CONCEPT OF REACTIVE POWER

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ABSTRACT

The notion of reactive power has been introduced long time ago because of its advantages to describe the behaviour of a power system. Among these advantages are the facts that the reactive power is a scalar quantity (therefore easy to manipulate) and that it is conserved over any network.
Over the time, the meaning of the original concept of reactive power has been lost and today the "reactive power" is used for the maximum reactive power that flows through a piece of electric network. The change of meaning is the origin of a few misconceptions that are present in almost every textbook today. This article reanalyzes the reactive power concept, identifies the present contradictions and defines clearly the differences between reactive and active power.

Keywords: Active power/real power, reactive power

I INTRODUCTION

Everyone uses today the concept of reactive power, from the physicist to the electrical engineer. We are so familiar with this term that we do not realize the contradictions associated with this concept.
If one follows any textbook (1 – 7), the concepts of active or real power and reactive power are introduced, usually at the beginning, as a product of the voltage and current measured at the same point of the circuit. The active power depends on the cosine function and the reactive power on the sinusoidal function where the angle is taken between the voltage and the current. When the average in time is calculated, the two powers behave quite differently; the average active power is well defined while the average reactive power is zero, no matter of the network or state of the system.
If for the active power exists a net flow from one point of the network to another, for the reactive power there is a continuously flow back and forth, but the net flow is zero for a complete cycle, as the amount of energy flowing in one direction for half a cycle is equal to the amount of energy flowing in the opposite direction in the next half of the cycle. The reactive power is exchanged by different parts of the network – capacitors and reactors – permanently, but is never consumed or produced. In reality, we can say that this reactive power is produced once, when the network is energized (after a collapse) and the same reactive power is consumed once, when the network collapses again. In between these two major events, the reactive energy stays constant. Of course the situation is changing when equipment is switched on or off.

The surprise comes when the discuss the power flow equations. A piece of the network is analyzed (usually a transmission line), and the active and reactive power at the ends are calculated. The contradiction appears when the so called "reactive power losses" formula is derived, as the reactive power at the receiving end is not equal with the reactive power at the sending end. The only explanation, in the light of energy conservation, is that the line itself is producing or consuming reactive power in order to balance the two ends. This statement contradicts the previous statement that the reactive power is neither produced nor consumed.

The conclusion that one can draw after reading the reactive power chapter of any textbook is that the theory predicts the reactive power oscillates without being produced or consumed but the practice shows the contrary as any utility can prove that they measure loss of reactive power on any line

II CLASSICAL RESULTS FOR REACTIVE POWER

2.1 Definitions Of Various Powers.

Consider a simple RLC series circuit, fig. 1, with resistance R, impedance L and capacitance C, supplied by an alternating power source with maximum voltage V_{max}. The voltage and current at a point are expressed by

$$v = V_{max} \cos(\omega t)$$
$$i = I_{max} \cos(\omega t - \theta) \quad , \qquad (1)$$

Where θ is the phase difference between the voltage and current due to the non-linear elements L and C.

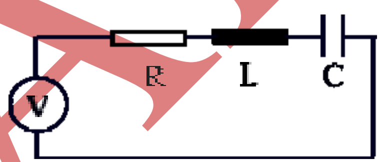

Fig. 1 RLC series circuit.

The instantaneous power is defined as

$$p = vi = V_{max} I_{max} \cos(\omega t)\cos(\omega t - \theta) . \qquad (2)$$

If the voltage v, current i and the instantaneous power p are represented on the same graph, regions of alternating negative and positive power would appear (fig. 2).

Using trigonometric identities, the expression (2) is reduced to

$$p = \frac{V_{max} I_{max}}{2} \cos\theta\,(1 + \cos 2\omega t) + \frac{V_{max} I_{max}}{2} \sin\theta \sin 2\omega t \quad . \qquad (3)$$

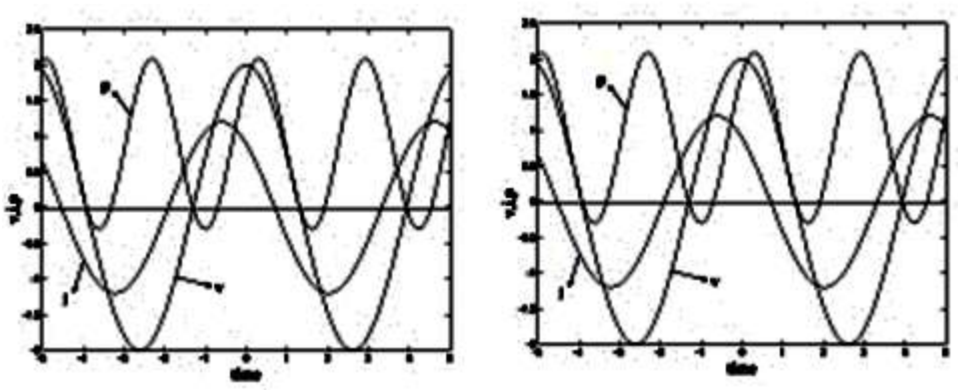

Fig. 2. The current, voltage and their product, instantaneous power.

