HIGHER ORDER SQUEEZING IN SIXTH HARMONIC GENERATION

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ABSTRACT

The quantum effect of squeezing is investigated in the higher order of field amplitude in sixth harmonic generation under the short time approximation. Squeezing is found to be dependent on coupling constant g and phase of the field amplitude. The effect of photon number on higher order squeezing and signal to noise ratio has also been investigated.

Keywords: Harmonic generation; Squeezing; Quantum optics

I INTRODUCTION

Since optical field obeys the laws of quantum mechanics, a minimum size of inherent quantum indeterminacy is always present with each excitation for single mode fields even in the absence of any external field. These are the vacuum fluctuations (zero–point fluctuations) of the electromagnetic field. The quantum limit can be circumvented by the use of squeezed states of light. Squeezed states comprise phase-dependent distributions of zero–point fluctuations such that the fluctuations in one quadrature are smaller than those of a coherent state, at the expense of increased fluctuations in the canonically conjugate quadrature, while preserving the Heisenberg limit on the uncertainty product. These states are called quadrature squeezed states of electromagnetic field. The earlier work is mainly devoted to second-order quantum squeezing. However, with the development of techniques for making higher-order correlation measurement in quantum optics, it was quite natural to turn the attention towards higher-order squeezing. Hillery [1] in 1987 introduced the concept of higher-order squeezing called ‘amplitude –squared squeezing’ different from that defined by Hong and Mandel [2].

Squeezed states are a unique set of quantum states with no classical analogue. Many theoretical and experimental developments on squeezed states have taken place in a number of optical processes such as harmonic generation [4-6], multiwave mixing processes [7-9], parametric amplification [10], intermediate states and superposed coherent states [11-13], Raman process [14] and optical Faraday rotation [15]. Squeezed states have become central to quantum optics through their promise of accuracy improved beyond the standard quantum limit in interferometers for detection of gravitational waves [16-17]. Besides their use in ultra-precise measurements, squeezed states are also gaining relevance in quantum information science. They have been used to construct entangled states of lights to demonstrate quantum teleportation [18], quantum computation [19], optical storage [20], dense coding [21].

As the number of applications hitting the limit to their sensitivity set by quantum noise grows and as more squeezing becomes possible, the breadth of potential of squeezed states can only continue to expand higher-order squeezing. In the present work we have reported that the generation of higher order squeezed state is possible by using sixth harmonic generation.

II DEFINITION OF SQUEEZING AND HIGHER ORDER SQUEEZING

Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature. Normal squeezing is defined in terms of the operators

$$X_1 = \frac{1}{2}\left(A + A^\dagger\right) \quad \text{and} \quad X_2 = \frac{1}{2i}\left(A - A^\dagger\right)$$

Where X_1 and X_2 are the real and imaginary parts of the field amplitude, respectively. A and $A^\dagger$ are slowly varying operators defined by $A = ae^{i\omega t}$ and $A^\dagger = a^\dagger e^{-i\omega t}$

The operators X_1 and X_2 obey the commutation relation

$$\left[X_1, X_2\right] = \frac{i}{2}$$

Which leads to uncertainty relation ($\hbar$ =1)

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

A quantum state is squeezed in X_i variable if

$$\Delta X_i < \frac{1}{2} \quad \text{for } i = 1 \text{ or } 2$$

III SQUEEZING OF FUNDAMENTAL MODE IN SIXTH HARMONIC GENERATION

Sixth harmonic generation model has been adopted from the works of Zhou yong et al. [22] and G.Wang and X. Wang et al. [23] is shown in figure 1. In this model, the interaction is looked upon as a process which involves the absorption of six photons, each having a frequency ω_1 going from state $|1\rangle$ to state $|2\rangle$ and emission of one photon of frequency ω_2 where $\omega_2 = 6\omega_1$.

