is fuzzy lower γ-continuous.

Definition 3.9. For a fuzzy multifunction $F : X \to Y$, fuzzy multifunction $\gamma clF : X \to Y$, $\beta clF : X \to Y$, $\alpha clF : X \to Y$, $sclF : X \to Y$ [6], $pclF : X \to Y$, $clF : X \to Y$ [6], $\delta pclF : X \to Y$ [8] are defined by $(\gamma clF)(x) = \gamma clF(x)$, $(\beta clF)(x) = \beta clF(x)$, $(\alpha clF)(x) = \alpha clF(x)$, $(sclF)(x) = sclF(x)$, $(pclF)(x) = pclF(x)$, $(clF)(x) = clF(x)$, $(\delta pclF)(x) = \delta pclF(x)$, for all $x \in X$.

Lemma 3.10. Let $F : X \to Y$ be a fuzzy multifunction. Then we have $(\gamma clF)^-(G) = F^-(G)$, for each fuzzy open set G of Y.

Proof. Let G be a fuzzy open set of Y. Let $x \in (\gamma clF)^-(G)$. Then $(\gamma clF)(x)qG \Rightarrow F(x)qG$ [Indeed, if $F(x)\bar{q}G$, then $[F(x)](y) + G(y) \le 1$, for each $y \in Y \Rightarrow F(x) \le 1_Y \backslash G \Rightarrow \gamma clF(x) \le \gamma cl(1_Y \backslash G) = 1_Y \backslash G$ (since G is fuzzy open $\Rightarrow G$ is fuzzy γ-open in Y) $\Rightarrow \gamma clF(x)\bar{q}G$, a contradiction.] $\Rightarrow x \in F^-(G)$.

Similarly we can prove that

Lemma 3.11. Let $F : X \to Y$ be a fuzzy multifunction. Then we have $(\beta clF)^-(G) = F^-(G)$, $(\alpha clF)^-(G) = F^-(G)$, $(sclF)^-(G) = F^-(G)$, $(pclF)^-(G) = F^-(G)$, $(clF)^-(G) = F^-(G)$, $(\delta pclF)^-(G) = F^-(G)$, for each fuzzy open set G of Y.

Theorem 3.12. For a fuzzy multifunction $F : X \to Y$, the following are equivalent :

(a) F is fuzzy lower γ-continuous.

(b) γclF is fuzzy lower γ-continuous.

(c) βclF is fuzzy lower γ-continuous.

(d) αclF is fuzzy lower γ-continuous.

(e) $sclF$ is fuzzy lower γ-continuous.

(f) $pclF$ is fuzzy lower γ-continuous.

(g) clF is fuzzy lower γ-continuous.

(h) $\delta pclF$ is fuzzy lower γ-continuous.

Proof. The proof follows from Lemma 2.10 and Lemma 2.11.

IV SOME RELATIONSHIP

In this section it has been shown that fuzzy upper (lower) γ-continuous multifunction may not be fuzzy upper (lower) semi-continuous, fuzzy upper (lower) quasi continuous, fuzzy upper (lower) δ-precontinuous multifunctions.

We first recall some theorems for ready references.

Theorem 4.1 [7]. A fuzzy multifunction $F : X \to Y$ is fuzzy upper (lower) quasi continuous iff $F^+(G)$ (resp., $F^-(G)$) is semiopen in X for every fuzzy open set G of Y.

Theorem 4.2 [18]. A fuzzy multifunction $F : X \to Y$ is fuzzy upper (lower) semi-continuous iff $F^+(G)$ (resp., $F^-(G)$) is open in X for every fuzzy open set G of Y.

Theorem 4.3 [8]. A fuzzy multifunction $F : X \to Y$ is fuzzy upper (lower) δ-precontinuous iff $F^+(G)$ (resp., $F^-(G)$) is δ-preopen in X for every fuzzy open set G of Y.

Remark 4.4. Since open set, semiopen set and δ-preopen set are γ-open, it is clear from Theorem 3.1, Theorem 3.2 and Theorem 3.3 that fuzzy upper (lower) quasi continuous, fuzzy upper (lower) semi-continuous and fuzzy upper (lower) δ-precontinuous multifunctions are fuzzy upper (lower) γ-continuous. But the converses are not true, in general, as seen from the following examples.

Example 4.5. fuzzy upper γ-continuity ⇏ fuzzy upper semi-continuity

Let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\emptyset, X\}$, $\tau_Y = \{0_Y, 1_Y, A, B\}$ where $A(y) = 0.35, B(y) = 0.4$, for all $y \in Y$. Then (X, τ) and (Y, τ_Y) are ordinary topological space and an fts respectively. Let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A, F(b) = B, F(c) = C$ where $C(y) = 0.6$, for all $y \in Y$. Now $F^+(A) = \{x \in X : F(x) \leq A\} = \{a\} \notin \tau$ and so F is not fuzzy upper semi-continuous.