It is customary to call the first part of (3) the instantaneous active power and the second part of (3) the instantaneous reactive power. There is a fundamental difference between the two powers that can be seen in fig. 3. The active power oscillates around an arbitrary average value while the reactive power oscillates around a zero average power as the average value of a cos and sin function is zero. This observation is valid under any conditions as long as the oscillations are sinusoidal. For a non-sinusoidal regime the problem is much more complicated and is not yet fully solved [8], [9].

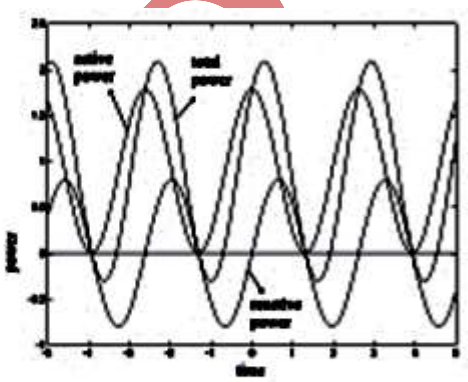

Fig. 3. The active, reactive and total power.

As is not practical to work with instantaneous quantities because they are difficult to measure, averaged values are introduced. The average value of the instantaneous active power is called the active power and is given by

$$P = VI \cos\theta \tag{4}$$

where V and I are the rms values. This quantity is obtained from averaging in time the first part of (3). Following the same method for the instantaneous reactive power, the average is zero and therefore is useless to introduce a quantity that is always zero. Instead, another quantity is introduced to describe the instantaneous reactive power that is transferred to the network and this quantity is the maximum of the instantaneous reactive power,

$$Q = VI \sin\theta \tag{5}$$

which is exactly the second part of (3) without the sin of the time.

In this way the symmetry between the active and reactive power is broken and therefore *P* and *Q* do not have the same meaning as it is inferred in many textbooks. This is the root of the misconceptions about reactive power. The above definitions are summarized in table 1.

Instantaneous power $p = V_{\max} I_{\max} \cos \omega t \cos(\omega t \varphi)$	
$p = \frac{V_{\max} I_{\max}}{2} \cos\varphi (1 + \cos 2\omega t) + \frac{V_{\max} I_{\max}}{2} \sin\varphi \sin 2\omega t$	
Instantaneous active power $\frac{V_{\max} I_{\max}}{2} \cos\varphi (1 + \cos 2\omega t)$	Instantaneous reactive power $\frac{V_{\max} I_{\max}}{2} \sin\varphi \sin 2\omega t$
Average active power $P = VI \cos\varphi$ called simply **active power**	Average reactive power =0 ignored usually.
Maximum instantaneous active power $VI \cos\varphi$ ignored usually as is the same quantity as P	Maximum instantaneous reactive power $Q = VI \sin\varphi$ called simply **reactive power**.

It can be seen from the table 1 that the active power describes an average power while the reactive power describes the maximum of the instantaneous power. As a consequence the two concepts cannot be treated on equal foot as they are not similar. The simplest solution to this problem would be to avoid the usage of reactive power term for *Q* but perhaps this choice would be difficult to be implemented in practice.

2.2 Physical Interpretation of The Power Terms

As both the voltage and current oscillate in time, it is natural to define an instantaneous power *p* as a product of the instantaneous voltage *v* and current *i*. The meaning of the instantaneous power is the rate of change of energy at an instant of time. It has been shown that the instantaneous power (2) can be expressed as a combination of two terms (3), the instantaneous active power and the instantaneous reactive power.

The instantaneous quantities are not of a great help in practice as they oscillate at a rate of 50 (60) times per second. Therefore, average values are rather preferred. An average value <A> of a measured quantity A is defined as the sum of the instantaneous value A over a certain period of time T divided by that period of time

$$\langle A \rangle = \frac{\int_0^T A dt}{T} \qquad (6)$$

But even the average value is not the complete indicator of a certain physical quantity. For example, a utility is happy to measure the average value of the active power but completely disappointed with the average value of the reactive power (=0). The fact that the average value of the active power is not zero means that the energy, in average, is flowing in a certain direction, therefore there is a net transfer of energy from one point of the network to another one.