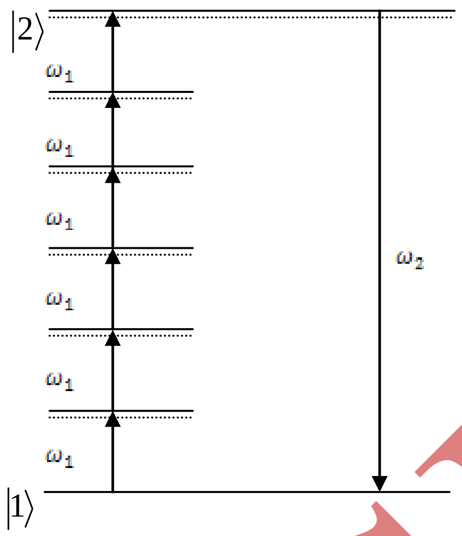

Fig. 1. Sixth Harmonic Generation Model.

The Hamiltonian for this process is given as follows ($\hbar = 1$)

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + g\left(a^6 b^\dagger + a^{\dagger 6} b\right) \tag{1}$$

in which g is a coupling constant for sixth harmonic generation.

$A = a\exp(i\omega_1 t)$ and $B = b\exp(i\omega_2 t)$ are the slowly varying operators at frequencies ω_1 and ω_2, $a\left(a^\dagger\right)$ and $b\left(b^\dagger\right)$ are the usual annihilation (creation) operators, respectively. The Heisenberg equation of motion for fundamental mode A is given as ($\hbar = 1$)

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + i\left[H, A\right] \tag{2}$$

Using Eq. (1) in Eq. (2), we obtain

$$\dot{A} = -6igA^{\dagger 5}B \tag{3}$$

Similarly,

$$\dot{B} = -igA^6 \tag{4}$$

By assuming the short time interaction of waves with the medium and expanding $A(t)$ by using Taylors series expansion and retaining the terms up to g^2t^2 as

$$\begin{aligned} A(t) = {} & A - 6igtA^{\dagger 5}B + 3g^2t^2\left[30(A^{\dagger 4}A5 + 10^{\dagger 3}A^4 + 40A^{\dagger 2}A^3\right. \\ & \left. + 60A^{\dagger 2}A^2 + 24A)B^\dagger B - A^{\dagger 5}A^6\right] \end{aligned} \tag{5}$$

For squeezing of field amplitude in fundamental mode A, the real quadrature component is

$$X_{1A} = \frac{1}{2}\left[A(t) + A^{\dagger}(t)\right] \tag{6}$$

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode *A* and the vacuum state for the harmonic mode *B* i.e.

$$|\psi\rangle = |\alpha\rangle|0\rangle \tag{7}$$

Using Eqs.(5) and (7), number of photons in mode A may be expressed as

$$\begin{aligned} N_{1A}(t) &= A^{\dagger}(t)A(t) \\ &= A^{\dagger}A - 6igt\left(A^{\dagger 6}B - A^{6}B^{\dagger}\right) - 6g^2t^2A^{\dagger 6}A^{6} \end{aligned} \tag{8}$$

$$\begin{aligned} N_{1A}^2(t) &= N_{1A}(t)N_{1A}(t) \\ &= A^{\dagger 2}A^2 + A^{\dagger}A - 6g^2t^2\left(2A^{\dagger 7}A^7 + 6A^{\dagger 6}A^6\right) \end{aligned} \tag{9}$$

and

$$\begin{aligned} N_{1A}^3(t) &= N_{1A}^2(t)N_{1A}(t) \\ &= A^{\dagger 3}A^3 + 3A^{\dagger 2}A^2 + A^{\dagger}A - 6g^2t^2\left(3A^{\dagger 8}A^8 + 27A^{\dagger 7}A^7 + 36A^{\dagger 6}A^6\right) \end{aligned} \tag{10}$$

Using Eqs.(5) and (7) the fourth-order amplitude of the fundamental mode is expressed as

$$\begin{aligned} A^4(t) = A^4 - 6igt\left(4A^{\dagger 5}A^3 + 30A^{\dagger 4}A^2 + 80A^{\dagger 3}A + 60A^{\dagger 2}\right)B - 3g^2t^2 \\ \left(4A^{\dagger 5}A^9 + 30A^{\dagger 4}A^8 + 80A^{\dagger 3}A^7 + 60A^{\dagger 2}A^6\right) \end{aligned} \tag{11}$$

$$\begin{aligned} A^{\dagger 4}(t) = A^{\dagger 4} + 6igt\left(4A^{\dagger 3}A^5 + 30A^{\dagger 2}A^4 + 80A^{\dagger}A^3 + 60A^2\right)B^{\dagger} - 3g^2t^2 \\ \left(4A^{\dagger 9}A^5 + 30A^{\dagger 8}A^4 + 80A^{\dagger 7}A^3 + 60A^{\dagger 6}A^2\right) \end{aligned} \tag{12}$$