But $F^+(A) = \{a\} \Rightarrow int(cl(\{a\})) = X \Rightarrow \{a\} \subseteq \left(cl\left(int(\{a\})\right)\right) \cup \left(int\left(cl(\{a\})\right)\right) \Rightarrow F^+(A)$ is γ-open in X.

Again $F^+(B) = \{a, b\} \subseteq int\left(cl(\{a, b\})\right) = X \Rightarrow \{a, b\} \subseteq \left(cl\left(int(\{a, b\})\right)\right) \cup \left(int\left(cl(\{a, b\})\right)\right) \Rightarrow F^+(B)$ is γ-open in X which shows that $F^+(V)$ is γ-open in X for all fuzzy open set V of $Y \Rightarrow F$ is fuzzy upper γ-continuous.

Example 4.6. fuzzy lower γ-continuity ⇏ fuzzy lower semi-continuity

Let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\emptyset, X\}$, $\tau_Y = \{0_Y, 1_Y, A, B\}$ where $A(y) = 0.35, B(y) = 0.5$, for all $y \in Y$. Then (X, τ) and (Y, τ_Y) are ordinary topological space and an fts respectively. Let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A, F(b) = B, F(c) = C$ where $C(y) = 0.6$, for all $y \in Y$. Now $F^-(A) = \{x \in X : F(x) q A\} = \emptyset \in \tau$ and $F^-(B) = \{c\}$, $int(cl(\{c\})) = X \Rightarrow F$ is fuzzy lower γ-continuous. But $F^-(B) \notin \tau$ and so F is not fuzzy lower semi-continuous.

Example 4.7. fuzzy upper γ-continuity ⇏ fuzzy upper quasi-continuity

Consider Example 3.5. Here $F^+(A) = \{a\} \nsubseteq cl(int(\{a\})) = \emptyset \Rightarrow F$ is not fuzzy upper quasi-continuous, though F is upper γ-continuous.

Example 4.8. fuzzy lower γ-continuity ⇏ fuzzy lower quasi-continuity

Consider Example 3.6. Here $F^-(B) = \{c\} \nsubseteq cl(int(\{c\})) = \emptyset \Rightarrow F^-(B)$ is not semiopen in $X \Rightarrow F$ is not fuzzy lower quasi-continuous, though it is fuzzy lower γ-continuous.

Example 4.9. fuzzy upper γ-continuity ⇏ fuzzy upper δ-precontinuity

Consider Example 3.5. Here $F^+(A) = \{a\}$. Now $\delta cl(\{a\}) = \{x \in X : \{a\} \cap (int(clU)) \neq \emptyset, U \in \tau$ and $x \in U\} = \{a\}$, $int(\delta cl(\{a\}) = \emptyset \nsupseteq \{a\} \Rightarrow \{a\}$ is not δ-preopen in $X \Rightarrow F$ is not fuzzy upper δ-precontinuous, though F is fuzzy upper γ-continuous.

Example 4.10. fuzzy lower γ-continuity ⇏ fuzzy lower δ-precontinuity

Consider Example 3.6. Here $F^-(B) = \{c\}$. Now $\delta cl(\{c\}) = \{c\} \Rightarrow int(\delta cl(\{c\})) = \emptyset \nsupseteq \{c\} \Rightarrow F$ is fuzzy lower δ-precontinuous, though F is fuzzy lower γ-continuous.

V FUZZY UPPER (LOWER) γ-CONTINUOUS MULTIFUNCTION: SOME CHARACTERIZATIONS AND APPLICATIONS

In this section fuzzy upper (lower) γ-continuous multifunction is characterized by fuzzy upper (lower) nbd of a fuzzy set and also some applications of these fuzzy multifunctions have been given.

Definition 5.1. A fuzzy set A in an fts Y is said to be a fuzzy lower (upper) nbd of a fuzzy set B in Y if there exists a fuzzy open set V of Y such that BqV (resp., $B \leq V$) and $V \bar{q} (1_X \setminus A)$.

Theorem 5.2. A fuzzy multifunction $F : X \to Y$ is fuzzy upper γ-continuous on X iff for each point $x_0 \in X$ and each fuzzy upper nbd M of $F(x_0)$, $F^+(M)$ is a γ-nbd of x_0.