The situation is different for the reactive power as its average is zero. The zero average doesn't mean that no energy is flowing, but the actual amount that is flowing for half a cycle in one direction, is coming back in the next half a cycle.

The following example clarifies the behaviour of reactive power. Consider a simple LC circuit as in fig. 4 at resonance ($X_L = X_C$). Suppose that the capacitor has been charged prior to its connection to the inductor.

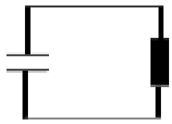

Fig. 4. An ideal LC circuit, no energy is lost due to heat.

Because there is no resistor in the circuit, the energy brought by the capacitor cannot be dissipated (radiation is considered small), thus it is conserved. Initially the energy is stored in the electric field of the capacitor but because the potential of the capacitor is bigger than the potential of the inductor, the energy starts flowing towards the inductor. After some time, the whole energy is stored in the inductor, in the magnetic field. Because the potential of the inductor is now higher, the energy starts flowing back to the capacitor. After some time the whole energy has been transferred back to the capacitor and the process repeats itself to infinity.

In the above example, the average reactive power is zero, the energy which left the capacitor during the first half of the cycle is coming back during the second half of the cycle. The same is happening in a real electric network. During one period the activity (flow of reactive energy) is large but the average reactive power do not provide any information. Another quantity must be introduced to describe the flow of reactive power, and this quantity is Q, the maximum of the instantaneous reactive power. This quantity measures the maximum reactive energy that flows during a cycle and therefore one gets a good estimate of how much energy is moving through the circuit even if the average reactive power is zero.

2.3 The Power Flow Equation

The equation of power flow relates the power transfer between two buses and the electrical data of the system. The electrical data comprises the receiving and sending bus voltages, the power angle between the two buses and the series impedance and natural capacitance of the transmission line connecting the two buses. We consider the PI model for a transmission line (fig. 6) and we express the reactive power at the two ends as a function of the voltages V_S and V_R and the characteristic of the line (R, X_L, X_C).

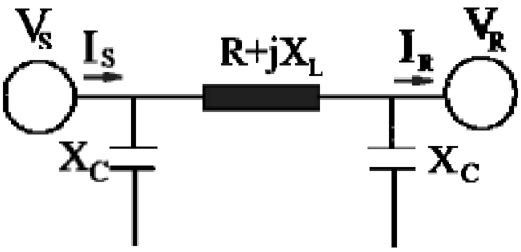

Fig. 6. Transmission Line Connecting Two Buses.

Using the phasor representation (bar symbol above the respective quantity) we have for the voltages

$$\begin{aligned} \bar{V}_S &= \bar{V}_R + \bar{I}_R \bar{Z} - \frac{\bar{V}_R \bar{Z}}{\bar{X}_C} \\ \bar{V}_R &= \bar{V}_S - \bar{I}_S \bar{Z} + \frac{\bar{V}_S \bar{Z}}{\bar{X}_C} \end{aligned} \tag{7}$$

And for the currents

$$\begin{aligned} \bar{I}_S &= \frac{\bar{V}_S}{\bar{X}_C} + \frac{\bar{V}_S - \bar{V}_R}{\bar{Z}} \\ \bar{I}_R &= -\frac{\bar{V}_R}{\bar{X}_C} + \frac{\bar{V}_S - \bar{V}_R}{\bar{Z}} \end{aligned} \tag{8}$$

The complex power for each end can be calculated by multiplying the voltage with the complex conjugate of the corresponding current. As we are interested to evaluate the reactive power Q (according with our definition the amplitude of the instantaneous reactive power), we take the complex part of the complex powers which are

$$\begin{aligned} Q_S &= \frac{V_S}{Z^2}\left[V_S X_L - X_L V_R \cos\theta + RV_R \sin\theta - \frac{V_S}{X_C} Z^2\right] \\ Q_R &= \frac{V_R}{Z^2}\left[-V_R X_L + X_L V_S \cos\theta - RV_S \sin\theta + \frac{V_R}{X_C} Z^2\right]. \end{aligned} \tag{9}$$