For fourth order squeezing, the real quadrature component in fundamental mode is given as

$$F_{1A}(t) = \frac{1}{2}\left[A^4(t) + A^{\dagger 4}(t)\right] \tag{13}$$

Using Eqs.(7) and (11) in Eq. (13), we get the expectation value as

$$\begin{aligned} \langle\psi|F_{1A}(t)|\psi\rangle^2 = \frac{1}{4}\Big[\alpha^8 + \alpha^{*8} + 2|\alpha|^8 - 6g^2t^2\{(4|\alpha|^{10} + 30|\alpha|^8 + 80|\alpha|^6 \\ + 60|\alpha|^4)\left(\alpha^8 + \alpha^{*8}\right) + 8|\alpha|^{18} + 60|\alpha|^{16} + 160|\alpha|^{14} + 120|\alpha|^{12}\}\Big] \end{aligned} \tag{14}$$

and

$$\begin{aligned} \langle\psi|F_{1A}^2(t)|\psi\rangle = \frac{1}{4}\Big[\alpha^8 + \alpha^{*8} + 2|\alpha|^8 + 16|\alpha|^6 + 72|\alpha|^4 + 96|\alpha|^2 + 24 \\ - 6g^2t^2\{(4|\alpha|^{10} + 70|\alpha|^8 + 560|\alpha|^6 + 2100|\alpha|^4 + 3360|\alpha|^2 \\ + 1680)(\alpha^8 + \alpha^{*8}) + 8|\alpha|^{18} + 108|\alpha|^{16} + 544|\alpha|^{14} + 896|\alpha|^{12}\}\Big] \end{aligned} \tag{15}$$

Therefore,

$$\left[\Delta F_{1A}\left(t\right)\right]^2 = \frac{1}{4}\Big[16|\alpha|^6 + 72|\alpha|^4 + 96|\alpha|^2 + 24 - 6g^2t^2\{(40|\alpha|^8 + 480|\alpha|^6 + 2040|\alpha|^4 + 3360|\alpha|^2 + 1680)(\alpha^8 + \alpha^{*8}) + 48|\alpha|^{16} + 384|\alpha|^{14} + 776|\alpha|^{12}\}\Big] \quad (16)$$

Using Eqs.(8) – (10), we get

$$\frac{1}{4}\left\langle 16N_{1A}^3\left(t\right) + 24N_{1A}^2\left(t\right) + 56N_{1A}\left(t\right) + 24\right\rangle = \frac{1}{4}[16|\alpha|^6 + 72|\alpha|^4 + 96|\alpha|^2 + 24 - 6g^2t^2(48|\alpha|^{16} + 384|\alpha|^{14} + 776|\alpha|^{12})] \quad (17)$$

Subtracting Eq. (17) from Eq. (16), we obtain

$$\left[\Delta F_{1A}\left(t\right)\right]^2 - \frac{1}{4}\left\langle 16N_{1A}^3\left(t\right) + 24N_{1A}^2\left(t\right) + 56N_{1A}\left(t\right) + 24\right\rangle = -120g^2t^2\left(|\alpha|^{16} + 12|\alpha|^{14} + 51|\alpha|^{12} + 84|\alpha|^{10} + 42|\alpha|^8\right)\cos 8\theta \quad (18)$$

The right hand side of Eq. (18) is negative and thus shows the existence of squeezing in the Fourth order of the fundamental mode for which $\cos 8\theta > 0$.

IV SIGNAL-TO-NOISE RATIO

Signal to noise ratio is defined as ratio of the magnitude of the signal to the magnitude of the noise. With the approximations $\theta = 0$ and $|gt|^2 = 1$, the maximum signal to noise ratio (in decibels) in higher order of field amplitude, is given below.

