

ENERGY MOMENTUM TENSOR OF DYONS

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ABSTRACT

The energy momentum tensor of generalized fields of dyons and energy momentum conservation laws of dyons has been developed in simple, compact and consistent manner. We have obtained the momentum operator, Hamiltonian, Poynting vector and Poynting theorem for generalized fields of dyons in a simple, unique and consistent way.

Keywords: Dyons, Electromagnetic Fields, Energy-Momentum Tensor, Hamiltonian, Poynting Vector.

I INTRODUCTION

The concept of the energy-momentum tensor in classical field theory has a long history, especially in Einstein's theory of gravity [1]. The energy-momentum tensor combines the densities and flux densities of energy and momentum of the fields into one single object. The problem of giving a concise definition of this object able to provide the physically correct answer under all circumstances, for an arbitrary Lagrangian field theory on an arbitrary space-time background, has puzzled physicists for decades. The classical field theory with space-time translation invariance has a conserved energy-momentum tensor [1, 2]. The classical Lagrangian in constructing the energy-momentum tensor, like the canonical energy-momentum tensor was noticed long ago. The question is based on the Noether theorem, according to which a field theory with space-time translation invariance has a conserved energy-momentum tensor. Belinfante [1, 2] and Rosenfield [3] who, in particular developed this strategy for Lorentz invariant field theories in flat Minkowski space-time to provide a symmetric energy-momentum tensor which, in the case of electrodynamics, is also gauge invariant and gives the physically correct expressions for the energy density and energy flux density i.e. Poynting vector as well as the momentum density and momentum flux density of the electromagnetic field. Callan et. al. [4] and Deser [5] proposed additional terms to define a energy-momentum tensor that, for dilatation invariant scalar field theories, is also traceless. The classical Lagrangian in constructing the energy-momentum tensor, like the canonical energy-momentum tensor was noticed long ago. The question is based on the Noether theorem, according to which a field theory with space-time translation invariance has a conserved energy-momentum tensor. Gotay and Marsden [6], which also provides an extensive list of references witnessing the long and puzzling history of the subject, is an exception. Their approach is perhaps the first systematic attempt to tackle the problem from a truly geometric point of view. In a classical field theories on arbitrary space-time

manifolds. Generically, space-time manifolds do not admit any isometries or conformal isometries at all, so there is no direct analogue of space-time translations, Lorentz transformations, nor are there any conserved quantities in the usual sense. The ordinary conservation law $\partial_\mu T^{\mu\nu} = 0$ for the energy-momentum tensor on flat space-time must be replaced by the covariant conservation law $\nabla_\mu T^{\mu\nu} = 0$ for its counterpart on curved space-time. The basic idea introduced by Gotay and Marsden [6], worked out in detail in using the modern geometric approach to general first order Lagrangian field theories and leading to an improved energy-momentum tensor which is both symmetric and gauge invariant. The energy-momentum tensor is a tensor quantity in physics that describes the density and flux of energy and momentum in space-time, generalizing the stress tensor of Newtonian physics [7]. It is an attribute of matter, radiation, and non-gravitational force fields. The energy-momentum tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity [1, 2]. The energy-momentum tensor is the conserved Noether current associated with space-time translations. When gravity is negligible and using a Cartesian coordinate system for space-time, the divergence of the non-gravitational energy-momentum tensor will be zero. In other words, non-gravitational energy and momentum are conserved [8], [9], [10]. In this paper, the energy momentum tensor of generalized fields of dyons and energy momentum conservation laws are discussed consistently for dyons. Here we have also discussed the momentum operator, Hamiltonian and Poynting vector for generalized electromagnetic fields in a manifest and consistent way.