Proof. Let F be fuzzy upper γ-continuous. Then for any $x_0 \in X$ and for any fuzzy upper nbd M of $F(x_0)$, there exists a fuzzy open set V of Y such that $F(x_0) \leq V$ and $V\bar{q}(1_X \backslash M) \Rightarrow V \leq M$. Since F is fuzzy upper γ-continuous, there exists $U \in \gamma O(X, x_0)$ such that $U \subseteq F^+(V) \Rightarrow F(U) \leq V \leq M \Rightarrow U \subseteq F^+(M)$. Therefore, $x_0 \in U \subseteq F^+(M)$ $\Rightarrow F^+(M)$ is a γ-nbd of x_0.

Conversely, let for any $x_0 \in X$ and any fuzzy open set V of Y with $F(x_0) \leq V$, we have $V\bar{q}(1_X \backslash V)$. Therefore, V is a fuzzy upper nbd of $F(x_0)$. Then by hypothesis, $F^+(V)$ is a γ-nbd of x_0. Then there exists $U \in \gamma O(X, x_0)$ such that $x_0 \in U \subseteq F^+(V) \Rightarrow F(U) \leq V \Rightarrow F$ is fuzzy upper γ-continuous.

Theorem 5.3. A fuzzy multifunction $F : X \to Y$ is fuzzy lower γ-continuous on X iff for each point $x_0 \in X$ and each fuzzy lower nbd M of $F(x_0)$, $F^-(M)$ is a γ-nbd of x_0.

Proof. Let F be fuzzy lower γ-continuous on X. Then for any $x_0 \in X$ and for any fuzzy lower nbd M of $F(x_0)$, there exists a fuzzy open set V of Y such that $F(x_0)qV$ and $V\bar{q}(1_X \backslash M) \Rightarrow V \leq M$. Since F is fuzzy lower γ-continuous, there exists $U \in \gamma O(X, x_0)$ such that $U \subseteq F^-(V) \subseteq F^-(M)$. Therefore, $x_0 \in U \subseteq F^-(M) \Rightarrow F^-(M)$ is a γ-nbd of x_0.

Conversely, let for any $x_0 \in X$ and any fuzzy open set V of Y with $F(x_0)qV$, we have $V\bar{q}(1_X \backslash V)$. Therefore, V is a fuzzy lower nbd of $F(x_0)$. Then by hypothesis, $F^-(V)$ is a γ-nbd of x_0. Then there exists $U \in \gamma O(X, x_0)$ such that $x_0 \in U \subseteq F^-(V) \Rightarrow F$ is fuzzy lower γ-continuous.

Definition 5.4 [11]. A topological space (X, τ) is said to be γ-compact if for every covering of X by γ-open sets in X has a finite subcovering.

Theorem 5.5. Let $F : X \to Y$ be a fuzzy upper γ-continuous surjective multifunction and $F(x)$ be a fuzzy compact set in Y for each $x \in X$. If X is γ-compact, then Y is fuzzy compact.

Proof. Let $\mathscr{A} = \{A_\alpha : \alpha \in \Lambda\}$ be a fuzzy open cover of Y. Now for each $x \in X$, $F(x)$ is fuzzy compact in Y and so there is a finite subset Λ_x of Λ such that $F(x) \leq \cup \{A_\alpha : \alpha \in \Lambda_x\}$. Let $A_x = \cup \{A_\alpha : \alpha \in \Lambda_x\}$. Then $F(x) \leq A_x$ where A_x is a fuzzy open set of Y. Since F is fuzzy upper γ-continuous, there exists $U_x \in \gamma O(X, x)$ such that $U_x \subseteq F^+(A_x)$. Then $\mathscr{U} = \{U_x : x \in X\}$ is a cover of X by γ-open sets of X. Since X is γ-compact, there exists finitely many points $x_1, x_2, \dots, x_n$ in X such that $X = \cup_{i=1}^{n} U_{x_i}$. As F is surjective, $1_Y = F(X) = F\left(\cup_{i=1}^{n} U_{x_i}\right) = \cup_{i=1}^{n} F(U_{x_i}) \leq \cup_{i=1}^{n} A_{x_i} = \cup_{i=1}^{n} \cup_{\alpha \in \Lambda_{x_i}} A_\alpha \Rightarrow Y$ is fuzzy compact.

Definition 5.6 [17]. An fts (Y, τ_Y) is said to be an FNC-space if every fuzzy regular open cover of Y has a finite subcover.

Remark 5.7. As every fuzzy regular open set is fuzzy open, we set the following theorem.

Theorem 5.8. Let $F : X \to Y$ be a fuzzy upper γ-continuous surjective multifunction and $F(x)$ be a fuzzy compact set in Y for each $x \in X$. If X is γ-compact, then Y is FNC-space.