Signal-to-noise ratio in fourth order of field amplitude is defined as

$$SNR = 20^{*}\log_{10}\frac{(F_{1A}(t))^2}{[\Delta F_{1A}(t)]^2} \quad (19)$$

Using Eqs. (14) and (16), SNR in fourth-order squeezing is expressed as

$$SNR_4 = 20^{*}\log_{10}\frac{(2|\alpha|^{10} + 15|\alpha|^8 + 40|\alpha|^6 + 30|\alpha|^4)}{(16|\alpha|^8 + 168|\alpha|^6 + 607|\alpha|^4 + 840|\alpha|^2 + 420)} \quad (20)$$

V RESULTS

The results show the presence of squeezing in fourth order of field amplitude in sixth harmonic generation. Taking $|gt|^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing, the variations of S_F is shown in Figure 2. Degree of squeezing is shown as a function of $|\alpha|^2$.

It is clear from Figure 2 that the squeezing increases non-linearly with $|\alpha|^2$. This confirms that the squeezed states are associated with the photon number in fundamental mode. The variation of SNR in fourth order of field amplitude for a squeezed state with photon number has also been shown in Figure 3.

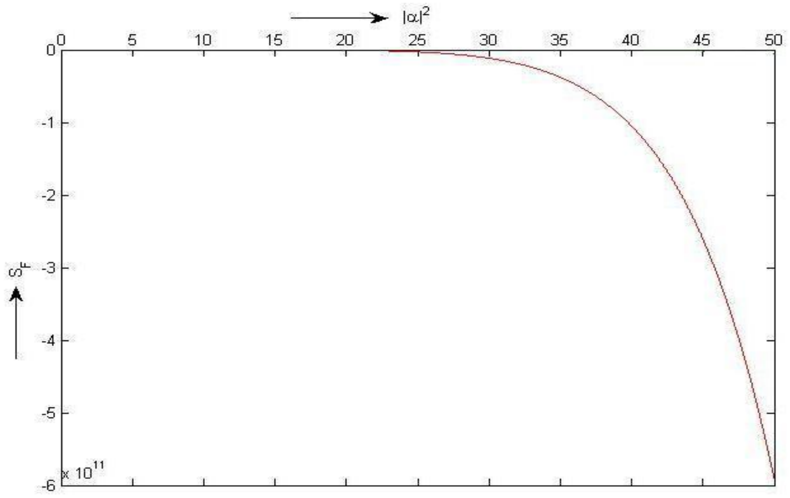

Fig. 2. Dependence of fourth-order amplitude squeezing on $|\alpha|^2$.

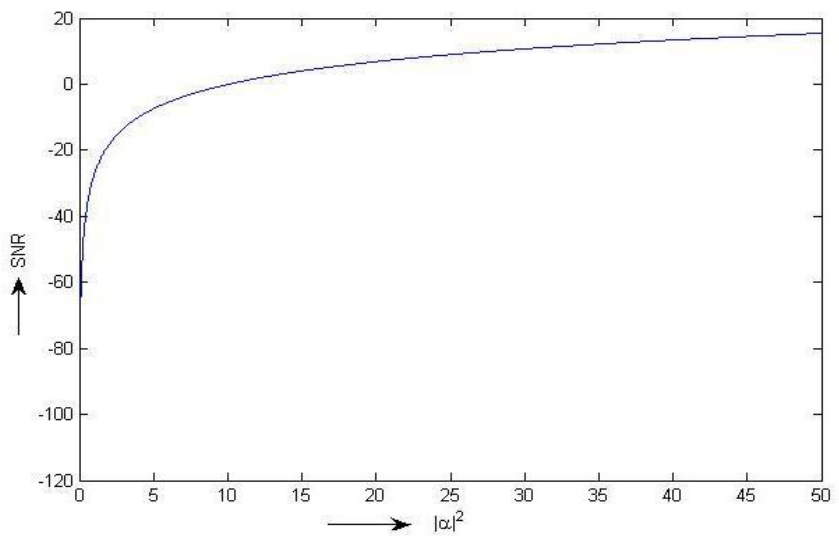

Fig. 3. Signal to noise ratio for fourth order squeezing.

VI CONCLUSION

It is shown that the selective phase values of field amplitude of fundamental mode during sixth harmonic generation lead to squeezing up to fourth order. Further, Figures 2-3, show that the degree of squeezing increases with increase in the order of field amplitude of the fundamental mode. This also establishes the fact that processes with higher order non linearity are more suitable for generation of squeezed light which is used in reducing noise in the output of certain non linear processes.

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