Considering a small resistance comparative with the inductance (R<<L), (9) can be simplified. This assumption does not affect the results as the reactive power is stored, absorbed or produced by the reactive part of the network (inductance or capacitance). The simplified equations for the reactive power at the two ends are then

$$\begin{aligned} Q_S &= \frac{V_S^2 - V_S V_R \cos\theta}{X_L} - \frac{V_S^2}{X_C} \\ Q_R &= \frac{-V_R^2 + V_S V_R \cos\theta}{X_L} + \frac{V_R^2}{X_C}\ . \end{aligned} \tag{10}$$

So far the standard procedure followed by textbooks to introduce the power flow equations was followed. As Q_R and Q_S are not equal, the reactive power loss is introduced as the difference of the two (10)

$$\Delta Q_{loss} = Q_S - Q_R . \tag{11}$$

The reactive power loss is explained to be the reactive power produced or absorbed by the line, depending on its sign. Accordingly, for a piece of electric network, the reactive power injected at one end will be the reactive power at the other end plus the reactive power produced or absorbed by the network element.

This unanimously accepted interpretation of reactive power loss contradicts the previous unanimously accepted interpretation that the reactive energy is neither consumed nor produced but oscillates among different part of the electric network. Here we would like to remind the reader that in fact not the power is lost, neither the power is flowing but rather the energy.

The confusion lies on the fact that the same term reactive power is used for the amplitude of the instantaneous reactive power and for the average of reactive power, two completely different concepts. The same confusion is avoided for the active power, as the amplitude of the instantaneous active power and the average of active power happen to be the same.

The confusion is removed by interpreting the two facts as following:

- The average of reactive power is zero, is interpreted as the energy is flowing for half a cycle in one direction and for the second half a cycle, the same amount of energy is flowing in the opposite direction. Therefore it is impossible to have a gain or loss of reactive energy (power);
- The “loss of reactive power” should be seen as a loss in the amplitude of the instantaneous reactive power that in fact is not a loss in real power.

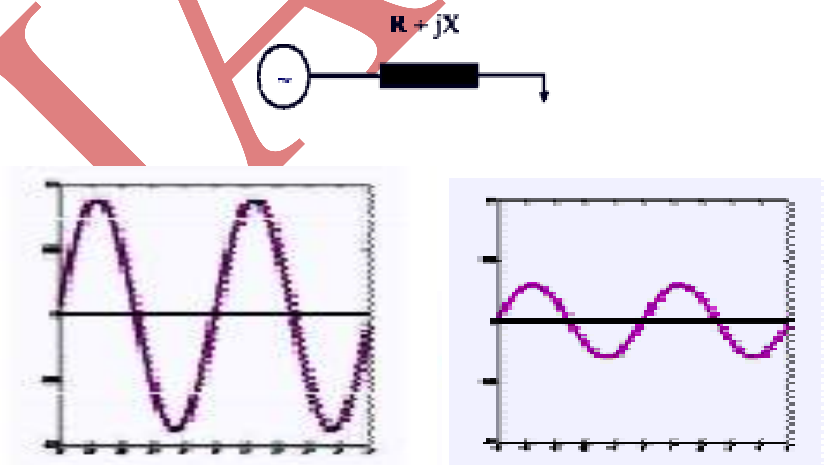

Fig. 7. The Reactive Power at the Two Ends of a Transmission Line.

The following example clarifies the idea. A simplified version of fig. 6 is used (fig. 7). For the sending and receiving ends, the instantaneous reactive powers at different points in space are represented. As one expects, the average is zero for both ends, no reactive energy is consumed or produced in time. The difference in the amplitude of the reactive power is exactly ΔQ.Therefore the so-called loss in reactive power is not a real loss but rather a loss in the amplitude of the reactive power as no reactive energy is lost.Why is this loss in amplitude introduced? Just because the average reactive power is zero always, a quantity to describe the "effort" done by the network (generation, loads) is necessary. This quantity is the amplitude of the reactive power, Q.

III CONCLUSION

The reactive power is very different of the active power and this fact induced some misconceptions about the reactive power itself. Attempts to clarify the reactive power notion [10 - 12] succeeded only partially to do so.
The interpretation presented in this paper for the reactive power, removes the contradictions found in many textbooks in power system. The reactive power is a complex and particular concept as its average in time is always zero. This lack of information of the average reactive power imposes the necessity to use another quantity to describe the flow, back and forth, of reactive power. This quantity has been identified as the amplitude of the instantaneous reactive power, *Q*. Thus by using the expression *reactive power* one should be very careful which of its senses is using. Not the same confusion is encountered for the active power as the amplitude of the instantaneous active power and the average active power are equal.

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