II ENERGY MOMENTUM TENSOR

The energy momentum tensor is a tensor quantity in physics that describes the density and flux of energy and momentum in space-time, generalizing the stress tensor of Newtonian physics. It is an attribute of matter, radiation, and non-gravitational force fields. The energy momentum tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity. The energy momentum tensor involves the use of super-scripted variables which are not exponents. If the components of the position four vector are given by $x^0 = t, x^1 = x, x^2 = y, x^3 = z$. The energy momentum tensor is defined as the tensor $T^{\alpha\beta}$ of rank two that gives the flux of the α^{th} component of the momentum vector across a surface with constant x^β coordinate. In the theory of relativity, this momentum vector is taken as the four-momentum. In general relativity, the energy momentum tensor is symmetric [7]

$$T^{\alpha\beta} = T^{\beta\alpha}; \quad (1)$$

The energy momentum tensor is of rank two; its components can be displayed in matrix form

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}; \quad (2)$$

The time component is the density of relativistic mass, i.e. the energy density divided by the speed of light squared [7, 8]. It is of special interest because it has a simple physical interpretation. In the case of a perfect fluid this component is

$$T^{00} = \rho; \quad (3)$$

For an electromagnetic field in otherwise empty space this component is given by

$$T^{00} = E^2 + H^2; \quad (4)$$

Where E and H are the electric and magnetic fields, respectively. The energy momentum tensor is the conserved Noether current associated with space-time translations. When gravity is negligible and using a Cartesian coordinate system for space-time, the divergence of the non-gravitational energy momentum will be zero. In other words, non-gravitational energy and momentum are conserved

$$T_{,\nu}^{\mu\nu} = \partial_\nu T^{\mu\nu} = 0; \quad (5)$$

In free space and flat space-time, the electromagnetic energy momentum tensor is given by [9, 10]

$$T^{\mu\nu} = \left[F^{\mu\alpha} F_\alpha^\nu - \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right]; \quad (6)$$

where $F^{\mu\nu}$ is the electromagnetic tensor. This expression is when using a metric of signature $(-, +, +, +)$. If using the metric with signature $(+, -, -, -)$, the expression for $T^{\mu\nu}$ will have opposite sign. $T^{\mu\nu}$ explicitly in matrix form [9, 10]

$$T^{\mu\nu} = \begin{pmatrix} (E^2 + H^2) & S_x & S_y & S_z \\ S_x & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}; \quad (7)$$

Where $\eta^{\mu\nu}$ is the Minkowski metric tensor of metric signature $(-, +, +, +)$, Poynting vector becomes

$$\vec{S} = \vec{E} \times \vec{H};$$

$$\sigma_{ij} = E_i E_j + B_i B_j - \frac{1}{2}(E^2 + B^2)\delta_{ij}. \quad (8)$$

is the Maxwell stress tensor. The flux of electromagnetic energy density is represents as [7, 8]

$$u_{em} = \frac{1}{2}(E^2 + H^2); \quad (9)$$

III ENERGY MOMENTUM TENSOR OF DYONS

The Lagrangian of generalized electromagnetic field for a minimally coupled source of electric and magnetic charges as [11]

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}M_{\mu\nu}M^{\mu\nu} + eA_\mu j^{\mu(e)} + gB_\mu j^{\mu(g)}; \quad (10)$$

We may now define the energy momentum tensor for generalized electromagnetic fields of dyons as [12]

$$\tilde{T}_\mu{}^\nu = \left(\partial_\mu A_\rho\right) \frac{\partial L}{\partial(\partial_\nu A_\rho)} + \left(\partial_\mu B_\rho\right) \frac{\partial L}{\partial(\partial_\nu B_\rho)} - \delta_\mu{}^\nu L; \quad (11)$$

Where the first part of right hand side of equation (11) appears due to the contribution of electric four-potential

$\{A_\mu\}$ so that we may calculate the value of $\frac{\partial L}{\partial(\partial_\nu A_\rho)}$ from the Lagrangian equation (10) as

$$\frac{\partial L}{\partial(\partial_\nu A_\rho)} = \frac{\partial\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)}{\partial(\partial_\nu A_\rho)} = -(\partial^\nu A^\rho - \partial^\rho A^\nu) = -F^{\nu\rho}; \quad (12)$$

Similarly, the second part of equation (11) appears due to the contribution of magnetic four-potential $\{B_\mu\}$ and accordingly we get

$$\frac{\partial L}{\partial(\partial_\nu B_\rho)} = \frac{\partial\left(-\frac{1}{4}M_{\mu\nu}M^{\mu\nu}\right)}{\partial(\partial_\nu B_\rho)} = -(\partial^\nu B^\rho - \partial^\rho B^\nu) = -M^{\nu\rho}. \quad (13)$$