Theorem 5.9. When X is product related to Y, a fuzzy multifunction $F : X \to Y$ is fuzzy upper γ-continuous if its fuzzy graph multifunction $G_F : X \to X \times Y$ is fuzzy upper γ-continuous.

Proof. Let G_F be fuzzy upper γ-continuous. Let $x \in X$ and V be any fuzzy open set in Y such that $F(x) \leq V$. Then $G_F(x) \leq X \times V$ and $X \times V$ is easily seen to be open in $X \times Y$. By hypothesis, there exists $U \in \gamma O(X, x)$ such that $G_F(U) \leq X \times V$. Now for any $z \in U$ and for any $y \in Y$, $[F(z)](y) = [G_F(z)](z, y) \leq (X \times V)(z, y) = V(y)$, i.e., $[F(z)](y) \leq V(y)$, for all $y \in Y \Rightarrow F(z) \leq V$, for any $z \in U \Rightarrow F(U) \leq V \Rightarrow F$ is fuzzy upper γ-continuous.

Definition 5.10 [2]. The γ-frontier of a subset A of a topological space X, denoted by $\gamma Fr(A)$, is defined by $\gamma Fr(A) = \gamma clA \cap \gamma cl(X \setminus A) = \gamma clA \setminus \gamma intA$.

Theorem 5.11. Let $F : X \to Y$ be a fuzzy multifunction. Let $A = \{x \in X : F$ is not fuzzy upper γ-continuous at $x\}$, $B = \cup \{\gamma Fr\left(F^{+}(V)\right) : F(x) \leq V$ and V is fuzzy open in $Y\}$. Then $A = B$.

Proof. Let $x \in X$ be such that F is not fuzzy upper γ-continuous at x. Then there exists a fuzzy open set V of Y with $F(x) \leq V$ such that $U \nsubseteq F^{+}(V)$, for all $U \in \gamma O(X, x) \Rightarrow U \cap \left(X \setminus F^{+}(V)\right) \neq \emptyset \Rightarrow x \in \gamma cl\left(X \setminus F^{+}(V)\right) = X \setminus \gamma int F^{+}(V) \Rightarrow x \notin \gamma int F^{+}(V)$, but $x \in F^{+}(V) \subseteq \gamma cl(F^{+}(V))$. Therefore, $x \in \gamma cl\left(F^{+}(V)\right) \setminus \gamma int(F^{+}(V)) = \gamma Fr(F^{+}(V))$.

Conversely, let $x \in X$ and V be a fuzzy open set of Y with $F(x) \leq V$ such that $x \in \gamma Fr(F^{+}(V))$. If possible, let F be fuzzy upper γ-continuous at x. Then there exists $U \in \gamma O(X, x)$ such that $U \subseteq F^{+}(V)$. Then $x \in U = \gamma int U \subseteq \gamma int(F^{+}(V)) \Rightarrow x \in \gamma int(F^{+}(V)) \Rightarrow x \notin \gamma Fr(F^{+}(V))$, a contradiction and hence F is not fuzzy upper γ-continuous at x.

Theorem 5.12. Let $F : X \to Y$ be a fuzzy multifunction. Let $A = \{x \in X : F$ is not fuzzy lower γ-continuous at $x\}$, $B = \cup \{\gamma Fr\left(F^{-}(V)\right) : F(x) q V$ and V is fuzzy open set of $Y\}$. Then $A = B$.

Proof. Let $x \in X$ be such that F is not fuzzy lower γ-continuous at x. Then there exists a fuzzy open set V of Y with $F(x) q V$ such that $U \nsubseteq F^{-}(V)$ i.e., $U \cap \left(X - (V)\right) \neq \emptyset$, for all $U \in \gamma O(X, x) \Rightarrow x \in \gamma cl\left(X \setminus F^{-}(V)\right) = X \setminus \gamma int F^{-}(V) \Rightarrow x \notin \gamma int F^{-}(V)$, but $x \in F^{-}(V) \subseteq \gamma cl(F^{-}(V))$. Therefore, $x \in \gamma cl\left(F^{-}(V)\right) \setminus \gamma int(F^{-}(V)) = \gamma Fr(F^{-}(V))$.

Conversely, let $x \in X$ and V be a fuzzy open set of Y with $F(x)qV$ such that $x \in \gamma Fr(F^{-}(V))$. If possible, let F be fuzzy lower γ-continuous at x. Then there exists $U \in \gamma O(X, x)$ such that $U \subseteq F^{-}(V)$. Then $x \in U = \gamma intU \subseteq \gamma int(F^{-}(V)) \Rightarrow x \in \gamma int(F^{-}(V)) \Rightarrow x \notin \gamma Fr(F^{-}(V))$, a contradiction and hence F is not fuzzy lower γ-continuous at x.