Inserting equations (12) and (13) into equation (11), we get

$$\begin{aligned} \tilde{T}_\mu{}^\nu &= -(\partial_\mu A_\rho)F^{\nu\rho} - (\partial_\mu B_\rho)M^{\nu\rho} \\ &+ \left[\frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} - eA_\sigma j_\sigma^{(e)} - gB_\sigma j_\sigma^{(g)} \right]; \end{aligned} \quad (14)$$

Which is not symmetric in the indices μ, ν . To make it symmetric one must add another appropriate rank two tensor which also obeys the same conservation law. So, the equation (14) reduces to

$$\begin{aligned} \tilde{T}^{\mu\nu} &= -(\partial_\mu A_\rho)F^{\nu\rho} - (\partial_\mu B_\rho)M^{\nu\rho} \\ &+ \eta^{\mu\nu} \left[\frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} - eA_\sigma j_\sigma^{(e)} - gB_\sigma j_\sigma^{(g)} \right]. \end{aligned} \quad (15)$$

Now adding the following new tensor,

$$\tilde{S}^{\mu\nu} = (\partial^\rho A^\nu)F_\rho{}^\mu + (\partial^\rho B^\nu)M_\rho{}^\mu; \quad (16)$$

With equation (16), we get the following expression for the energy momentum tensor of generalized electromagnetic fields of dyons as [12]

$$\begin{aligned} T^{\mu\nu} &= \tilde{T}^{\mu\nu} + \tilde{S}^{\mu\nu} \\ T^{\mu\nu} &= -F^{\nu\rho}F_\rho{}^\mu - M^{\nu\rho}M_\rho{}^\mu + \eta^{\mu\nu} \left[\frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} - eA_\sigma j_\sigma^{(e)} - gB_\sigma j_\sigma^{(g)} \right]. \end{aligned} \quad (17)$$

Thus equation (17) provides the field equations associated respectively with the dynamics of electric and magnetic charges of dyons after taking care the usual method of variations with respect to potential. Equation (17) may further be decomposed to

$$\begin{aligned} \tilde{T}^{\mu\nu} &= -(\partial_\nu A_\rho)F^{\mu\rho} - (\partial_\nu B_\rho)M^{\mu\rho} \\ &+ \eta^{\mu\nu} \left[\frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}M_{\sigma\rho}M^{\sigma\rho} - eA_\sigma j_\sigma^{(e)} - gB_\sigma j_\sigma^{(g)} \right]; \end{aligned} \quad (18)$$

and

$$\tilde{S}^{\mu\nu} = (\partial_\rho A_\nu) F^{\mu\rho} + (\partial_\rho B_\nu) M^{\mu\rho}. \quad (19)$$

Hence we may write the co variant derivative of energy momentum tensor (17) as [12]

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= \partial_\mu \left[-(\partial_\nu A_\rho) F^{\mu\rho} - (\partial_\nu B_\rho) M^{\mu\rho} \right] + \partial_\mu \delta_\nu^\mu \left[\frac{1}{4} F_{\sigma\rho} F^{\sigma\rho} + \frac{1}{4} M_{\sigma\rho} M^{\sigma\rho} - e A_\sigma j_\sigma^{(e)} - g B_\sigma j_\sigma^{(g)} \right] \\ &+ \partial_\mu \left[(\partial_\rho A_\nu) F^{\mu\rho} + (\partial_\rho B_\nu) M^{\mu\rho} \right] \\ &= -(\partial_\nu \partial_\mu A_\rho) F^{\mu\rho} - (\partial_\nu A_\rho) \partial_\mu F^{\mu\rho} - (\partial_\nu \partial_\mu B_\rho) M^{\mu\rho} - (\partial_\nu B_\rho) \partial_\mu M^{\mu\rho} \\ &+ \frac{1}{2} (\partial_\nu F_{\sigma\rho}) F^{\sigma\rho} + \frac{1}{2} (\partial_\nu M_{\sigma\rho}) M^{\sigma\rho} - 2\partial_\nu (A_\sigma j_\sigma^{(e)}) - 2\partial_\nu (B_\sigma j_\sigma^{(g)}) \\ &+ \partial_\rho (\partial_\mu A_\nu) F^{\mu\rho} + (\partial_\rho A_\nu) \partial_\mu F^{\mu\rho} + \partial_\rho (\partial_\mu B_\nu) M^{\mu\rho} + (\partial_\rho B_\nu) \partial_\mu M^{\mu\rho} \\ &= -(\partial_\nu \partial_\mu A_\rho) F^{\mu\rho} - (\partial_\nu \partial_\mu B_\rho) M^{\mu\rho} \\ &+ \frac{1}{2} (\partial_\nu F_{\sigma\rho}) F^{\sigma\rho} + \frac{1}{2} (\partial_\nu M_{\sigma\rho}) M^{\sigma\rho}. \end{aligned} \quad (20)$$

If we substitute

$$\begin{aligned} \partial_\mu A_\rho &= \frac{1}{2} (\partial_\mu A_\rho - \partial_\rho A_\mu) + \frac{1}{2} (\partial_\mu A_\rho + \partial_\rho A_\mu) \\ \partial_\mu B_\rho &= \frac{1}{2} (\partial_\mu B_\rho - \partial_\rho B_\mu) + \frac{1}{2} (\partial_\mu B_\rho + \partial_\rho B_\mu) \end{aligned} \quad (21)$$

into equation (20), we get

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= T_{\mu\nu,\nu} = -\frac{1}{2} (\partial_\nu F_{\mu\rho}) F^{\mu\rho} - \frac{1}{2} (\partial_\nu M_{\mu\rho}) M^{\mu\rho} + \frac{1}{2} (\partial_\nu F_{\mu\nu}) F^{\mu\rho} + \frac{1}{2} (\partial_\nu M_{\mu\nu}) M^{\mu\rho} \\ \partial_\mu T^{\mu\nu} &= T_{\mu\nu,\nu} = 0; \end{aligned} \quad (22)$$

Which represents the continuity equation of energy momentum tensor of dyonic fields. Let us write the energy momentum tensor of dyons in an alternative form as [12] - [20]

$$\begin{aligned} T^{\mu\nu} &= -F^{\nu\rho} F_\rho^\mu - M^{\nu\rho} M_\rho^\mu \\ &+ \eta^{\mu\nu} \left(\frac{1}{4} F_{\sigma\rho} F^{\sigma\rho} + \frac{1}{4} M_{\sigma\rho} M^{\sigma\rho} - e A_\sigma j_\sigma^{(e)} - g B_\sigma j_\sigma^{(g)} \right); \end{aligned} \quad (23)$$

Where

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (24)$$

Equation (23) may further be decomposed to momentum and energy operates as

$$T^{\sigma\nu} = -F^{\nu\rho}F_{\rho}^{\sigma} - M^{\nu\rho}M_{\rho}^{\sigma} + \eta^{0\nu} \left(\frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}M_{\sigma\rho}M^{\sigma\rho} - eA_{\sigma}j_{\sigma}^{(e)} - gB_{\sigma}j_{\sigma}^{(g)} \right); \quad (25)$$

$$T^{00} = -F^{0i}F_i^0 - M^{0i}M_i^0 + \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}M_{\sigma\rho}M^{\sigma\rho} - eA_{\mu}j_{\sigma}^{(e)} - gB_{\mu}j_{\sigma}^{(g)}. \quad (26)$$

For $i=1$, we may write

$$-F^{01}F_1^0 - M^{01}M_1^0 = -E(-E) - H(-H) = E^2 + H^2 = u; \quad (27)$$

where $u = E^2 + H^2$ is known as energy density of the generalized electromagnetic field of dyons [12] - [20]. So, equation (26) reduces to [12]

$$T^{00} = (E^2 + H^2) + \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} + \frac{1}{4}M_{\sigma\rho}M^{\sigma\rho} - eA_{\sigma}j_{\sigma}^{(e)} - gB_{\sigma}j_{\sigma}^{(g)}; \quad (28)$$