REFERENCES

[1] Abd El-Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A.; β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ*. 12 (1983), 77 – 90.

[2] Abd El-Monsef, M.E. and Nasef, A.A.; On multifunctions, *Chaos, Solitons and Fractals* 12 (2001), 2387-2394.

[3] Andrijevič, D. ; On b-open sets, *Mat. Bech*. 48 (1996), 59-64.

[4] Azad, K.K. ; On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, *J. Math. Anal. Appl*. 82 (1981), 14-32.

[5] Bhattacharyya, Anjana and Mukherjee, M.N. ; On fuzzy δ-almost continuous and δ^*-almost continuous functions, *J. Tripura Math. Soc*. 2 (2000), 45-57.

[6] Bhattacharyya, Anjana ; Concerning almost quasi continuous fuzzy multifunctions, *Universitatea Din Bacău Studii Şi Cercetări Ştiinţifice*, Seria : Matematică Nr. 11 (2001), 35-48.

[7] Bhattacharyya, Anjana : Upper and lower weakly quasi continuous fuzzy multifunctions, *Analele Universităţii Oradea, Fasc. Matematica*, Tom XX (2013), Issue No. 8, 5-17.

[8] Bhattacharyya, Anjana : On upper and lower δ-precontinuous fuzzy multifunctions, *International Journal of Advancements in Research and Technology* Vol. 2, Issue 7 (2013), 328-333.

[9] Bin Shahna, A.S. ; On fuzzy strong semicontinuity and fuzzy precontinuity, *Fuzzy Sets and Systems* 44 (1991), 303-308.

[10] Chang, C.L. ; Fuzzy topological spaces, *J. Math. Anal. Appl*. 24 (1968), 182-190.

[11] El-Atik, A.A. ; A study of some types of mappings on topological spaces, *Master's Thesis, Faculty of Science, Tanta University, Tanta, Egypt*, 1997.

[12] Fath Alla, M.A. ; On fuzzy topological spaces, *Ph.D. Thesis, Assiut Univ., Sohag, Egypt* (1984).

[13] Ganguly, S. and Saha, S. ; A note on δ-continuity and δ-connected sets in fuzzy set theory, *Simon Stevin* 62 (1988), 127-141.

[14] Ganguly, S. and Saha, S. ; A note on compactness in fuzzy setting, *Fuzzy Sets and Systems* 34 (1990), 117-124.

[15] Levin, N. ; Semiopen sets and semicontinuity in topological spaces, *Amer. Math. Monthly* 70 (1963), 36-41.

[16] Mashhour, A.S., Abd El-Monsef, M.E. and El-Deeb, S.N. ; On precontinuous and weak precontinuous mappings, *Proc. Phys. Soc. Egypt* 53 (1982), 47-53.

[17] Mukherjee, M.N. and Ghosh, B. ; On nearly compact and θ-rigid fuzzy sets in fuzzy topological spaces, *Fuzzy Sets and Systems* 43 (1991), 57-68.

[18] Mukherjee, M.N. and Malakar, S. ; On almost continuous and weakly continuous fuzzy multifunctions, *Fuzzy Sets and Systems* 41 (1991), 113-125.

[19] Nanda, S. ; Strongly compact fuzzy topological spaces, *Fuzzy Sets and Systems* 42 (1991), 259-262.

[20] Njåstad, O. ; On some classes of nearly open sets, *Pacific J. Math*. 15 (1965), 961-970.

[21] Papageorgiou, N.S. ; Fuzzy topology and fuzzy multifunctions, *J. Math. Anal. Appl*. 109 (1985), 397-425.

[22] Pu, Pao Ming and Liu, Ying Ming ; Fuzzy topology I, Neighbourhood structure of a fuzzy point and Moore-Smith convergence, *J. Math. Anal. Appl*. 76 (1980), 571-599.

[23] Raychaudhuri, S. ; Concerning δ-almost continuity and δ-preopen sets, *Bull. Inst. Math. Acad. Sinica* 21(4) (1993), 357-366.

[24] Stone, M.H. ; Applications of the theory of Boolean ring to general topology, *TAMS* 41 (1937), 375.

[25] Veličko, N.V. ; H-closed topological spaces, *Amer. Math. Soc. Transl*. 78 (1968), 103-118.

[26] Zadeh, L.A. ; Fuzzy sets, *Inform. Control* 8 (1965), 338-353.