If we put $\mu = 0$ and $\nu = 3$ in equation (23) we get

$$T^{03} = -F^{3\rho}F_{\rho}^0 - M^{3\rho}M_{\rho}^0; \quad (29)$$

Where

$$\begin{aligned} F^{3\rho}F_{\rho}^0 &= F^{31}F_1^0 + F^{32}F_2^0; \\ M^{3\rho}M_{\rho}^0 &= M^{31}M_1^0 + M^{32}M_2^0. \end{aligned} \quad (30)$$

Then equation (29) becomes,

$$\begin{aligned}
 T^{03} &= -(H_2)(-E_1) - (-H_1)(-E_2) - (-E_2)(-H_1) - (E_1)(-H_2) \\
 T^{03} &= H_2E_1 - H_1E_2 - E_2H_1 + H_2E_1 \\
 T^{03} &= 2(H \cdot E - E \cdot H) = 2(\vec{E} \times \vec{H})
 \end{aligned} \tag{31}$$

Where $T^{0i} = S_i = 2(\vec{E} \times \vec{H})$ is known as Poynting vector for dyons showing that T^{0i} is proportional to the i^{th} component of Poynting vector $\vec{S} = 2(\vec{E} \times \vec{H})$. As such the conservation law becomes

$$\partial_\nu T^{\mu\nu} = 0; \tag{32}$$

where

$$\partial_\nu T^{0\nu} = \frac{\partial}{\partial t} T^{00} + \nabla_i T^{0i} = 0; (\forall i=1,2,3) \tag{33}$$

Substituting the value of T^{00} and T^{0i} into equation (33), we get

$$\frac{\partial}{\partial t} \left[(E^2 + H^2) + \frac{1}{4} F_{\sigma\rho} F^{\sigma\rho} + \frac{1}{4} M_{\sigma\rho} M^{\sigma\rho} - e A_\sigma j_\sigma^{(e)} - g B_\sigma j_\sigma^{(g)} \right] + \nabla \cdot \vec{S} = 0;$$

which can further be reduced to

$$\frac{\partial w}{\partial t} + \nabla \cdot \vec{S} = 0. \tag{34}$$

Equation (34) represents the Poynting Theorem for generalized fields of dyons.

IV CONCLUSION

The Lagrangian density equation (10) of generalized electromagnetic field for a minimally coupled source of electric and magnetic charges in abelian gauge. We have discussed the energy momentum tensor of dyons which is given by equation (11) and its further consequences. It is shown that the energy momentum tensor equation (11) contains the contribution of electric and magnetic four-potential respectively giving rise the variation of Lagrangian in equations (12) and (13). Inserting equations (12) and (13) into equation (11) we have obtained the general expression (14) for energy-momentum tensor dyons and it is shown that the expression equation (14) is no more symmetric and to make it symmetric we need the rank two tensor. Accordingly the equation (14) has been converted to rank two tensor by expression equation (15) and it is concluded that the energy momentum tensor will become symmetric unless and until we add an extra term given by equation (16). As such we have obtained symmetric term energy momentum

tensor for generalized fields of dyons equation (17). It is concluded that the symmetric energy momentum tensor equation (17) of dyons provides the field equation of dyons as the combination of field equations respectively due to the dynamics of electric and magnetic charges after taking care the usual method of variation. In order to obtain the energy momentum conservation laws we have decomposed the equation (17) of energy momentum tensor of dyons in terms of equations (18) and (19). It is shown by equation (22) which is obtained by using equations (20) and (21) that the covariant derivative of energy momentum tensor is vanishing and thus leads the law of energy-momentum conservation. Alternatively we have obtained the energy and momentum components of energy momentum tensors given by equation (25) and (26) which is further expanded in terms of energy and momentum densities respectively given by equations (28) and (31). Equation (31) describes the Poynting vector while the conservation of work energy theorem is expressed by equation (32). Thus the Poynting vector is nothing but the energy flux or the momentum of radiation flowing into or out of a volume. This leads to an increase or decrease in energy of radiation. But in the case of generalized electromagnetic fields of dyons not only the energy is conserved but momentum is also conserved. This shows that the rate of change of total momentum in a volume is because of "momentum flux flowing in and out of the system". The conservation law of energy momentum tensor is described as equation (32). As such equation (34) represents the Poynting theorem for generalized fields of dyons.

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